Discussion 3: Graphical Convolution and CT Fourier Series

1. Graphical Convolution

Consider Problem 3e from PS2.

Given input, $x(t) = \delta(t) + \frac{1}{2}\delta(t-1) + \frac{1}{4}\delta(t-2)$, and impulse response, $h(t) = e^{-t}u(t)$, find the output using the convolution integral,

$$y(t) = \int_{-\infty}^{\infty} x(t-\tau)h(\tau)d\tau$$

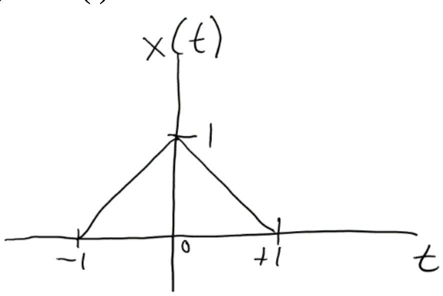
Using the convolution integral, we found that this convolution has the form:

$$y(t) = e^{-t}u(t) + \frac{1}{2}e^{-(t-1)}u(t-1) + \frac{1}{4}e^{-(t-2)}u(t-2)$$

The "flip-and-slide" method of convolution is like watching the input "slide into" the system. If we flip $x(\tau)$ about $\tau = 0$ and slide it to the right, we can see that the first impulse "hits" $h(\tau)$ at $t = 0$. At $t = 1$, we've slid the flipped version of $x(\tau)$ by 1 unit, and the 2nd impulse "hits" $h(\tau)$. At $t = 2$, we've slid the flipped version of $x(\tau)$ by 2 units, and the 3rd impulse "hits" $h(\tau)$.

Practice the following graphical convolutions:

(a) Let h(t) be a time-shifted delta function: $h(t) = \delta(t-3)$. Let x(t) be a "tent function":



Find x(t)*h(t) graphically.

(b) Let x[n] be a ramp starting at zero ($x[n] = n \cdot u[n]$) and let h[n] be two delta functions:

$$h[n] = \delta[n] + \delta[n-1]$$

Find x(t)*h(t) graphically.

Check your graphical result with the analytical result.

2. Continuous Time Fourier Series

The Fourier Series coefficients for a given periodic signal, $x(t)$, are given by the analysis equation:

$$a_k = \frac{1}{T}\int_{<T>} x(t)e^{-jk\frac{2\pi}{T}t}dt$$

Where T is the fundamental period of $x(t)$. The signal can be written using the Fourier coefficients in the synthesis equation:

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\frac{2\pi}{T}t}dt$$

Prove the following properties of CT Fourier Series:

(a) Linearity: Given $x(t) \leftrightarrow a_k$ and $y(t) \leftrightarrow b_k$ both periodic with fundamental period T, show $z(t) = Ax(t) + By(t) \leftrightarrow c_k = Aa_k + Bb_k$

(b) Time-shifting: Given $x(t) \leftrightarrow a_k$ with fundamental period T, $y(t) = x(t - t_0)$, show $y(t) \leftrightarrow b_k = e^{-jk\frac{2\pi}{T}t_0}a_k$

(c) Time-scaling: Given $x(t) \leftrightarrow a_k$ with fundamental period T, $y(t) = x(\alpha t)$, show $y(t) \leftrightarrow b_k = a_k$

(d) Consider a quarter-period cosine:

$$x_1(t) = \begin{cases} \cos(\omega_0 t) & \frac{2\pi}{\omega_0}\left(n - \frac{1}{8}\right) < t < \frac{2\pi}{\omega_0}\left(n + \frac{1}{8}\right) \\ 0 & otherwise \end{cases}$$

This looks like a “truncated” cosine. Find the Fourier Series coefficients of this signal.