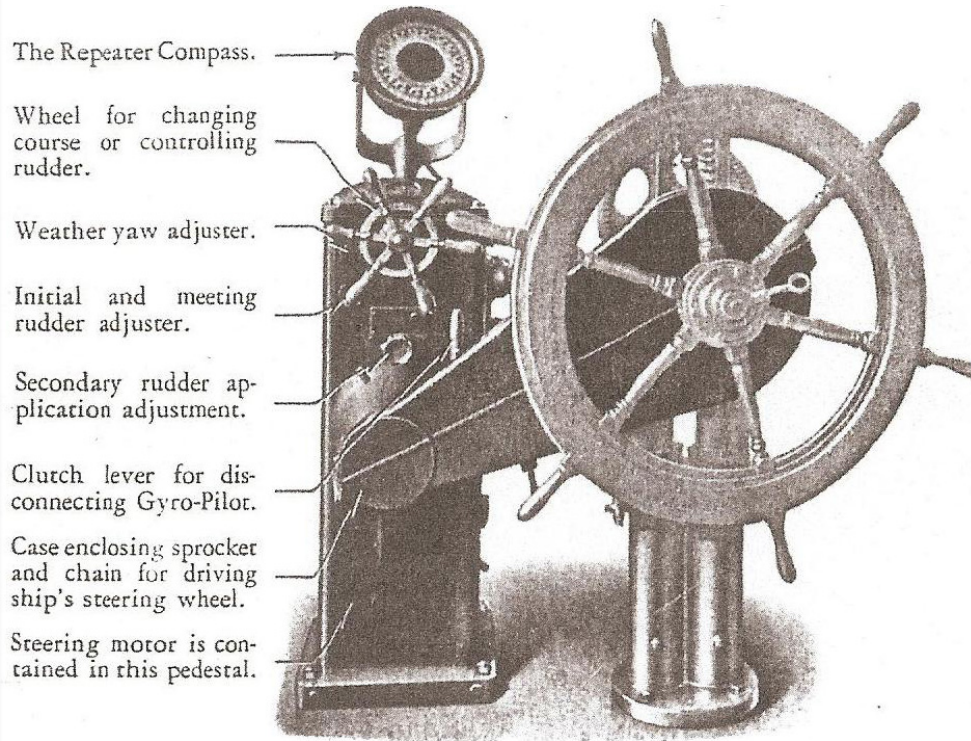


AUTONOMOUS DRIVING

a brief EE120+ level review



The Sperry gyropilot (ca. 1922)



Google car (2012)



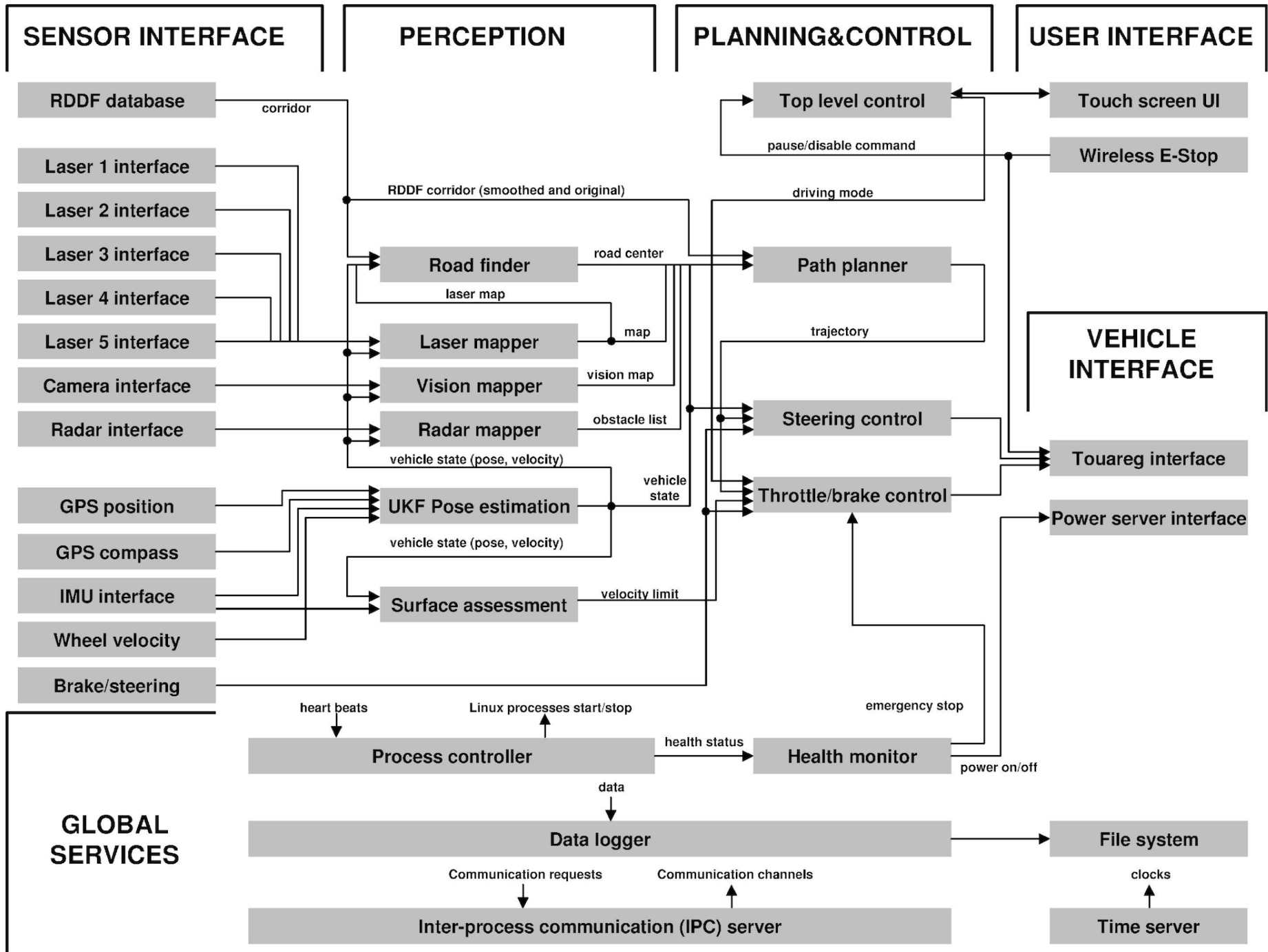
ca. 1957

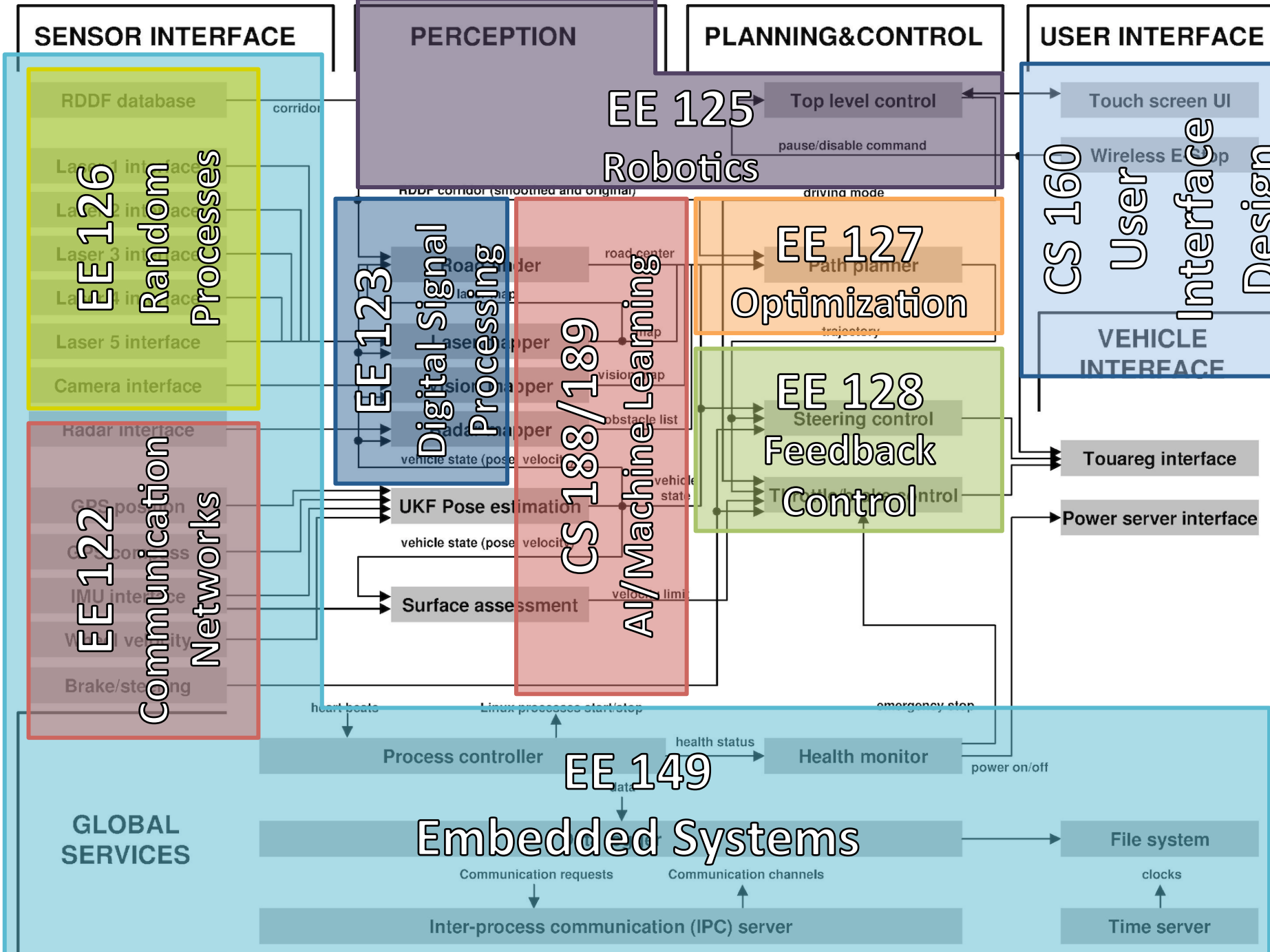
The idea of self-driving cars has existed for a long time. Recent progress was enabled by computation, sensors, radar, mapping, machine learning, and catalyzed by the DARPA Grand Challenges.

Stanley: The winner of the 2005 DARPA Grand Challenge



Credit: Thrun, Journal of Field Robotics, 23(9), pp. 661-692, 2006. DOI: 10.1002/rob.20147





Development steps – automated driving

Degree of automation ↑

- Single sensor
- Sensor-data fusion
- Sensor-data fusion + map



ACC/lane keeping support

Only longitudinal or lateral control



Integrated cruise assist

Partially automated longitudinal and lateral guidance in driving lane
Speed range 0-130 kph



Highway assist

Partially automatic longitudinal and lateral guidance
Lane change after driver confirmation
Supervision of surrounding traffic (next lane, ahead, behind)



Highway pilot

Highly automated longitudinal and lateral guidance with lane changing
Reliable environment recognition, including in complex driving situations
No permanent supervision by driver

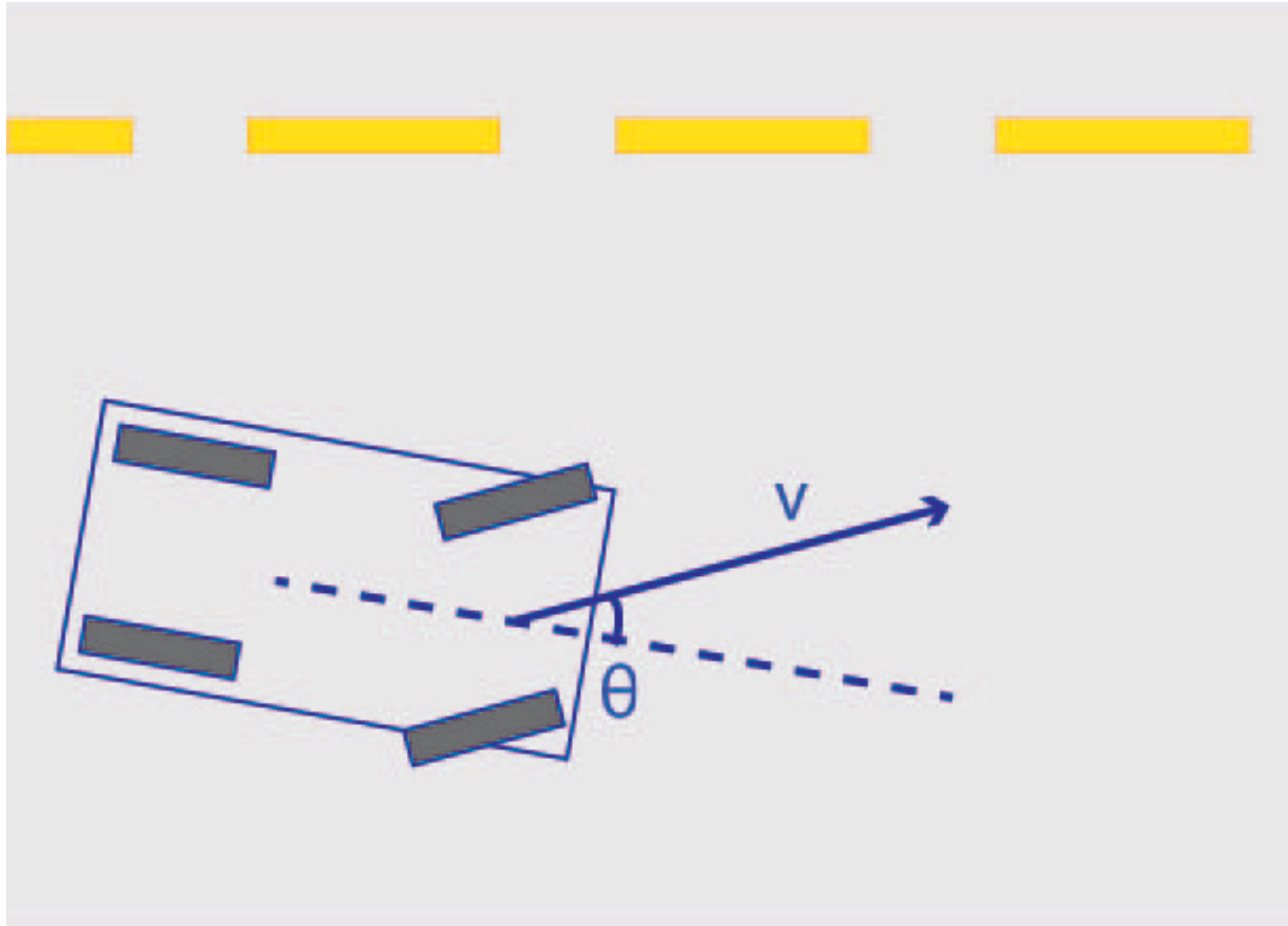


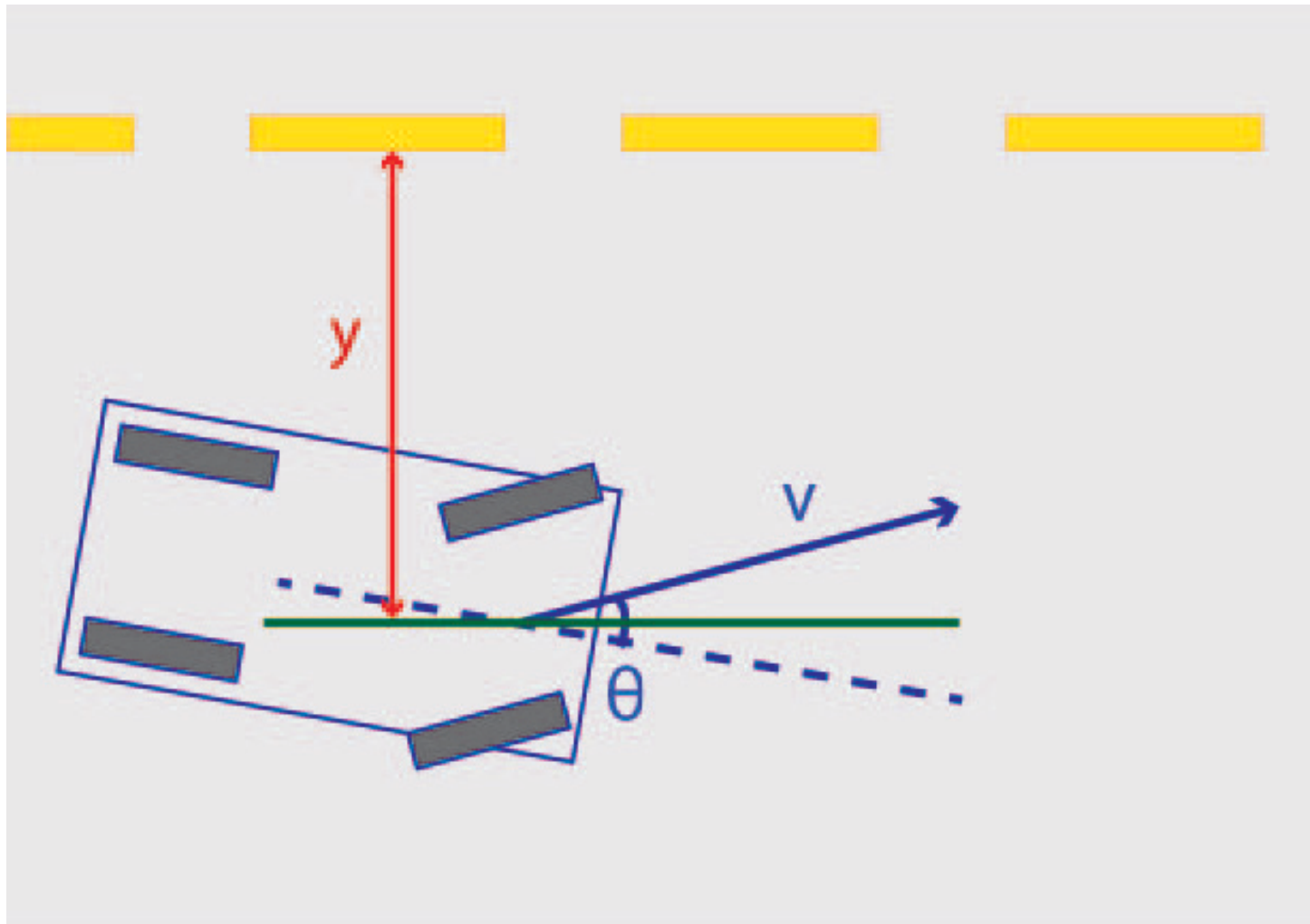
Auto pilot

Door-to-door commuting (e.g. to work) in urban traffic
Strictest safety requirements
No supervision by driver

