Professor Fearing EECS120/Discussion 2 GSI: Alanna Tran

1. **Unit Impulse Function**

The unit impulse (Dirac delta) has the following properties:

$$\delta(t) = \begin{cases} 0, t \neq 0 \\ \infty, t = 0 \end{cases}$$

$$\int_{-\epsilon}^{\epsilon} \delta(t) dt = 1$$

Remark 1 An ordinary function which is 0 everywhere except for a single point would have an integral value of 0 (in the Riemann integral sense). Thus, $\delta(t)$ cannot be defined like an ordinary function, but it can be defined mathematically (in a weak sense) by

$$\int_{-\infty}^{\infty} \phi(t)\delta(t) dt = \phi(0)$$

Delayed Delta

$$\int_{-\infty}^{\infty} \phi(t)\delta(t - t_0) dt = \phi(t_0)$$

$*$ *taken from Lillian Ratliff, Discussion 2 2014*

2. **Convolution**

Discrete Time

$$x[n] * h[n] = \Sigma_{k=-\infty}^{\infty} x[k]h[n-k]$$

Continuous Time

$$x(t) * h(t) = \int_{\tau=-\infty}^{\infty} x(\tau)h(t-\tau) d\tau$$

Prove the following properties:

a. Identity

i. $x(t) * \delta(t)$

ii. $x(t) * \delta(t - t_0)$

b. Commutative

$x(t) * h(t) = h(t) * x(t)$

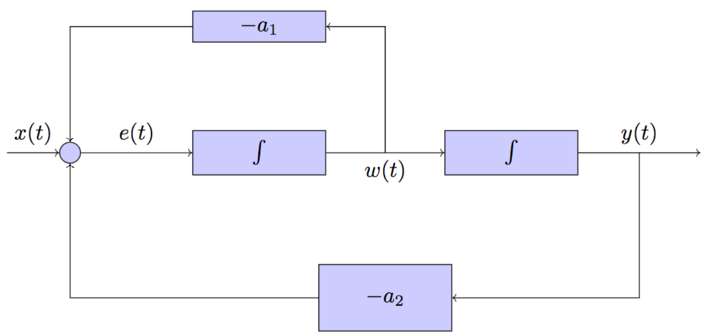
3. **B.I.B.O. Stability** *Bounded Input Bounded Output Stability*

For a system H, it is said to be B.I.B.O. stable if for any bounded input x the output y is bounded.

$$||x(t)|| \leq B < \infty$$

a. Given a system with impulse response $h(t) = (e^{-t} + e^{t})u(t)$, determine if the system is B.I.B.O. stable.

4. **Linear Differential Equations**



a. Write a differential equation that relates output $y(t)$ and input $x(t)$.

b. Consider the CT system whose input and output are related by:

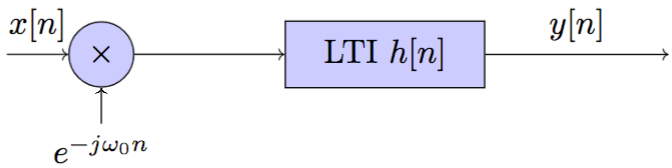
$$\frac{dy(t)}{dt} + ay(t) = x(t)$$

where a is constant. Find $y(t)$ with the initial condition $y(0) = y_0$ and with input

$$x(t) = Ke^{-bt}u(t)$$

5. **LCCDE**

Consider a system S with input $x[n]$ and output $y[n]$ related according to the block diagram in the figure below. The input $x[n]$ is multiplied by $e^{-j\omega_0 n}$ and the product is passed through a stable LTI system with impulse response $h[n]$.



a. Is the system S linear? Time invariant?

b. Is the system S stable?

c. Specify a system C such that the block diagram in the figure below represents an alternative way of expressing the input-output relationship of the system S.