Discussion 6: Discrete-Time Fourier Transform Practice

1. Circular Convolution

Multiplication property of DTFT: $x_1[n]x_2[n] \overset{DTFT}{\longleftrightarrow} \frac{1}{2\pi}\int_{<2\pi>} X_1(e^{j\theta})X_2(e^{j(\omega-\theta)})d\theta$

Consider the functions:

$$x_1[n] = \frac{\sin\left(\frac{\pi n}{4}\right)}{\pi n} \qquad x_2[n] = \cos\left(\frac{\pi n}{2}\right)$$

Let $y[n] = x_1[n]x_2[n]$. Find the DTFT of $y[n]$, $Y(e^{j\omega})$ using the multiplication property for DTFT.

2. LDEs and DTFTs

Consider the LDE: $y[n] - 1.2y[n-1] + 0.36y[n-2] = x[n] + x[n-1]$.

i. Find the frequency response $H(e^{j\omega})$
ii. Use $H(e^{j\omega})$ to calculate the impulse response, $h[n]$, of the system.

3. DTFT Practice

(a) Find DTFT for $h[n]$: $h[n] = \left(\frac{1}{2}\right)^n e^{j2n} u[n+2]$

(b) Given the following properties of $x[n]$ and its DTFT $X(e^{j\omega})$, find $x[n]$.

i. $x[n]$ real
ii. $x[n] = 0$ for $n > 0$
iii. $x[0] > 0$
iv. $Im\{X(e^{j\omega})\} = \sin(\omega) - \sin(2\omega)$
v. $\frac{1}{2\pi}\int_{-\pi}^{\pi} |X(e^{j\omega})|^2 d\omega = 3$