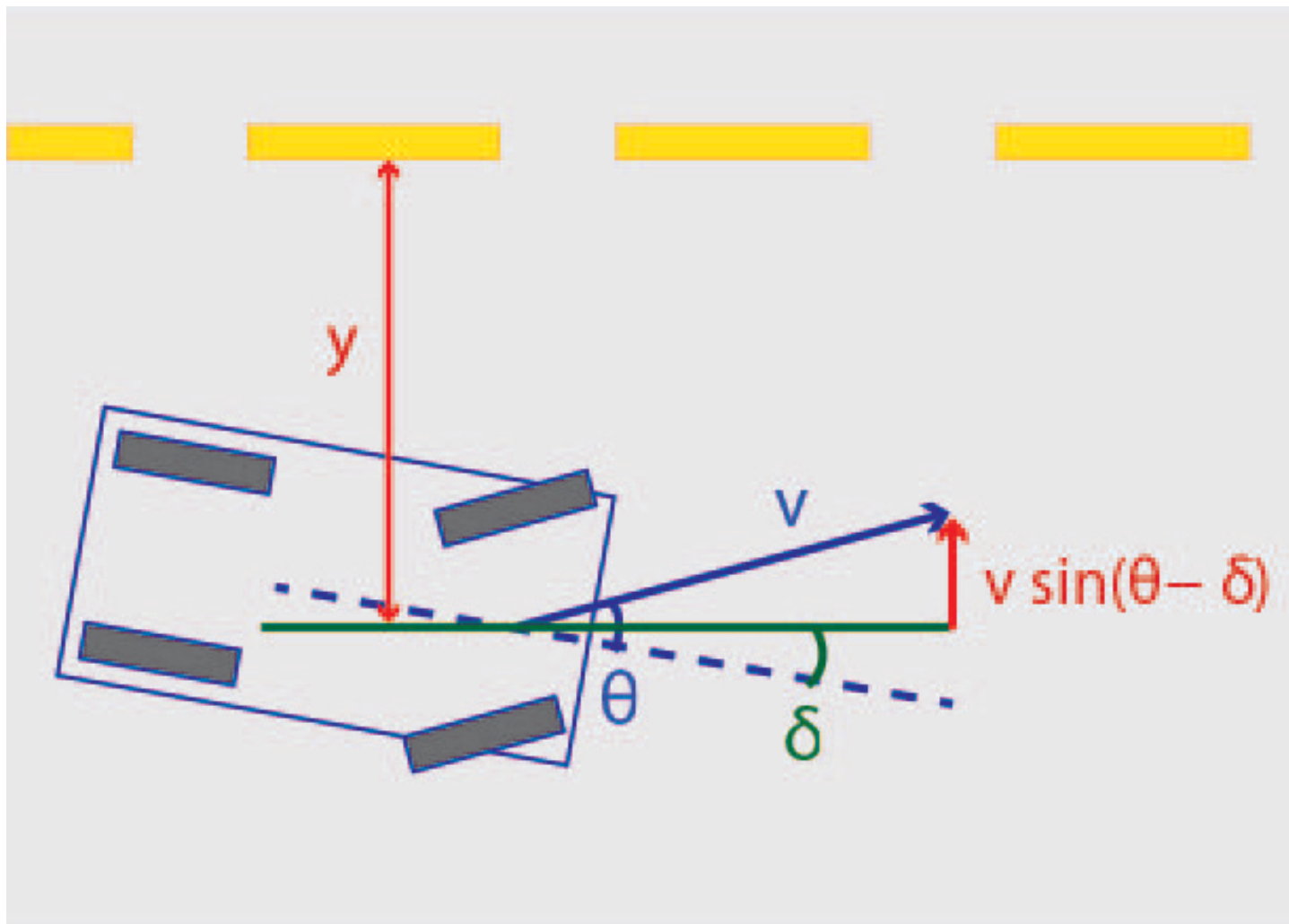


Topic 1: Steering Control





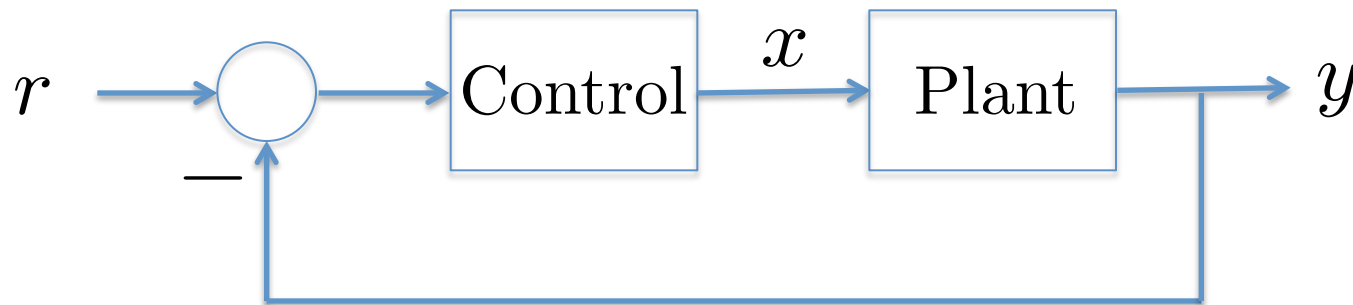
Goal: Maintain $y(t)$ at desired value



$$\frac{dy(t)}{dt} = -v \sin(\theta - \delta)$$

Define $x \triangleq \delta - \theta$. Then, the 'plant' is:

$$\frac{dy(t)}{dt} = v \sin(x(t)) \quad (\text{nonlinear!})$$



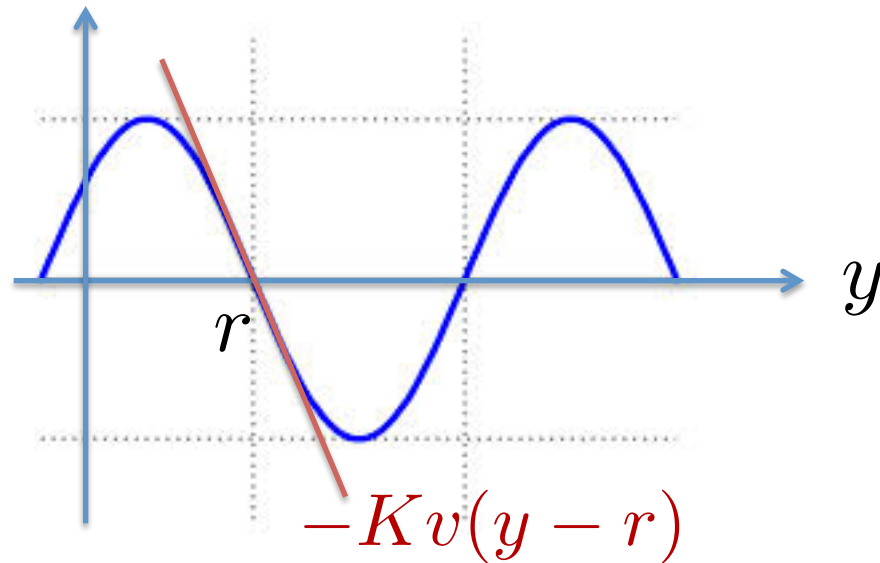
First, try the constant gain control

$$x = K(r - y)$$

Closed-loop: $\frac{dy}{dt} = v \sin(K(r - y))$

Near $y = r$,

$$\frac{dy}{dt} = v \sin(K(r - y)) \approx -Kv(y - r)$$



Therefore, for constant r :

$$y(t) - r \approx (y(0) - r)e^{-Kvt}$$

if $y(0) - r$ small

Root Locus Interpretation

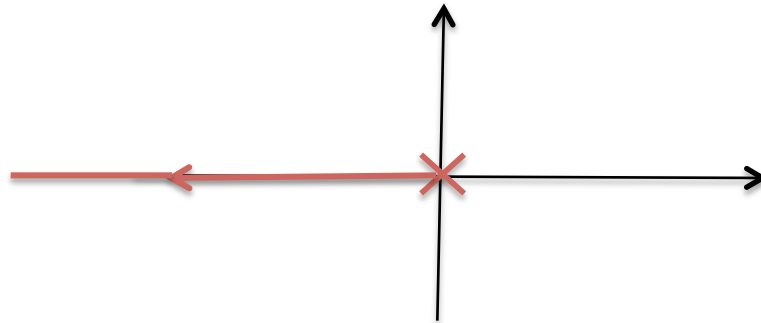
Linearized plant model:

$$\frac{dy}{dt} = v \sin(x) \approx vx$$

$$H_p(s) = \frac{v}{s}$$

Closed-loop poles are the roots of:

$$1 + KH_p(s) = 0 \quad \Rightarrow \quad s = -Kv$$



Steady-state error = 0 ($H_p(s)$ has pole at 0)

The controller above does not obey the physical steering limit: $|x| < 90^\circ$

Instead, try the nonlinear controller:

$$x = \tan^{-1}(K(r - y))$$

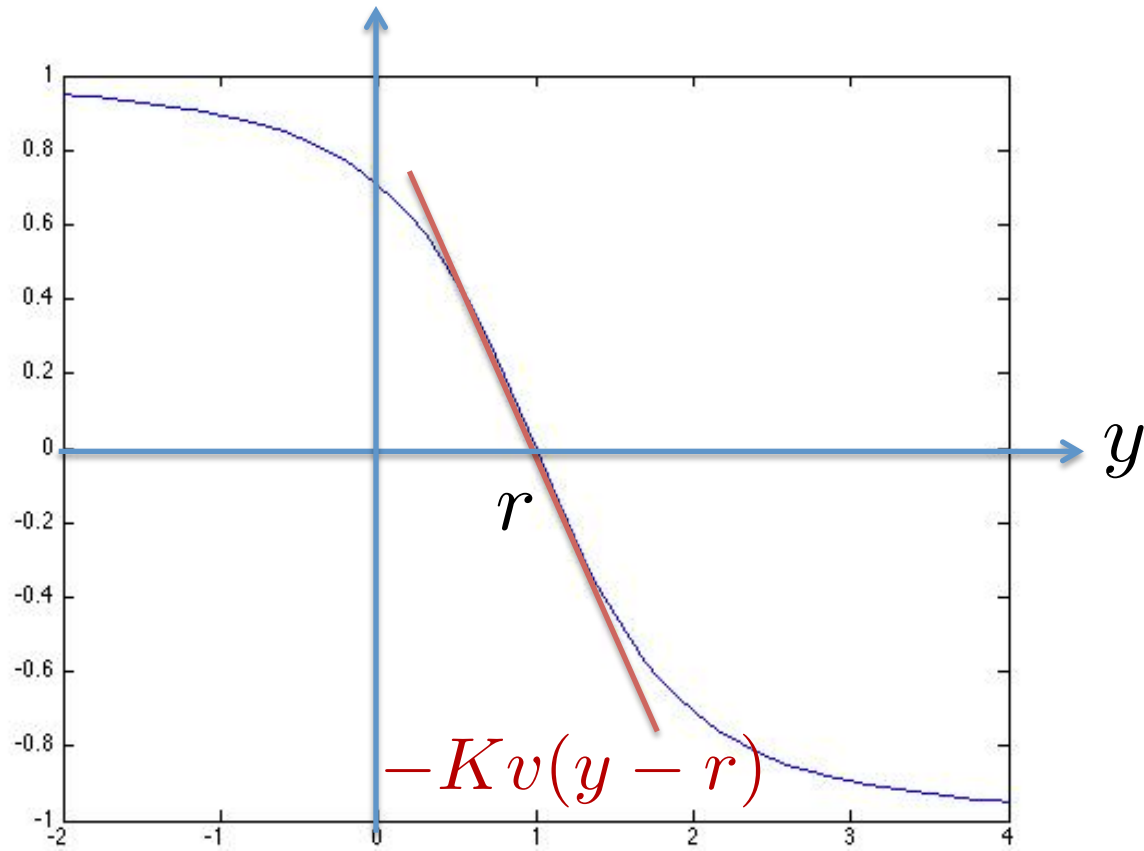
Closed-loop:

$$\frac{dy}{dt} = v \sin(\tan^{-1}(K(r - y)))$$

Substitute: $\sin(\tan^{-1}(u)) = \frac{u}{\sqrt{1+u^2}}$

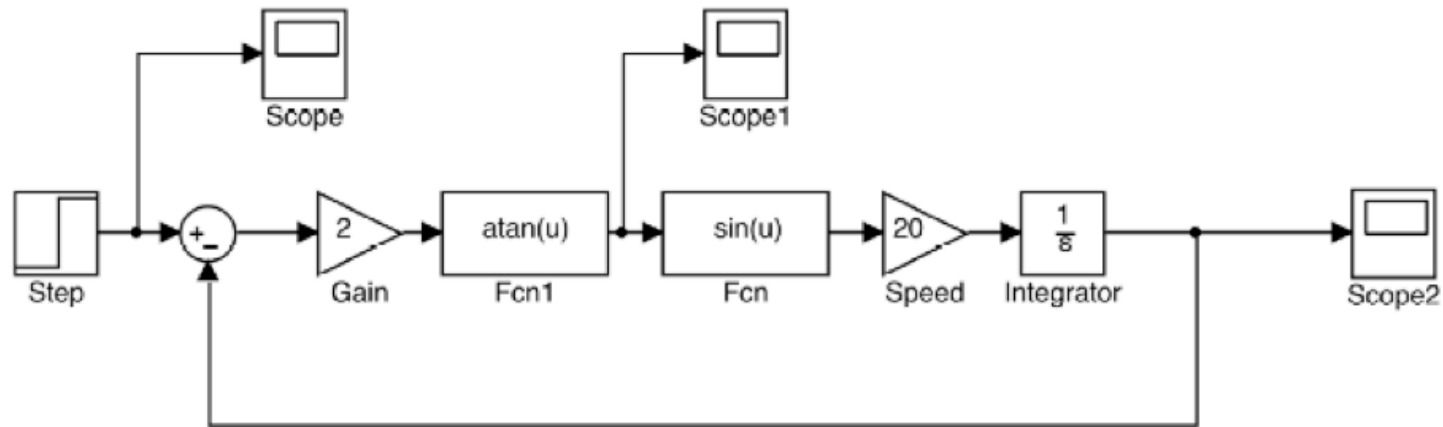
$$\frac{dy}{dt} = v \frac{K(r - y)}{\sqrt{1 + K^2(r - y)^2}}$$

$$\frac{dy}{dt} = v \frac{K(r - y)}{\sqrt{1 + K^2(r - y)^2}}$$

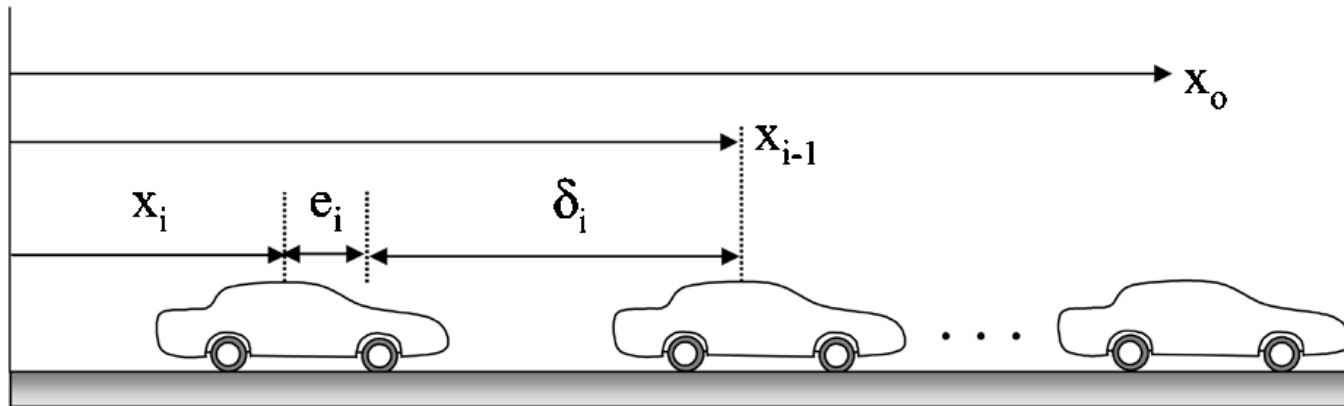


$y(t) \rightarrow r$ for all $y(0)$

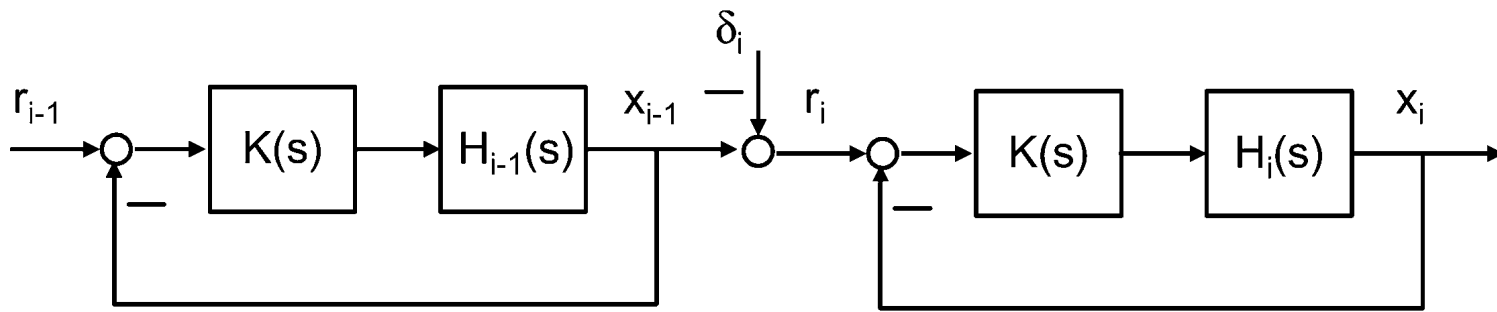
Simulink Diagram:



