Professor Fearing EECS120/Discussion 5 Ming Jin

Note: $\Pi(t) = u(t+\frac{1}{2}) - u(t-\frac{1}{2})$, and $comb(t) = \sum_{n=-\infty}^{\infty} \delta(t-n)$.

1. Fourier Transforms

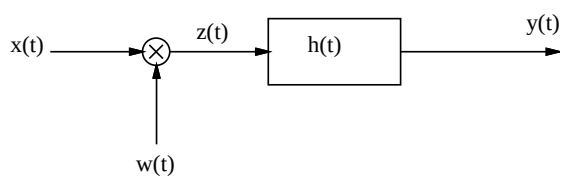
Given $x(t) = cos(\pi t)$. Sketch $z(t), y(t)$ and the Fourier transforms $Z(j\omega), Y(j\omega)$ for the following, referring to the block diagram below. Sketch should label key heights and frequencies.

$$a. \quad w(t) = \Pi(t) \qquad h(t) = \sum_{n=-\infty}^{\infty} \delta(t-2n) \tag{1}$$

$$b. \quad w(t) = \Pi(t) * \sum_{n=-\infty}^{\infty} \delta(t-2n) \qquad h(t) = \delta(t) \tag{2}$$

$$c. \quad w(t) = \Pi(t) \qquad h(t) = \sum_{n=-\infty}^{\infty} \delta(t-n) \tag{3}$$

$$d. \quad w(t) = [2\Pi(t) * \sum_{n=-\infty}^{\infty} \delta(t-2n)] - 1 \qquad h(t) = \delta(t) \tag{4}$$



2. Fourier Transforms

In the homework, you will show that given $X(j\omega) = \mathcal{F}\{x(t)\}$, $\mathcal{F}\{X(t)\} = 2\pi x(-\omega)$.
Use the duality property to find the following Fourier Transform pairs:

a. If $X_1(j\omega) = \frac{(\alpha+j\omega)}{\omega_0^2+(\alpha+j\omega)^2}$, find $\mathcal{F}\{X_1(t)\}$. Hint: $\mathcal{F}^{-1}\{X_1(j\omega)\} = u(t)e^{-\alpha t}\cos(\omega_0 t)$

b. If $x_2(t) = u(t)\cos(\omega_0 t)$, find $\mathcal{F}^{-1}\{x_2(\omega)\}$.

Hint: $\mathcal{F}\{x_2(t)\} = \frac{\pi}{2}\left[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)\right] + \frac{j\omega}{\omega_0^2-\omega^2}$

c. Find $\mathcal{F}\{u(t)\}$, where $u(t)$ is the step function.

Hint: $\mathcal{F}\{\frac{2}{jt}\} = -2\pi Sgn(\omega)$, where $Sgn(\omega) = -1$ for $\omega < 0$ and 1 for $\omega >= 0$, and $\mathcal{F}\{\delta(t)\} = 1$.