Discussion 4: Fourier Transform Properties and Practice

1. FT Properties

Definition: Fourier Transform $X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$

Do the following proofs:

(a) Prove the linearity property of the FT: $ax(t) + by(t) \leftrightarrow aX(j\omega) + bY(j\omega)$

(b) Prove the time-shifting property of the FT: $x(t - t_0) \leftrightarrow e^{-j\omega t_0}X(j\omega)$

(c) Prove the convolution property of the FT: $x(t) * y(t) \leftrightarrow X(j\omega)Y(j\omega)$

(d) Prove the time-scaling property of the FT: $x(at) \leftrightarrow \frac{1}{|a|}X\left(\frac{j\omega}{a}\right)$

Definition: Inverse Fourier Transform $x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)e^{+j\omega t}d\omega$

Do the following proofs:

(e) Prove the frequency-shifting property of the IFT: $X\big(j(\omega - \omega_0)\big) \leftrightarrow e^{j\omega_0 t}x(t)$

(f) Prove the convolution property of the IFT: $\frac{1}{2\pi}X(j\omega) * Y(j\omega) \leftrightarrow x(t) \cdot y(t)$

2. FT Practice

(a) What is the Fourier Transform of $x(t) = e^{-\alpha t}u(t)$; $X(j\omega) = FT\{x(t)\}$? What restrictions are there on the parameter α?

(b) What is the Fourier Transform of a rectangle function, $x(t) = \Pi\left(\frac{t}{2}\right)$?

(c) Find the Fourier Transform of $x(t)$ using FT properties (O.W. Table 4.1) and FT pairs (O.W. Table 4.2):

$$x(t) = \frac{\sin(t)\sin\left(\frac{t}{2}\right)}{\pi t^2}$$