Topic 2: Vehicle Following



Goal: Maintain the gap $x_i - x_{i-1}$ at δ_i ; thus the reference for vehicle i is $r_i = x_{i-1} - \delta_i$



Vehicle Model

$$H(s) = e^{-T_d s} \frac{k}{s^2 + 2\zeta\omega_n s + \omega_n^2} \frac{1}{s}$$

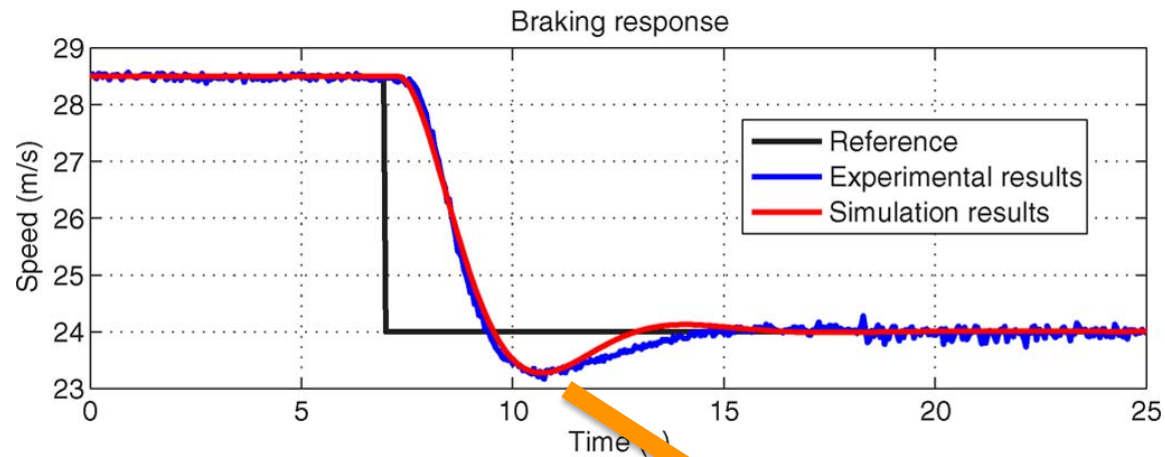
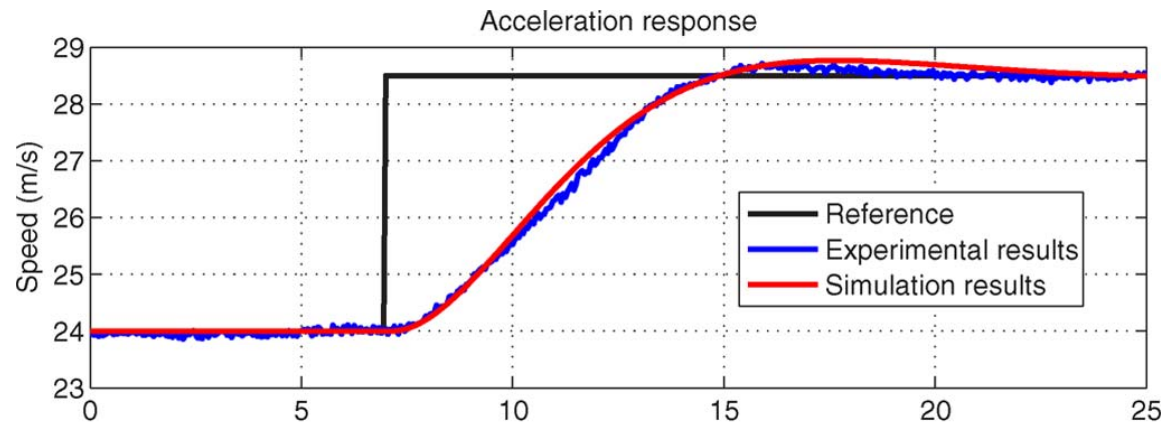
$\triangleq H_v(s)$ (velocity)

position

Model validated and parameters identified for experimental vehicles by PATH (Partners for Advanced Transit and Highways) researchers at UC Berkeley



	k	ζ	ω_n	T_d
Accelerating	0.156	0.661	0.396	0.146
Braking	1.136	0.5	1.067	0.287



overshoot due to engine braking

Credit: (Milanes et al., 2014)

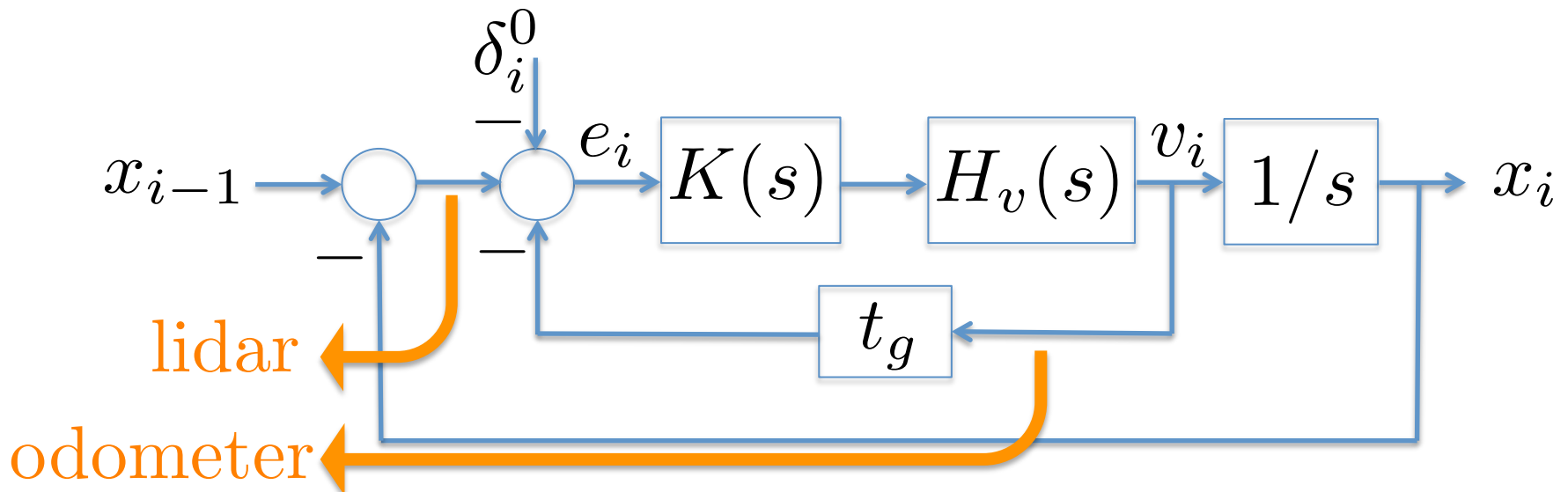
Time Gap Control

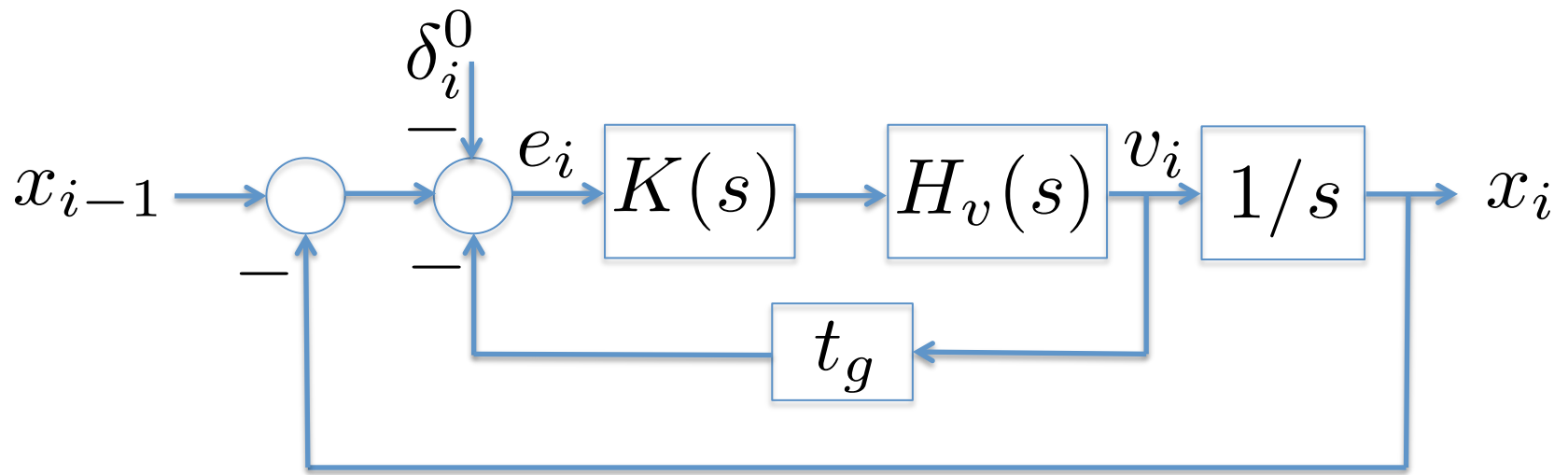
Larger δ_i with increasing speed:

$$\delta_i = t_g v_i(t) + \delta_i^0$$

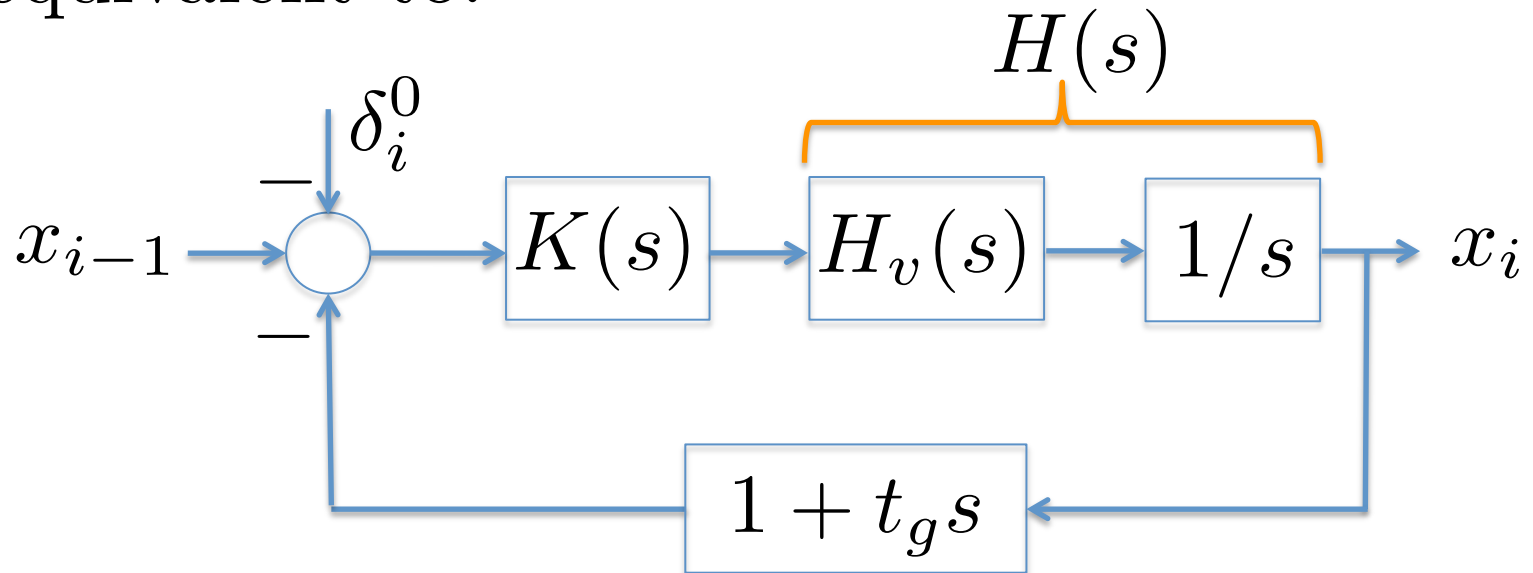
where t_g is a fixed time gap (e.g., $t_g = 1$ sec means about one vehicle length per 10 mph).

Error: $e_i = x_{i-1} - x_i - v_i t_g - \delta_i^0$





is equivalent to:



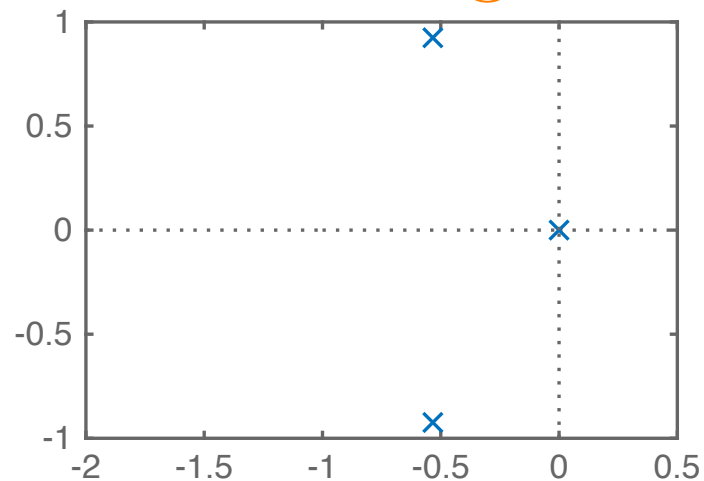
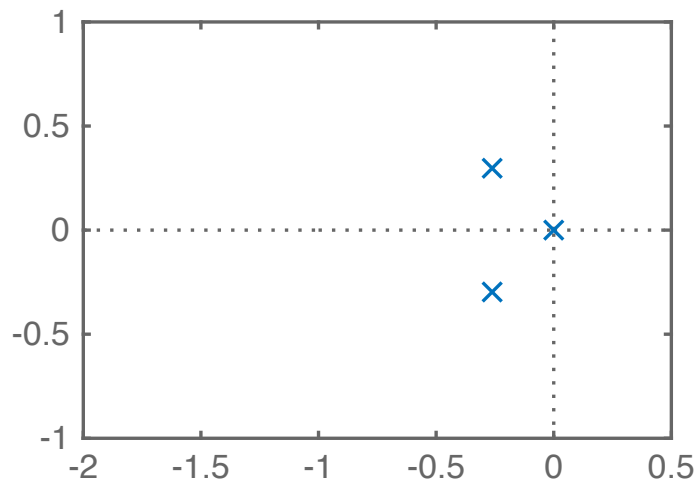
Note $t_g = 0$ recovers the fixed distance control.

Sketch root locus for cst.gain control and $T_d = 0$

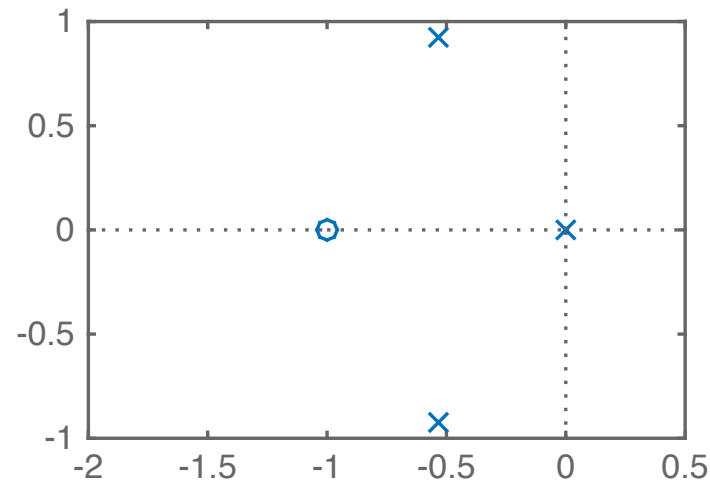
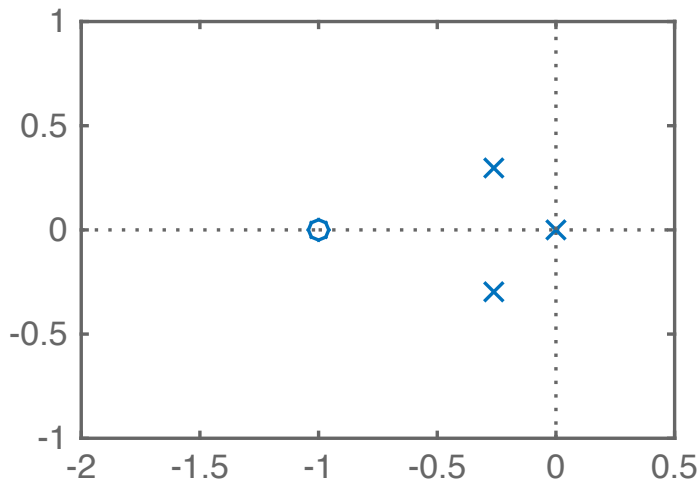
acceleration

braking

$t_g = 0$



$t_g = 1$



All-pass approximation for delay:

$$e^{-T_d s} \approx \frac{1 - \frac{T_d}{2} s}{1 + \frac{T_d}{2} s}$$

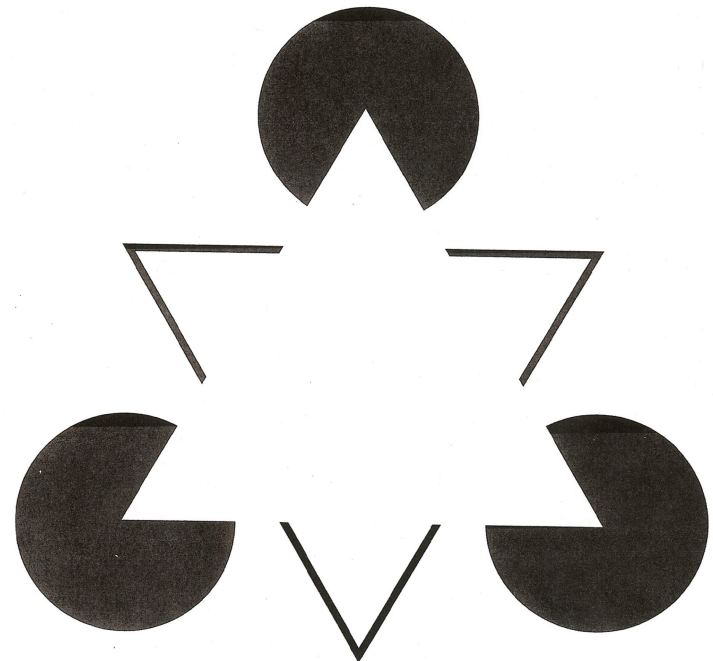
Augment transfer function with this all pass
and design controller using Matlab rltool

Topic 3: Edge Detection

- Detection of object boundaries is critical in computer vision (e.g., detecting lane markings).
- Visual cortex detects light-dark discontinuities. The brain interpolates between contours.



Credit: (H. Cheng, 2011)

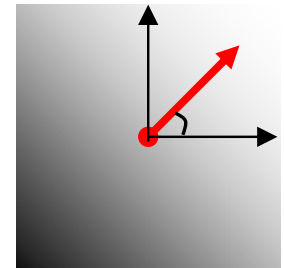
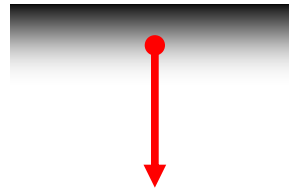
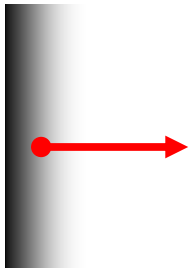


Credit: (Kanizsa, 1979)

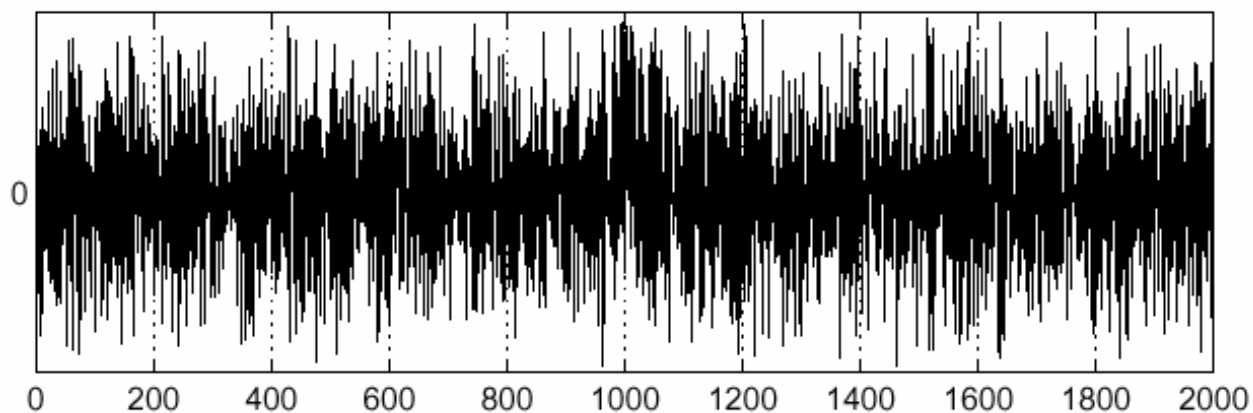
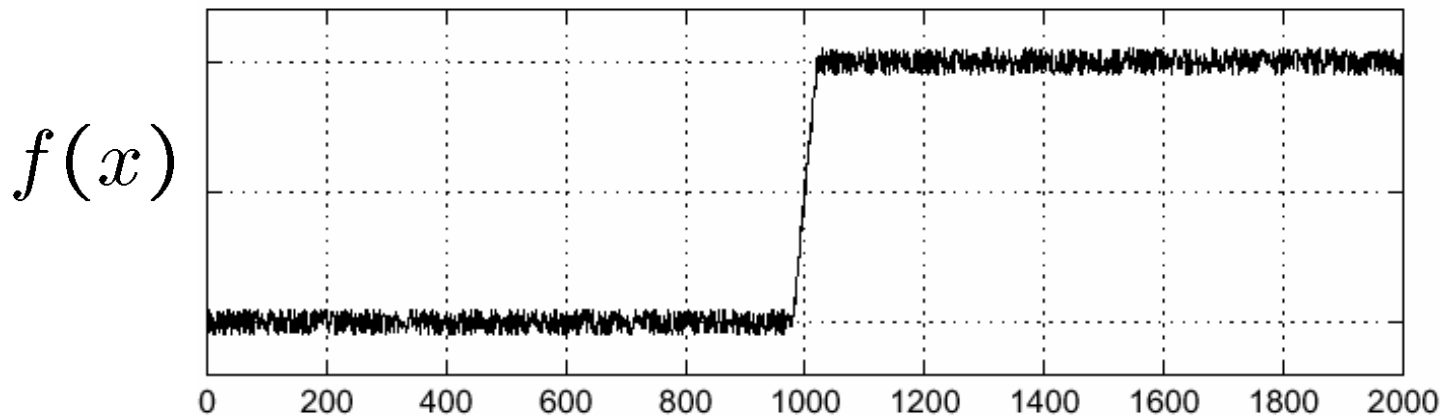
Edge detection algorithms seek sharp gradients in the image.

$$\nabla f(x, y) = \begin{bmatrix} \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} \end{bmatrix}$$

The gradient points in the direction of most rapid change in intensity:



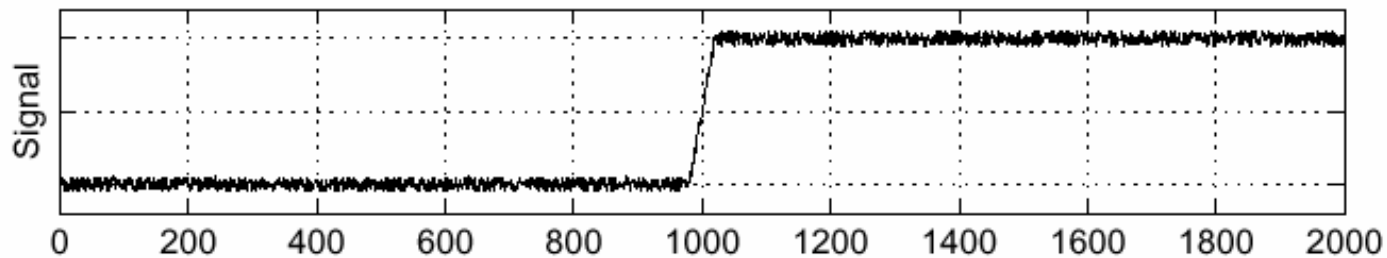
Problem: Algorithms based on derivatives are sensitive to noise



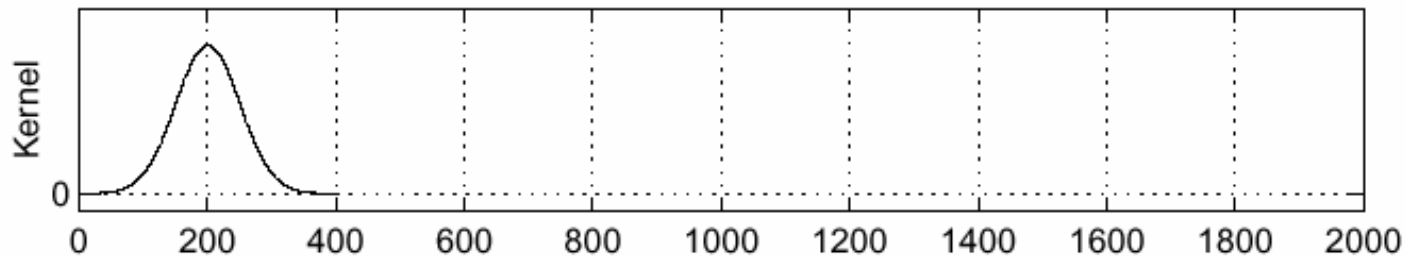
Credit: Efros, Computational Photography

Solution: Low-pass filter before differentiation

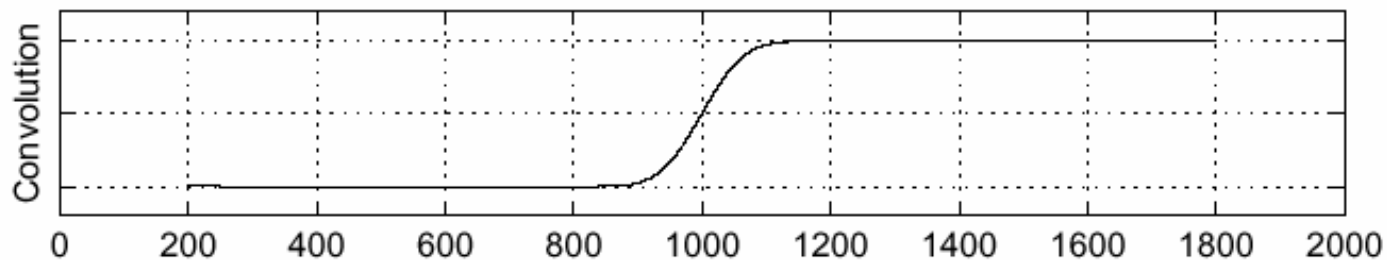
$f(x)$



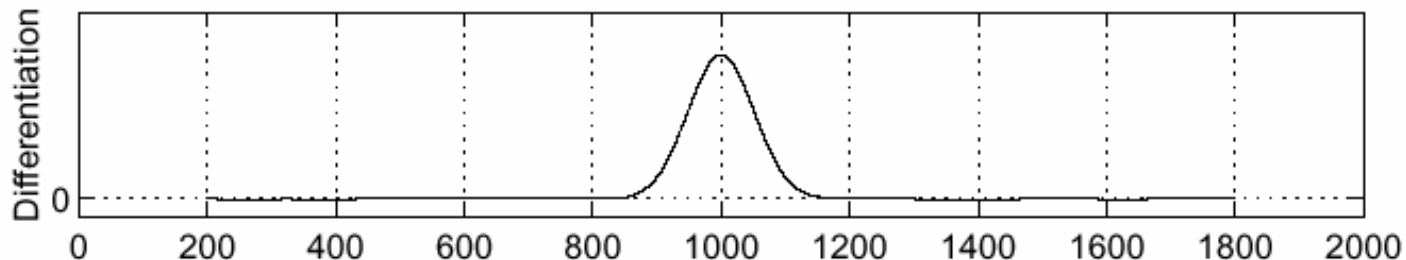
$h(x)$



$h(x) * f(x)$



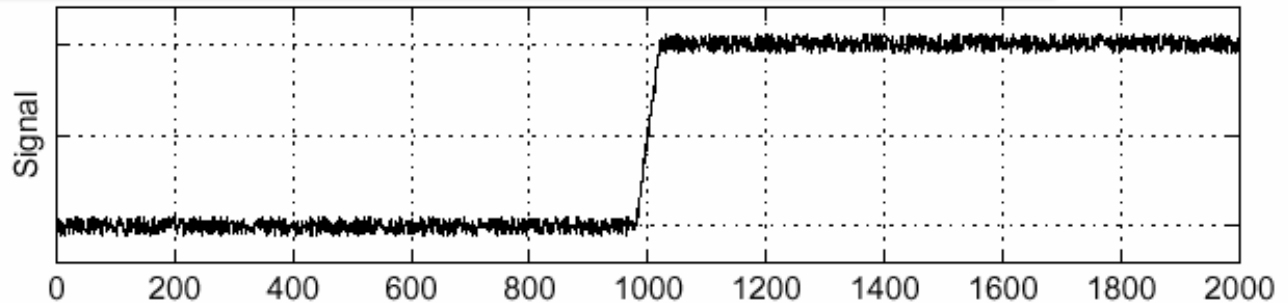
$\frac{d}{dx} \{h * f\}$



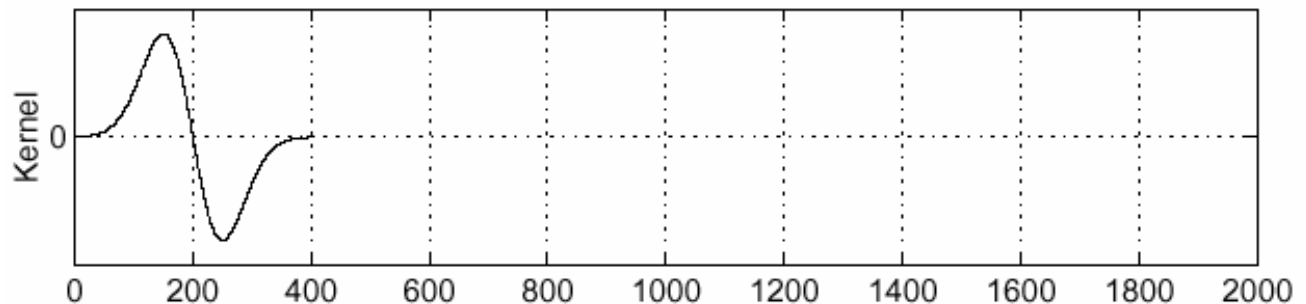
Save one operation using the property:

$$\frac{d}{dx} \{h(x) * f(x)\} = \frac{dh(x)}{dx} * f(x)$$

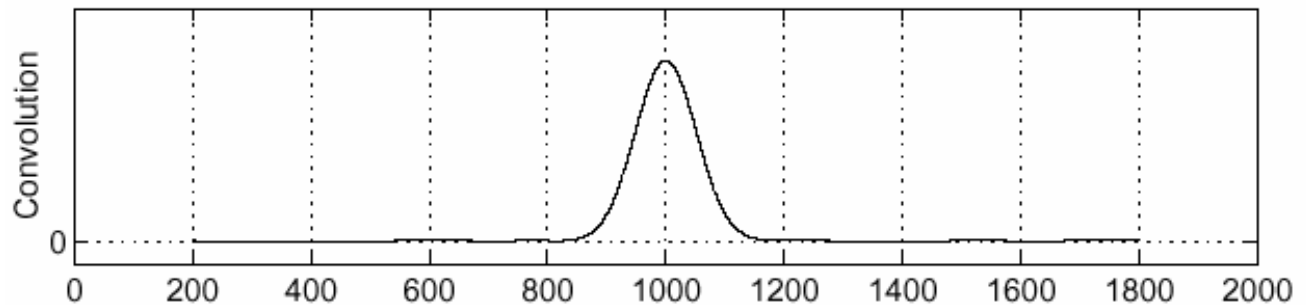
$f(x)$



$\frac{dh(x)}{dx}$

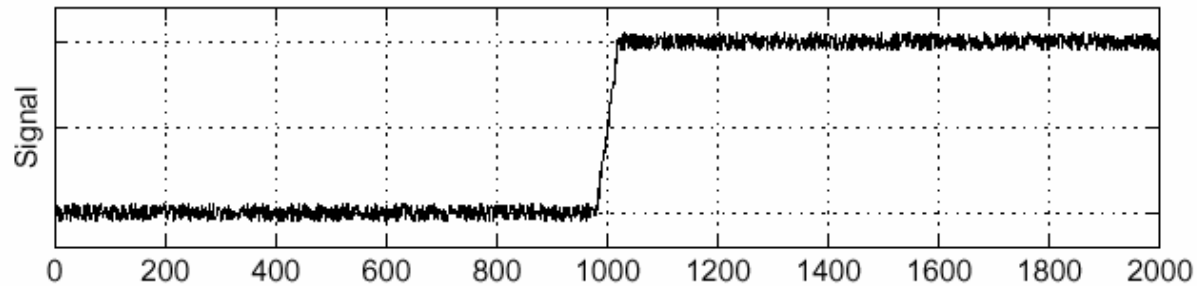


$\frac{dh(x)}{dx} * f(x)$

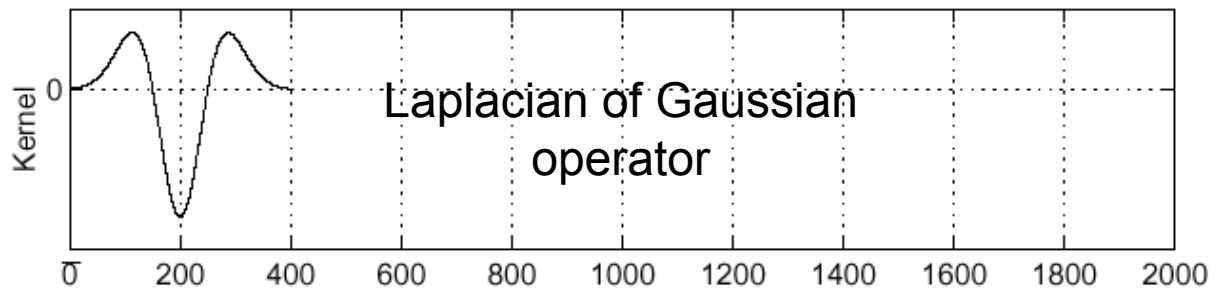


The peaks of $\frac{d}{dx} \{h * f\} = \frac{dh}{dx} * f$ are the zero crossings of $\frac{d^2}{dx^2} \{h * f\} = \frac{d^2h}{dx^2} * f$

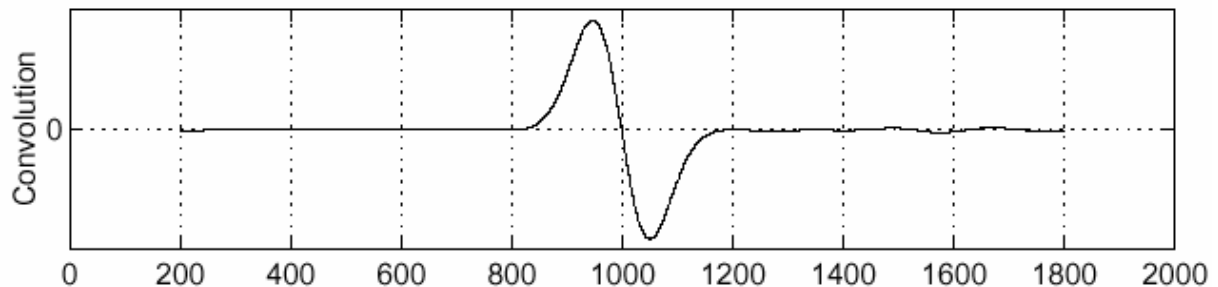
$f(x)$



$\frac{d^2h}{dx^2}$



$\frac{d^2h}{dx^2} * f$

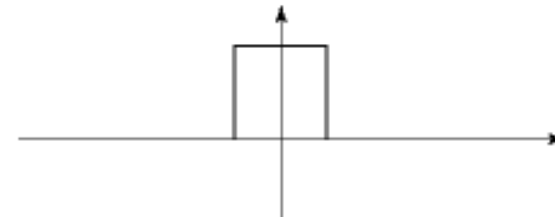
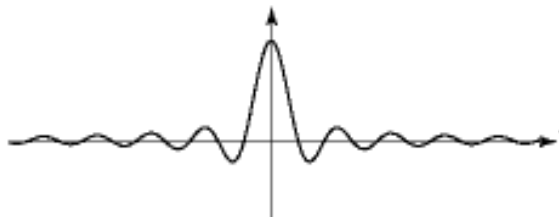
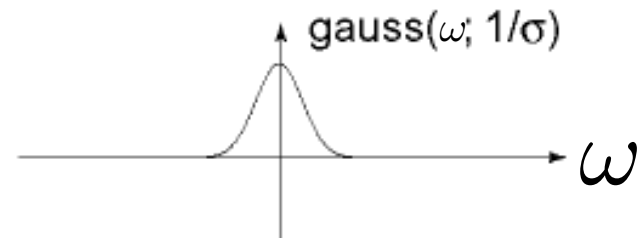
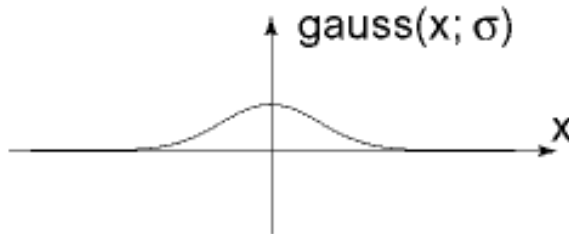
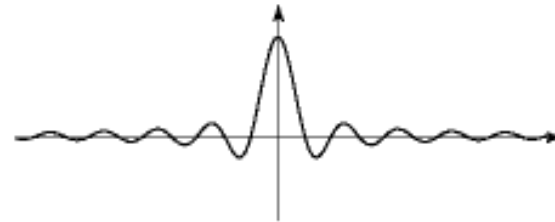
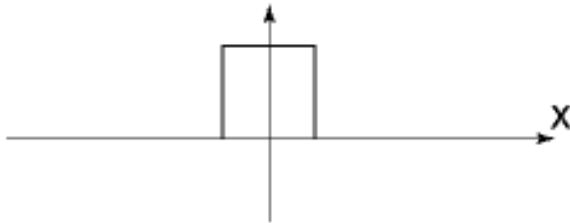


Gaussian Filter

Removes noise while minimizing spatial smoothing

Spatial domain

Frequency domain



Credit: Efros, Computational Photography

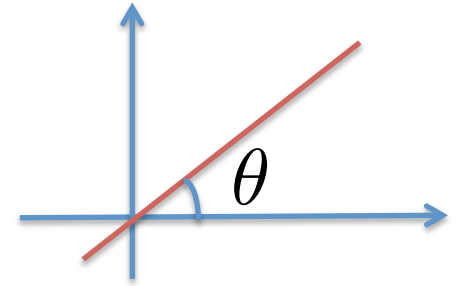
2D Gaussian Filter

$$h(x, y) = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2 + y^2}{2\sigma^2}}$$

Directional derivatives along $[\cos \theta \ \sin \theta]$:

First derivative:

$$\begin{aligned} D_\theta f(x, y) &= \nabla f \cdot [\cos \theta \ \sin \theta] \\ &= \frac{\partial f(x, y)}{\partial x} \cos \theta + \frac{\partial f(x, y)}{\partial y} \sin \theta \end{aligned}$$



Second derivative:

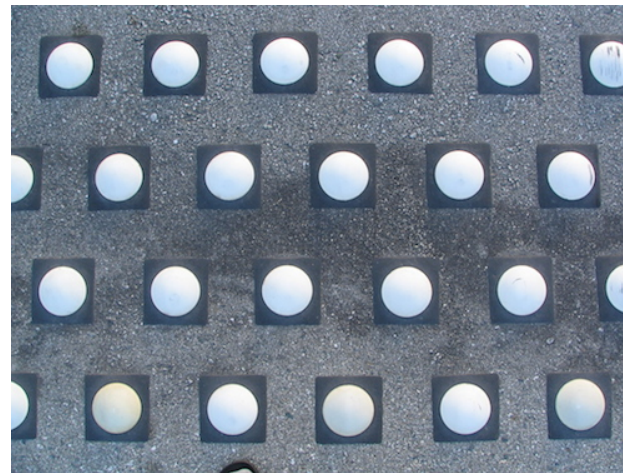
$$D_\theta^2 f(x, y) = \frac{\partial^2 f(x, y)}{\partial x^2} \cos^2 \theta + \frac{\partial^2 f(x, y)}{\partial x \partial y} \cos \theta \sin \theta + \frac{\partial^2 f(x, y)}{\partial y^2} \sin^2 \theta$$

Therefore, we can detect edges in the θ direction from the zero crossings of:

$$D_{\theta}^2 \{h(x, y) * f(x, y)\} = D_{\theta}^2 h(x, y) * f(x, y)$$

Directional derivatives are suitable to detect lanes or stop markings:

Choose direction θ to be orthogonal to them.



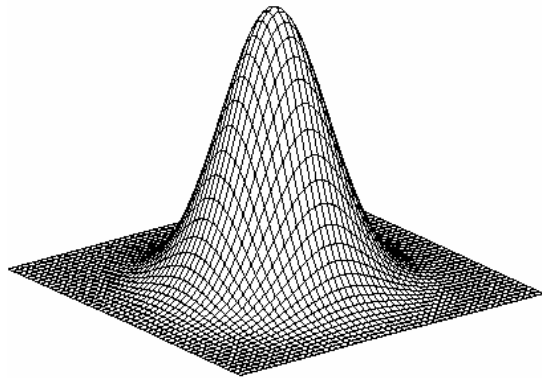
What about isotropic edges?

When there is no directionality (e.g., circular objects, such as Botts dots), use the Laplacian:

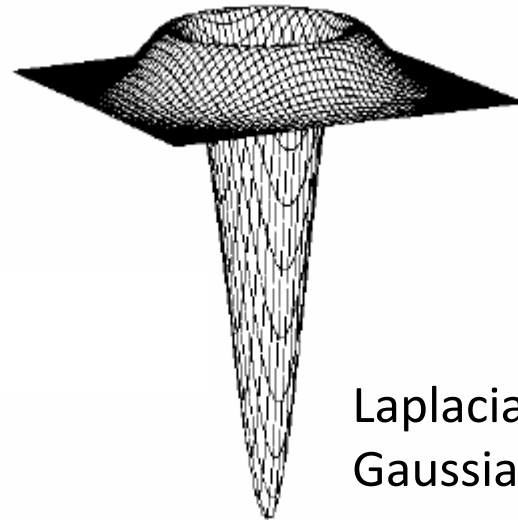
$$\nabla^2 f(x, y) = \frac{\partial^2 f(x, y)}{\partial x^2} + \frac{\partial^2 f(x, y)}{\partial y^2}$$

which is invariant under rotations of the image.
Look for zero crossings of:

$$\nabla^2 \{h(x, y) * f(x, y)\} = \nabla^2 h(x, y) * f(x, y)$$



Gaussian



Laplacian of
Gaussian

Finite Difference Approximations of Derivatives

Estimate derivatives from samples of $f(x)$:

$$\left. \frac{df(x)}{dx} \right|_{x=n\Delta} \approx \frac{1}{\Delta} \{f(n\Delta + \Delta) - f(n\Delta)\}$$

(forward difference)

$$\frac{1}{\Delta} \{f(n\Delta) - f(n\Delta - \Delta)\}$$

(backward difference)

$$\frac{1}{2\Delta} \{f(n\Delta + \Delta) - f(n\Delta - \Delta)\}$$

(central difference)

Taylor series to assess the approximation accuracy:

$$f(n\Delta + \Delta) = f(n\Delta) + \Delta f'(n\Delta) + \frac{\Delta^2}{2} f''(n\Delta) + \mathcal{O}(\Delta^3)$$

$$f(n\Delta - \Delta) = f(n\Delta) - \Delta f'(n\Delta) + \frac{\Delta^2}{2} f''(n\Delta) + \mathcal{O}(\Delta^3)$$

Forward difference:

$$\frac{1}{\Delta} \{f(n\Delta + \Delta) - f(n\Delta)\} = f'(n\Delta) + \mathcal{O}(\Delta)$$

Backward difference:

$$\frac{1}{\Delta} \{f(n\Delta) - f(n\Delta - \Delta)\} = f'(n\Delta) + \mathcal{O}(\Delta)$$

Central difference:

$$\frac{1}{2\Delta} \{f(n\Delta + \Delta) - f(n\Delta - \Delta)\} = f'(n\Delta) + \mathcal{O}(\Delta^2)$$

smaller error

Second Derivative Approximation

$$\frac{1}{\Delta^2} \{f(n\Delta + \Delta) - 2f(n\Delta) + f(n\Delta - \Delta)\} = f''(n\Delta) + \mathcal{O}(\Delta^2)$$

Finite Difference Approximations in 2D

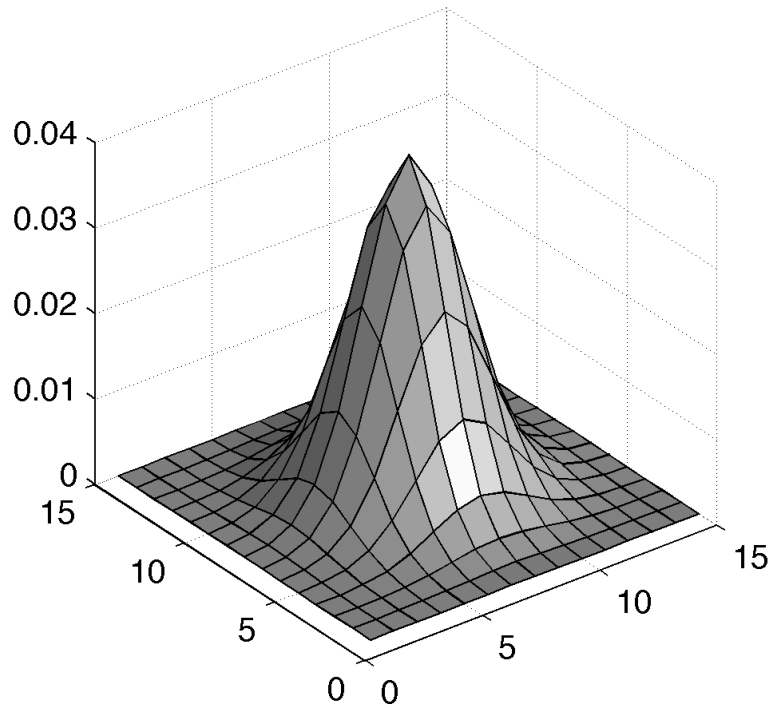
$$\frac{\partial}{\partial x} f(x, y) \Big|_{(n_1\Delta, n_2\Delta)} \approx \frac{1}{\Delta} \{f(n_1\Delta + \Delta, n_2\Delta) - f(n_1\Delta, n_2\Delta)\}$$

$$\frac{\partial}{\partial y} f(x, y) \Big|_{(n_1\Delta, n_2\Delta)} \approx \frac{1}{\Delta} \{f(n_1\Delta, n_2\Delta + \Delta) - f(n_1\Delta, n_2\Delta)\}$$

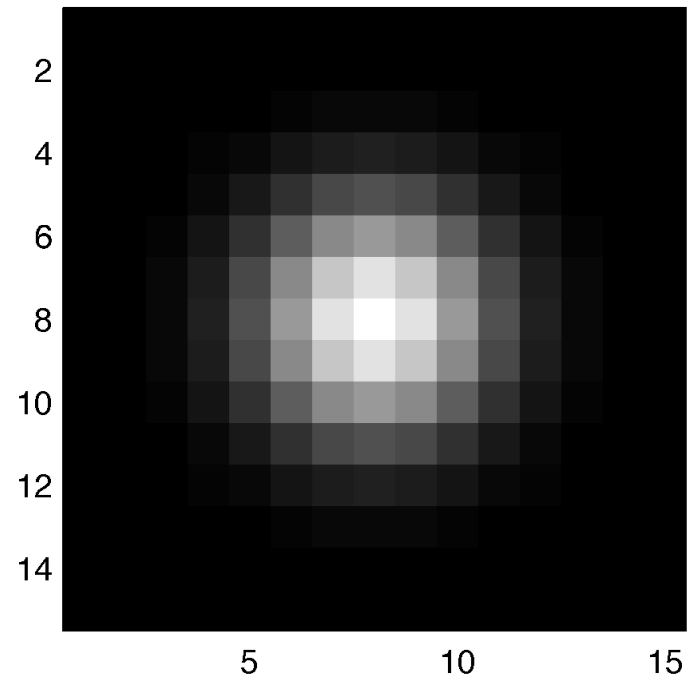
$$\begin{aligned} \nabla^2 f(x, y) \approx & \frac{1}{\Delta^2} \{f(n_1\Delta + \Delta, n_2\Delta) + f(n_1\Delta - \Delta, n_2\Delta) \\ & f(n_1\Delta, n_2\Delta + \Delta) + f(n_1\Delta, n_2\Delta - \Delta) \\ & - 4f(n_1\Delta, n_2\Delta)\} \end{aligned}$$

Gaussian Filter and its Derivatives in MATLAB

Gaussian Filter



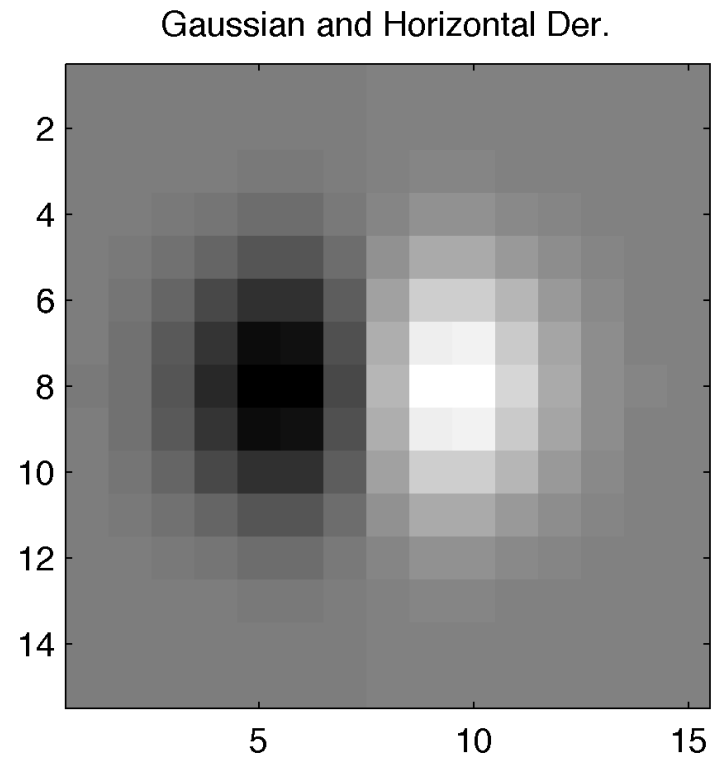
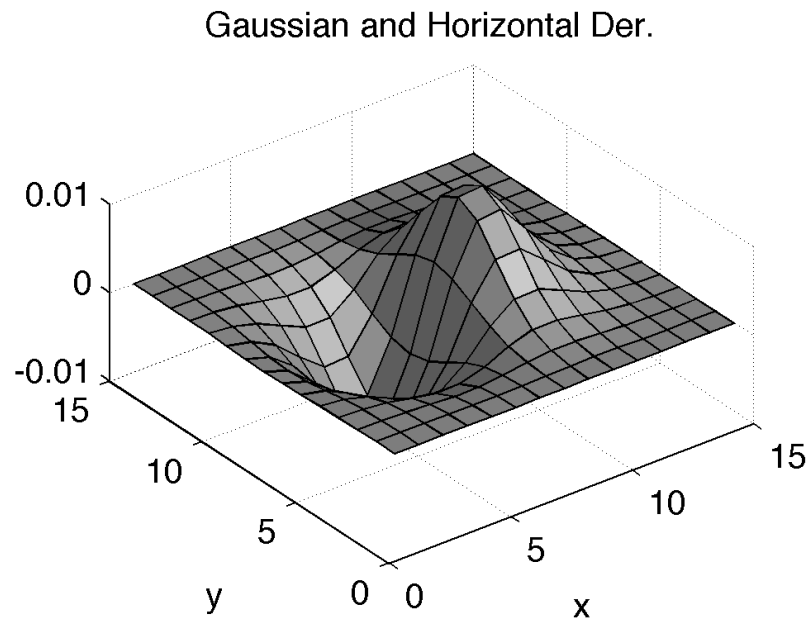
Gaussian Filter



Gaussian Filter, 15x15 pixels, $\sigma=2$

```
>> g=fspecial('gaussian',15,2);
```

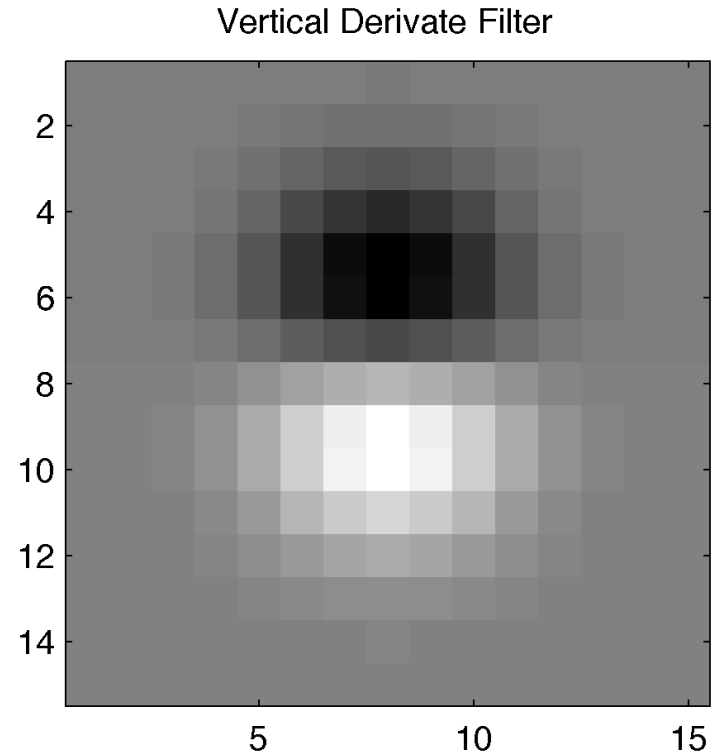
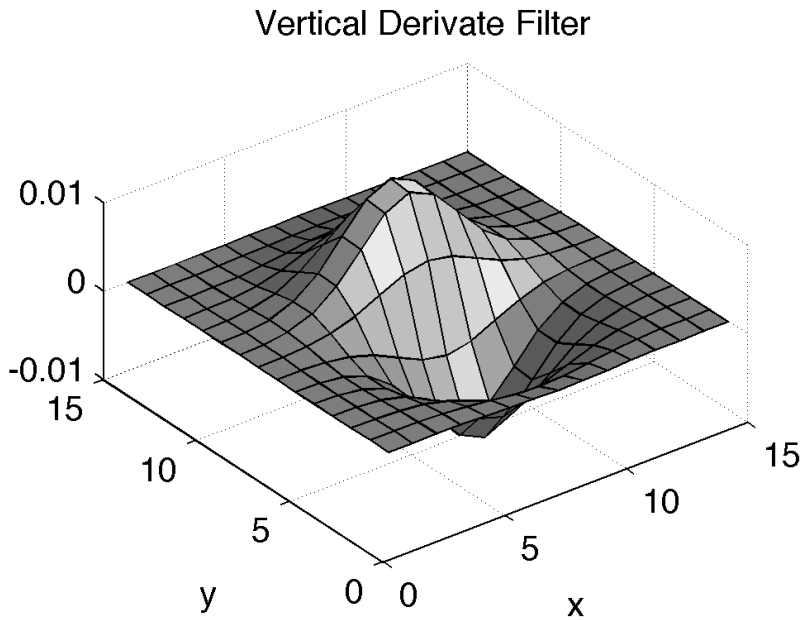
$$h(x, y) = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}}$$



Combine Gaussian Filter with Horizontal Derivative

```
>> dx=conv2(g,[-1,1],'same');
```

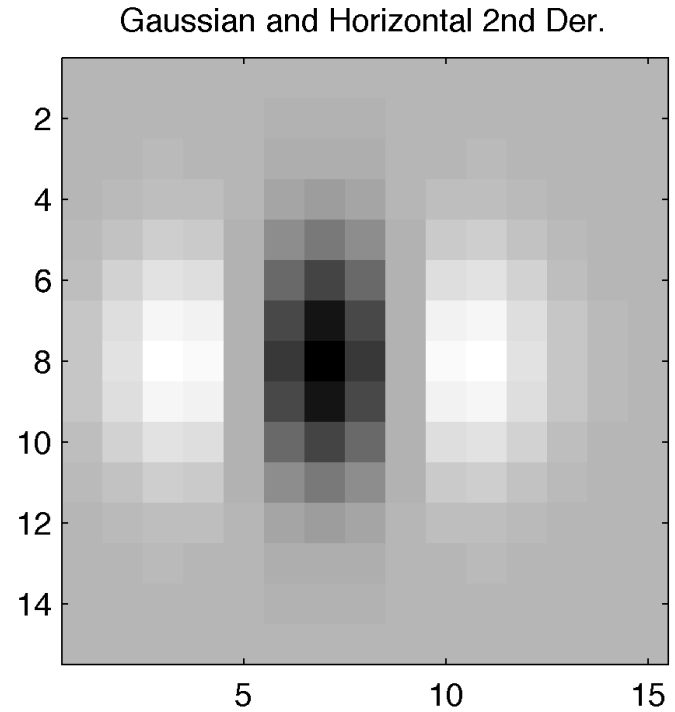
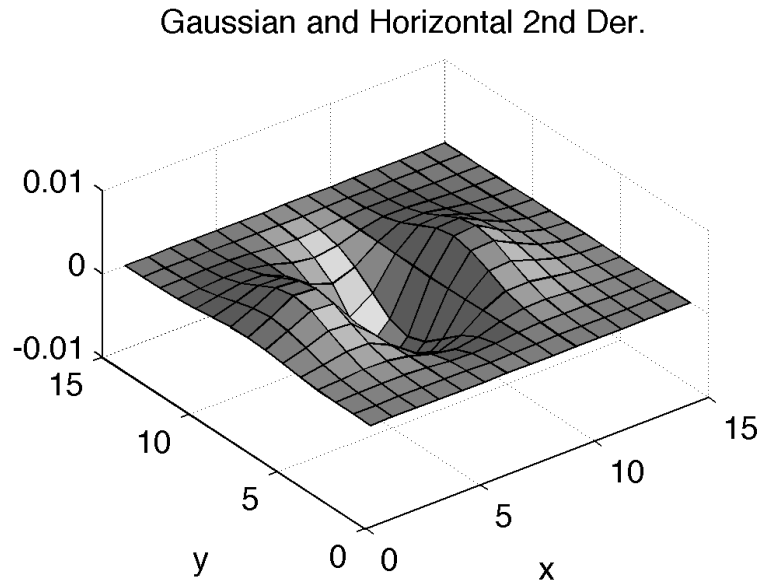
$$\frac{\partial}{\partial x} h(x, y)$$



Vertical Derivative Filter (with Gaussian)

```
>> dy=conv2(g,[-1;1],'same');
```

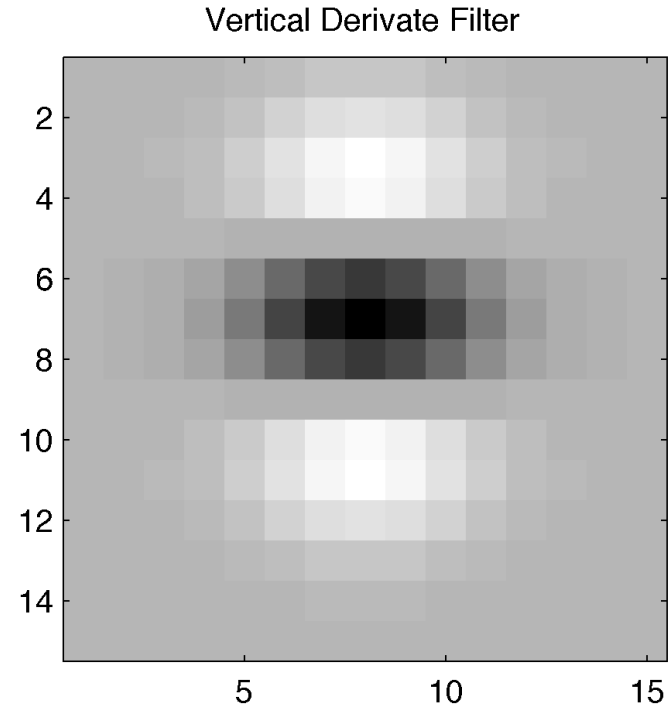
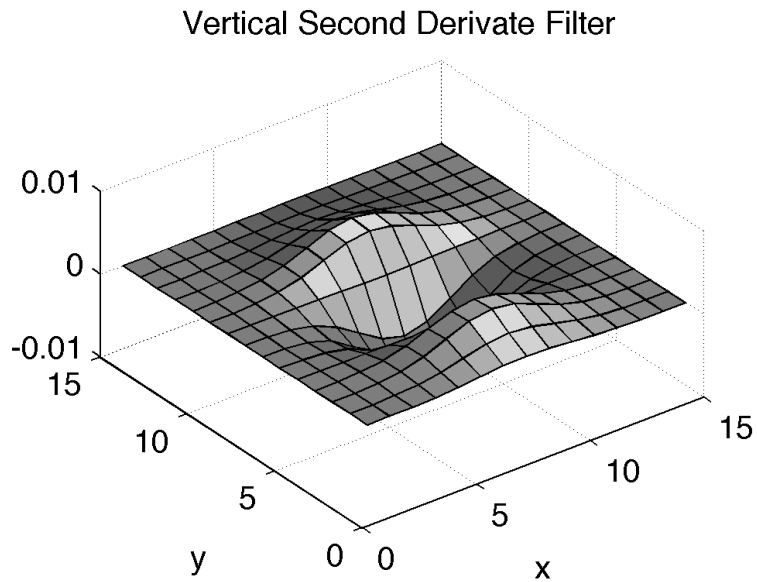
$$\frac{\partial}{\partial y} h(x, y)$$



Second Derivative of Gaussian in Horizontal Dir.

```
>> ddx=conv2(dx,[-1,1],'same')
```

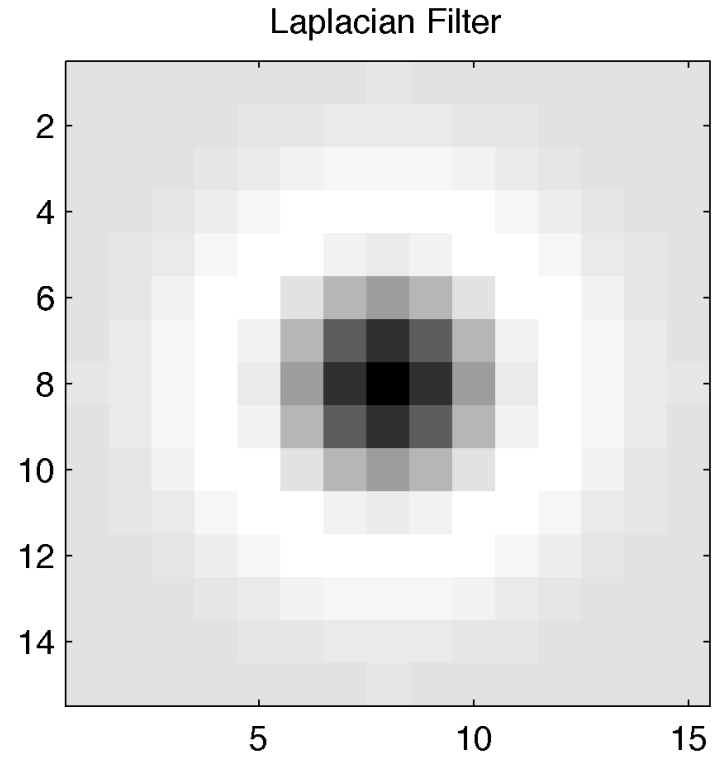
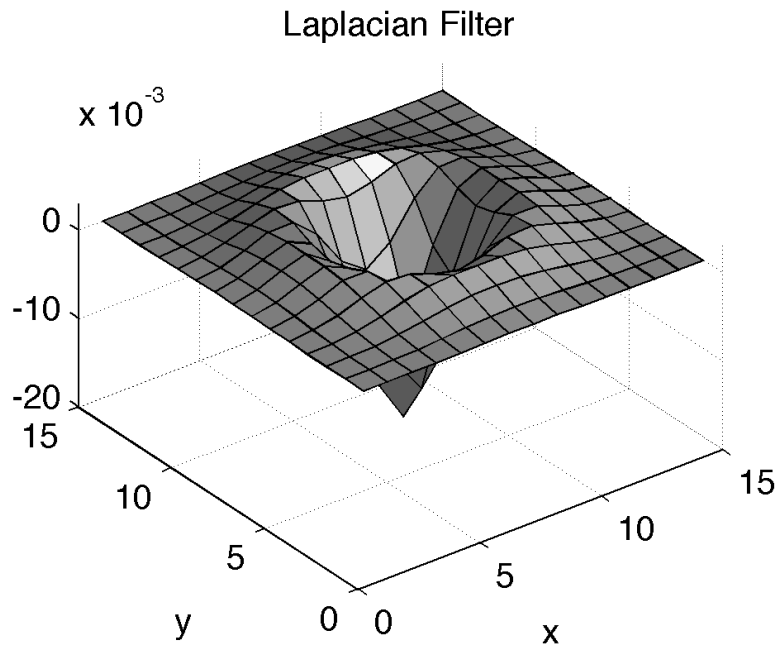
$$\frac{\partial^2}{\partial x^2} h(x, y)$$



Vertical Second Derivative Filter (with Gaussian)

```
>> dy=conv2(g,[-1;1],'same');
```

$$\frac{\partial^2}{\partial y^2} h(x, y)$$

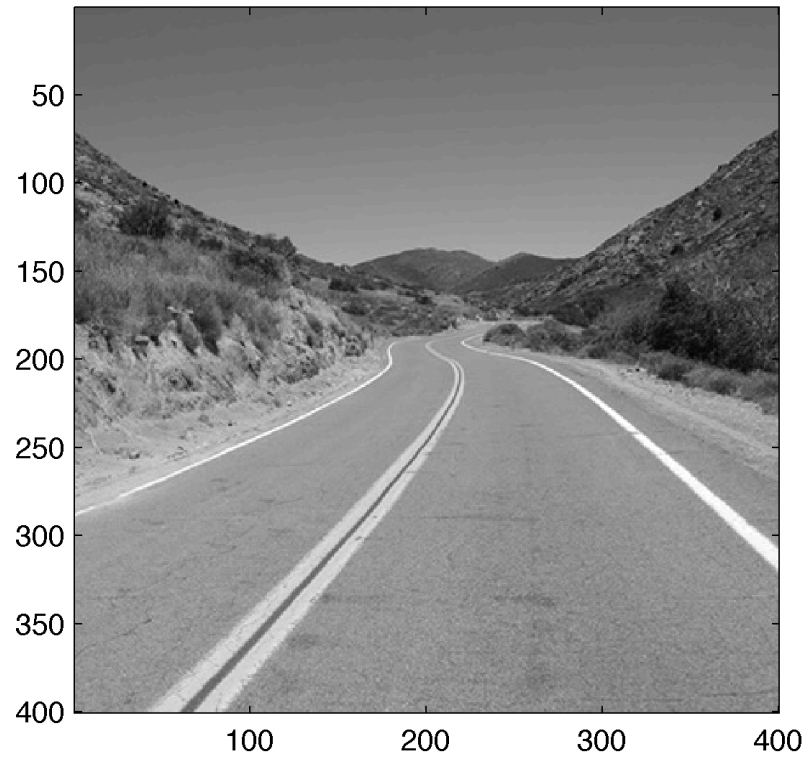


Laplacian Filtering

```
>> lg=fspecial('log',15,2);
```

$$\nabla^2 h(x, y)$$

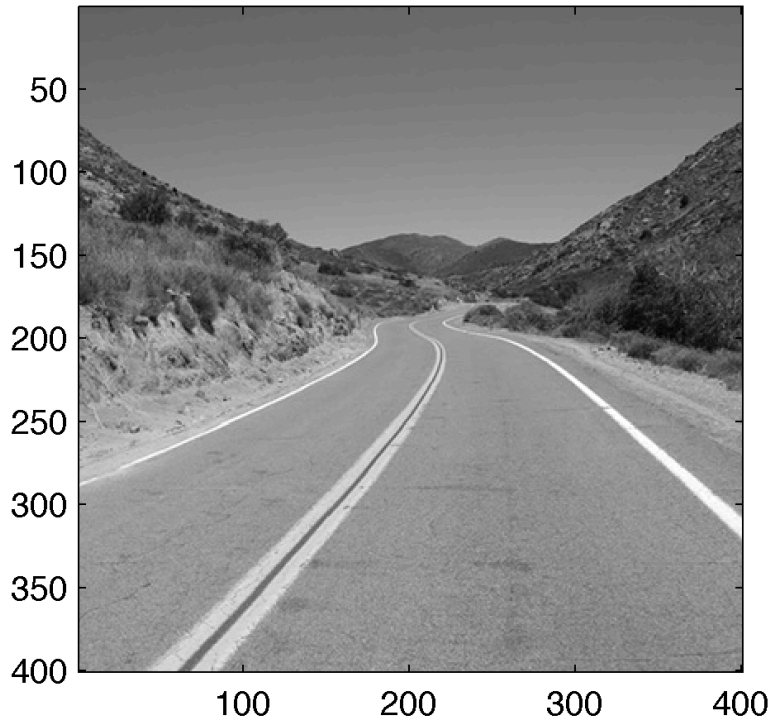
Edge Detection Examples with MATLAB



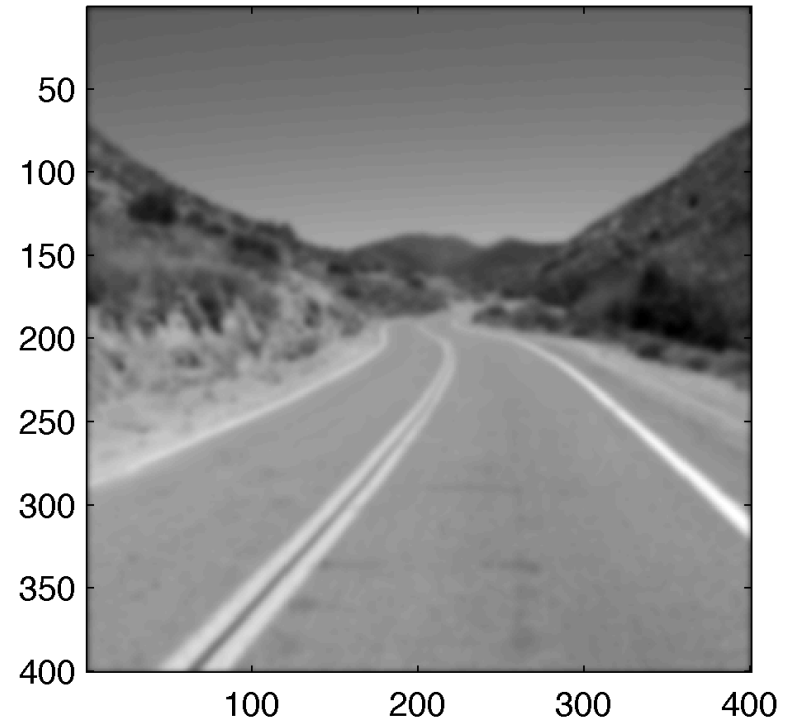
Load An Image

```
>> road=imread('road.jpg');  
>> imagesc(road); %Scale values and display image
```

Original Image



Gaussian Filtered Image

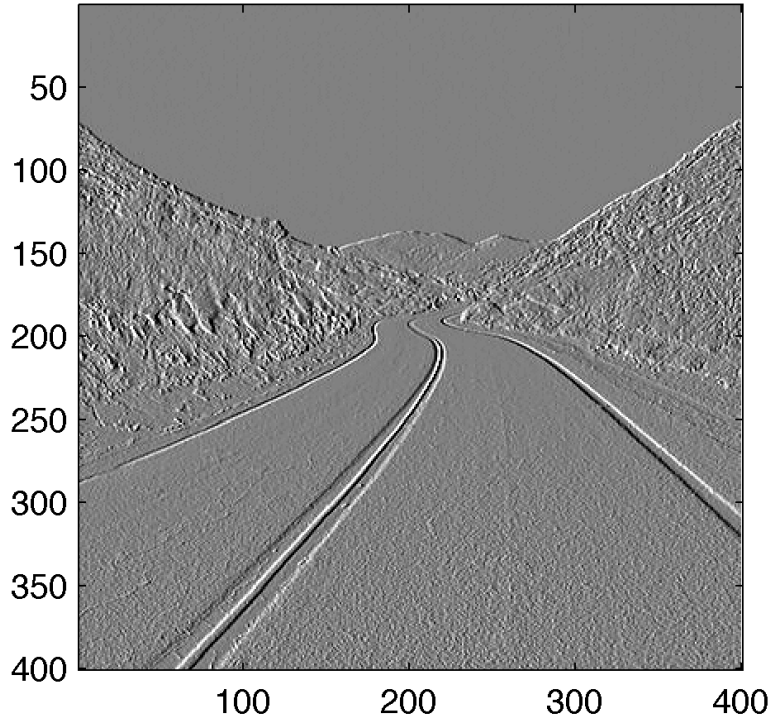


Gaussian Filtering

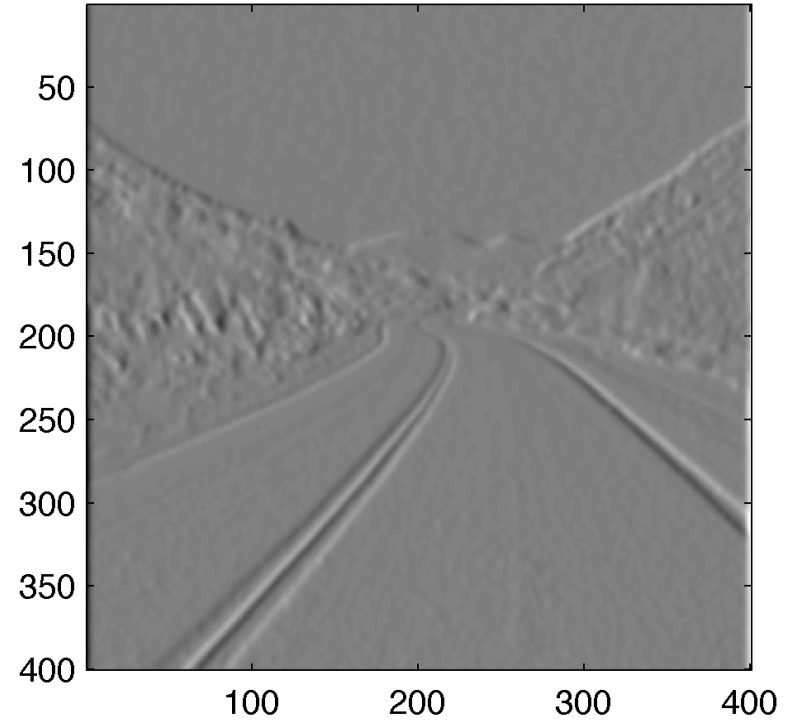
```
>> groad=conv2(road,g,'same');
```

$$f(x, y) \text{ vs. } h(x, y) * f(x, y)$$

Horizontal Derivative (noisy)



Horizontal Derivative after Filtering (Less Noise)



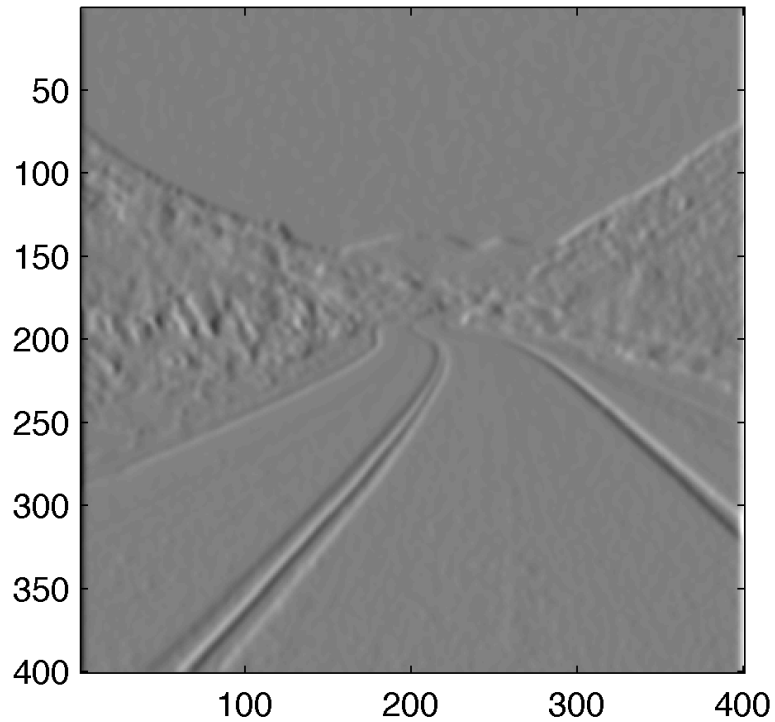
Edge Detection Along Horizontal Direction

```
>> conv2(road,[-1,1],'same')
```

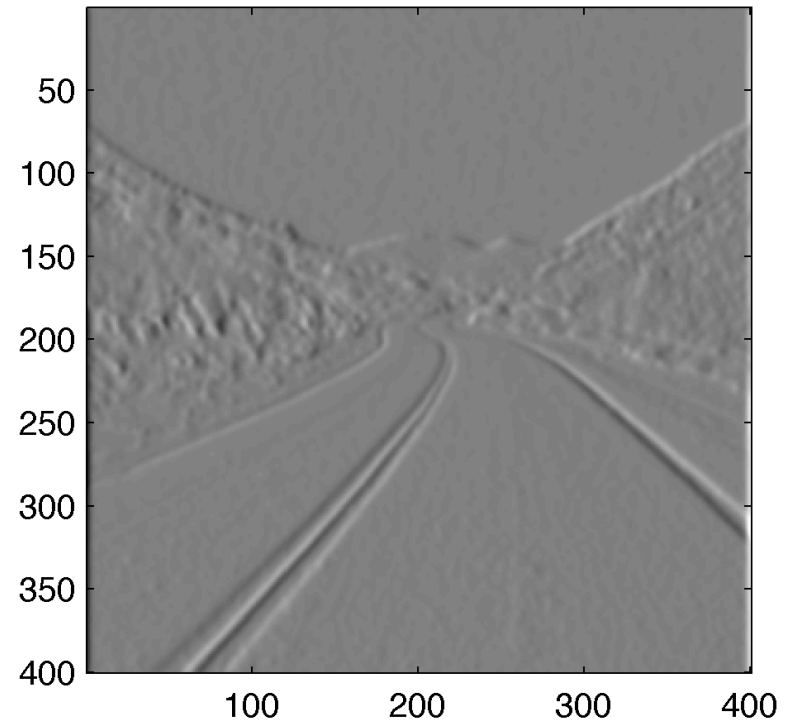
```
>> conv2(groad,[-1,1],'same')
```

$$\frac{\partial}{\partial x} f(x, y) \text{ vs. } \frac{\partial}{\partial x} \{h(x, y) * f(x, y)\}$$

Gaussian Filter, Then Derivative



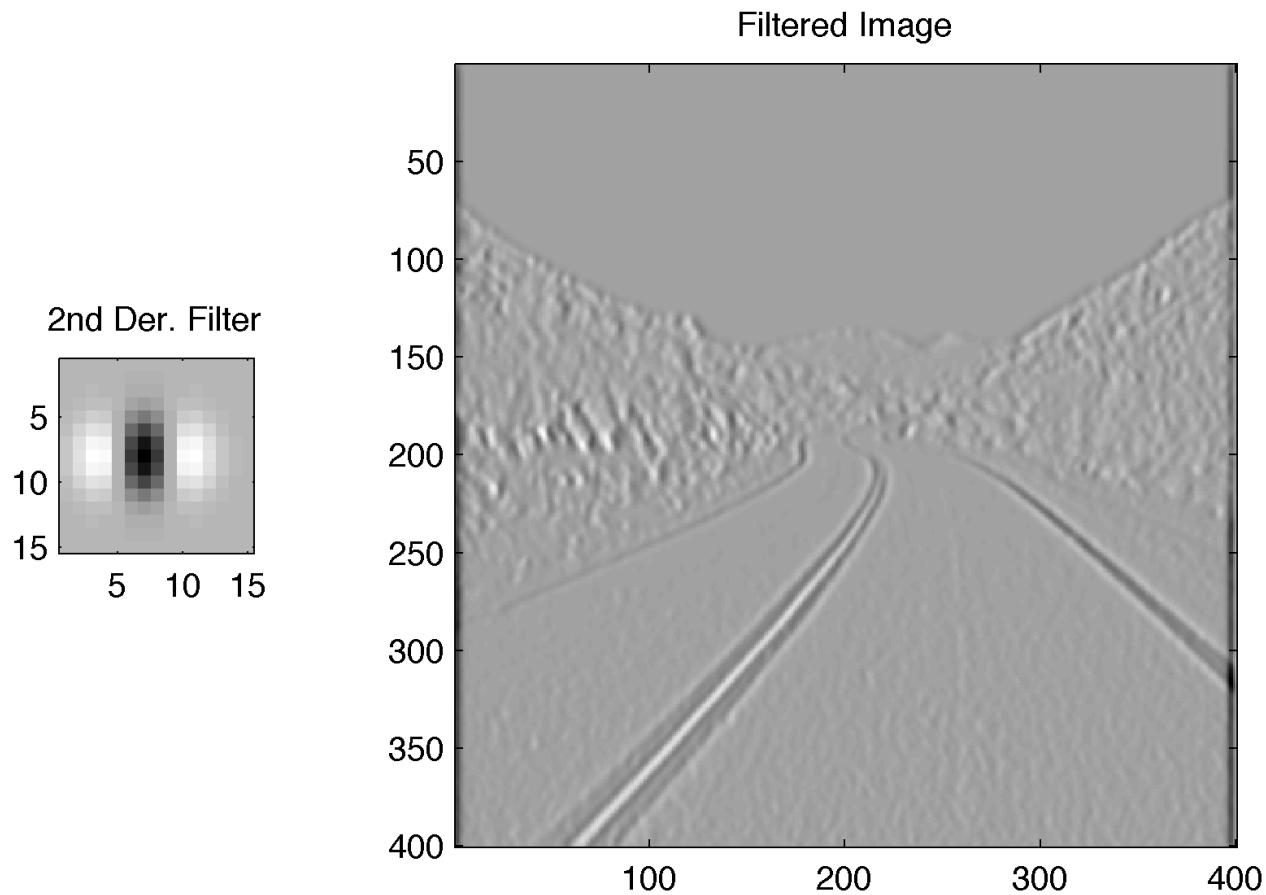
Combined Filter



Same Result

```
>> conv2(groad,[-1,1],'same')  
>> conv2(road,dx,'same')
```

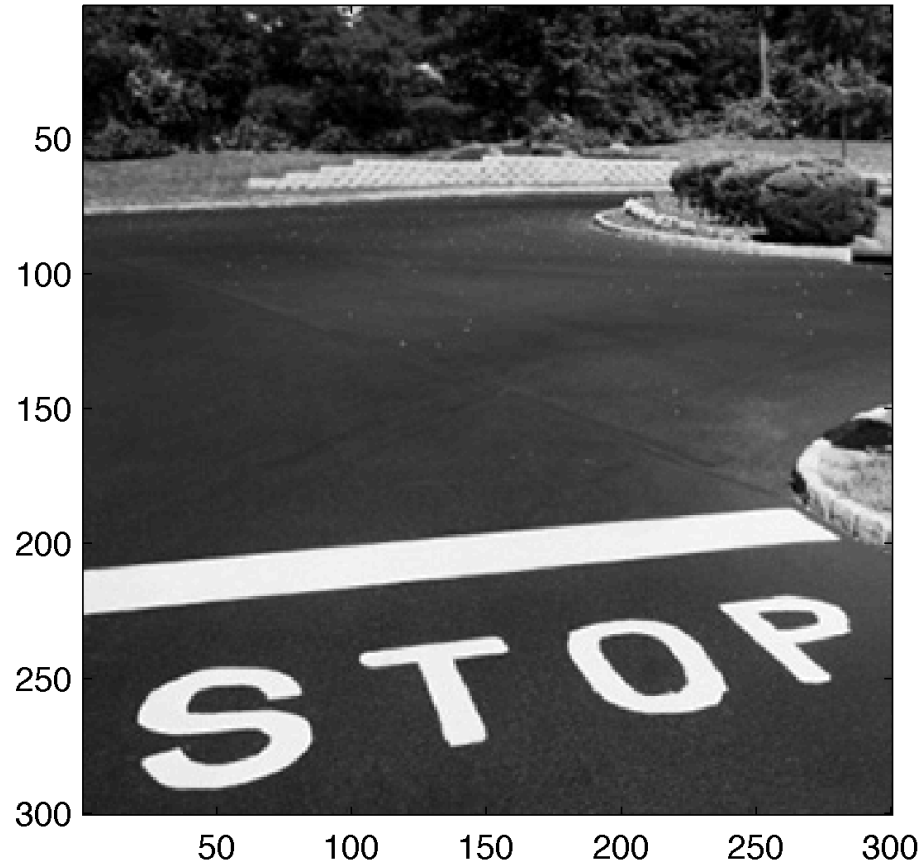
$$\frac{\partial}{\partial x} \{h * f\} = \frac{\partial h}{\partial x} * f$$



Second Derivative in Horizontal Direction

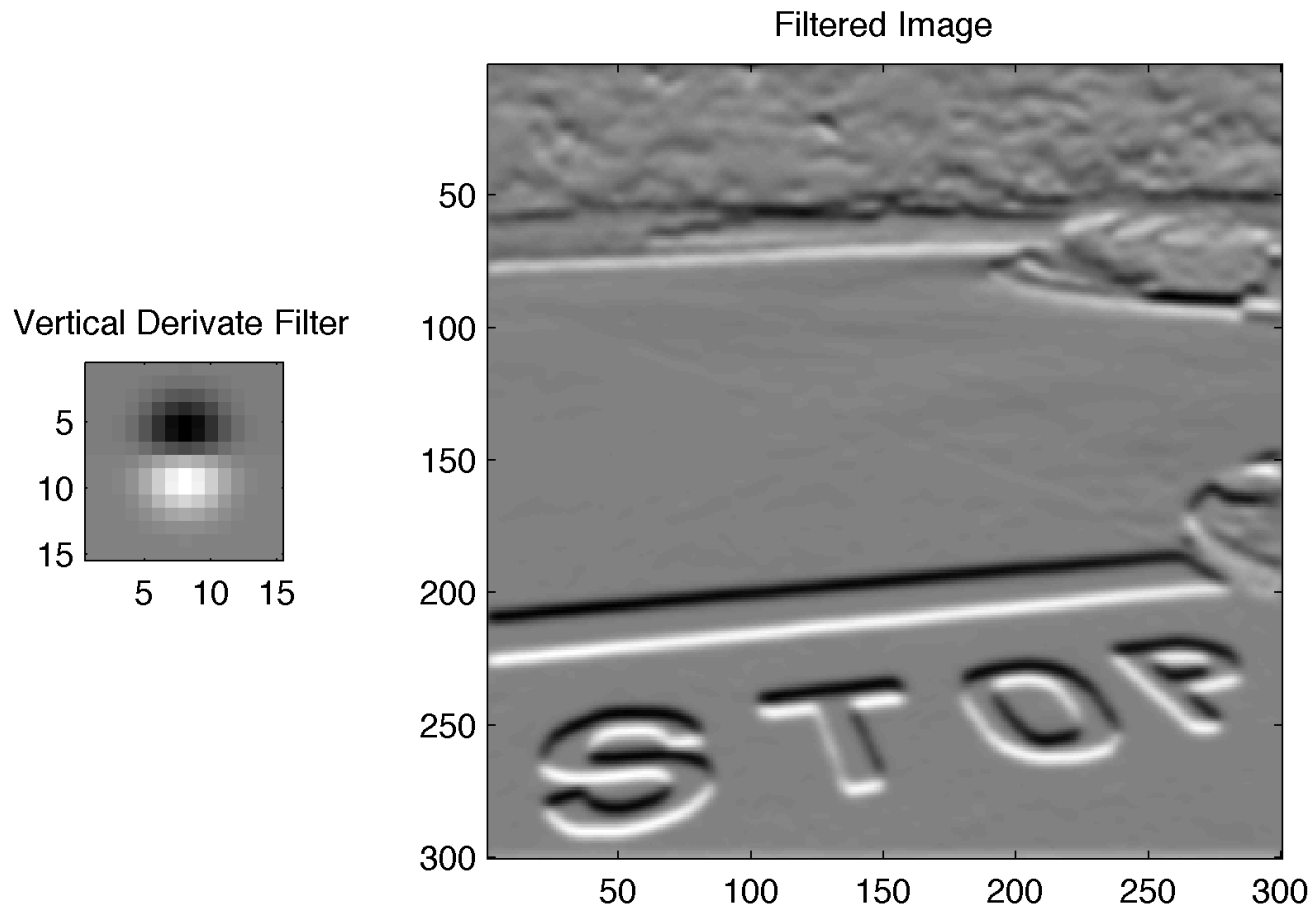
```
>> conv2(road,ddx,'same')
```

$$\frac{\partial^2}{\partial x^2} h(x, y) * f(x, y)$$



Stop Line Detection

```
>> stop_im = imread('stop.jpg')
```

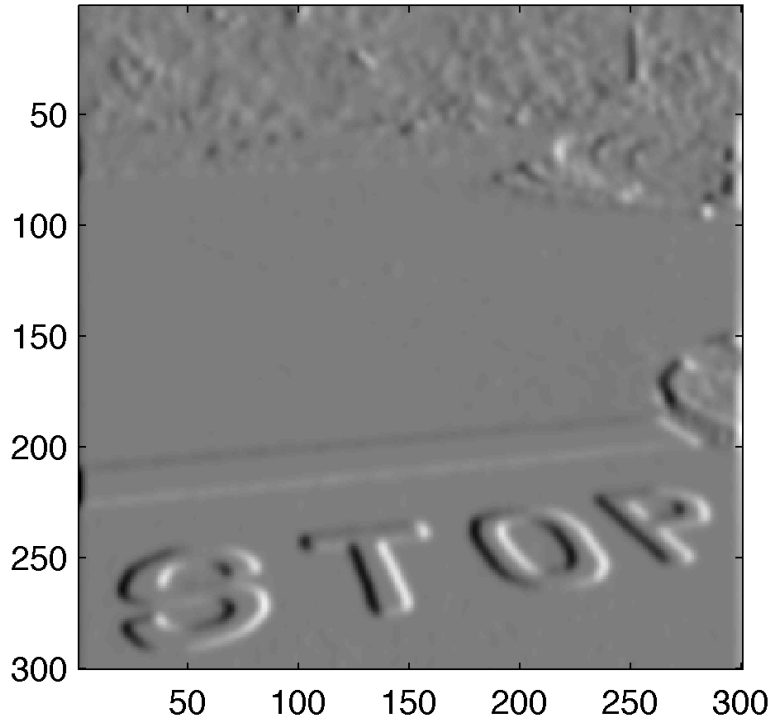


Detecting Stop Bar using Vertical Derivative

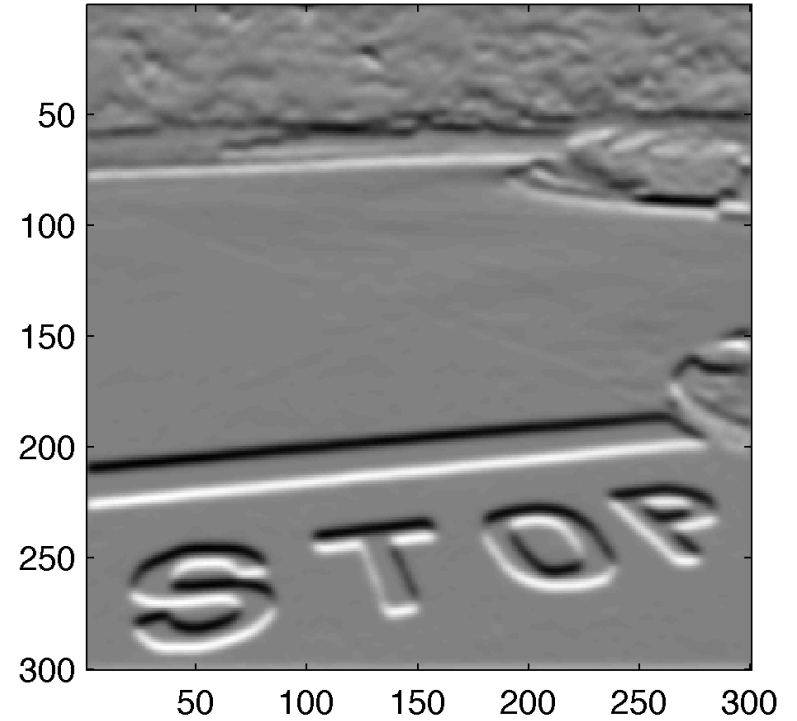
```
>> conv2(stop_im,dy,'same')
```

$$\frac{\partial h}{\partial y} * f$$

dx Filtered Image



dy Filtered Image

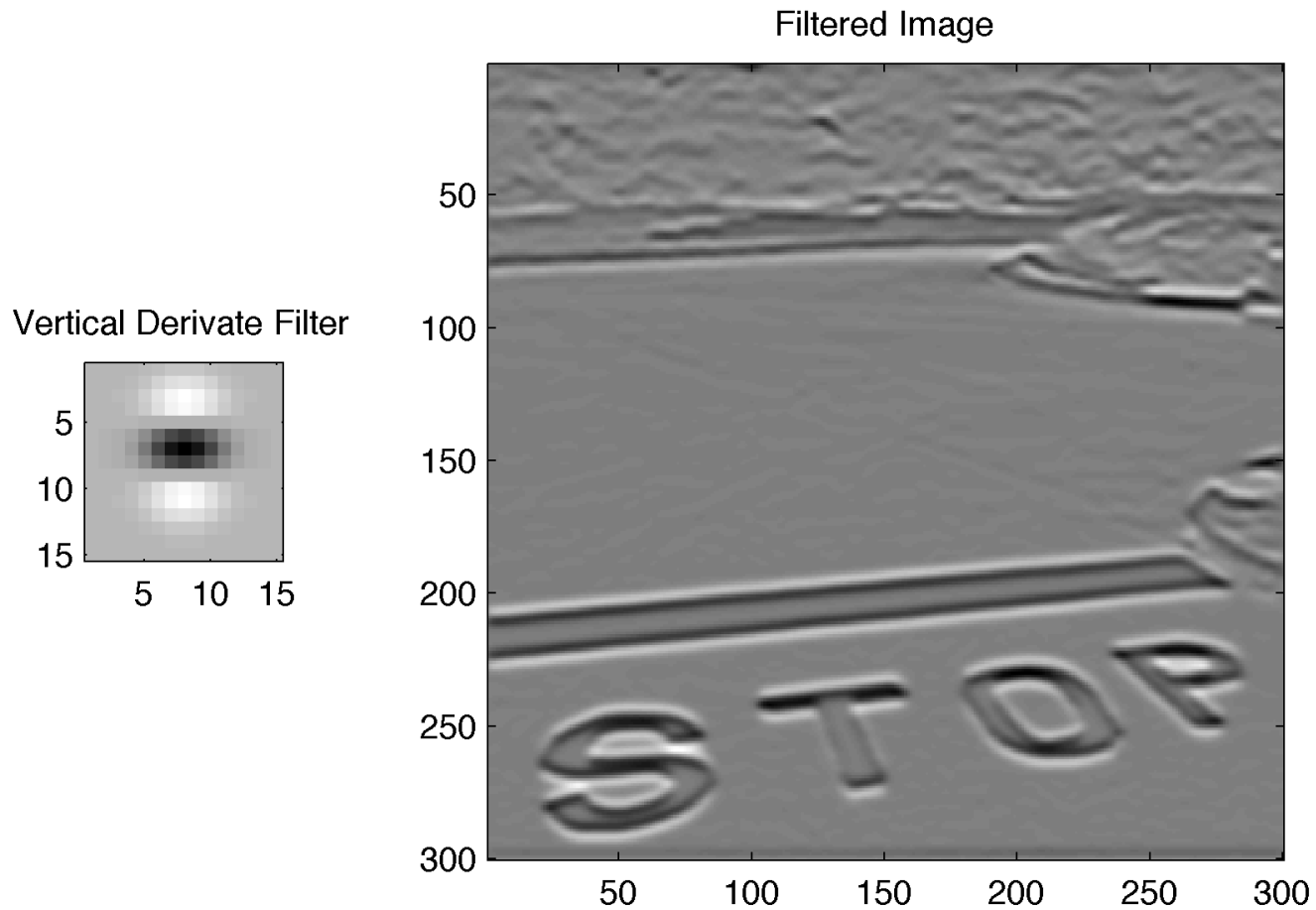


Comparing Horizontal and Vertical Derivative

```
>> conv2(stop_im,dx,'same')
```

```
>> conv2(stop_im,dy,'same')
```

$$\frac{\partial h}{\partial x} * f \text{ vs. } \frac{\partial h}{\partial y} * f$$

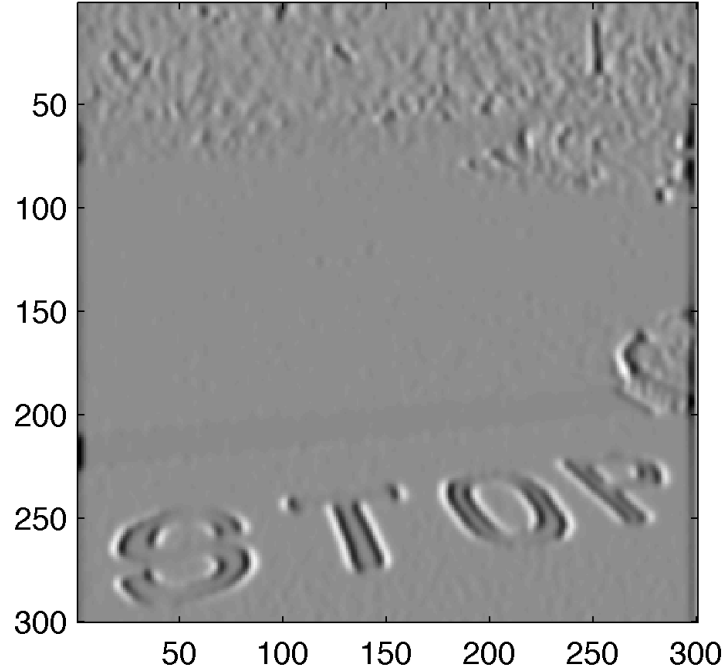


Detecting Stop Bar using Vertical Second Derivative

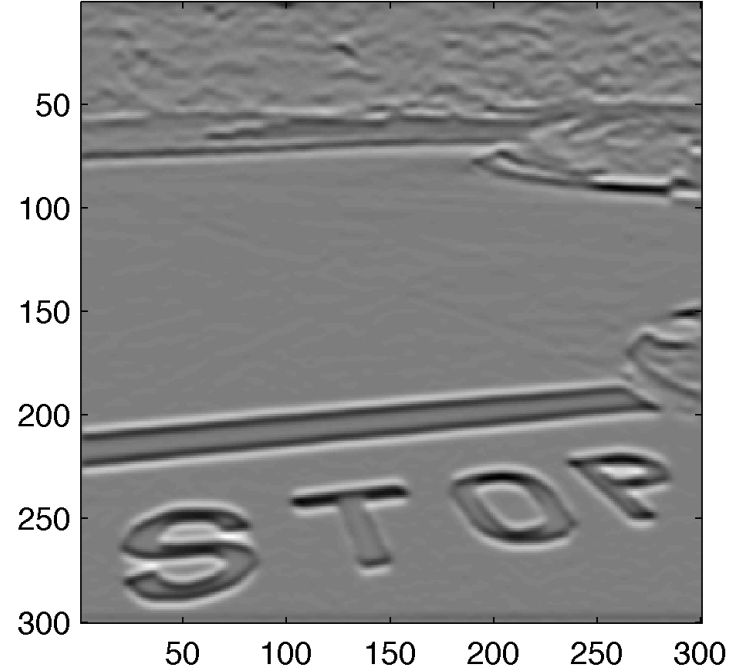
```
>> conv2(stop_im,dy,'same')
```

$$\frac{\partial^2}{\partial y^2} h(x, y) * f(x, y)$$

dx^2 Filtered Image



dy^2 Filtered Image

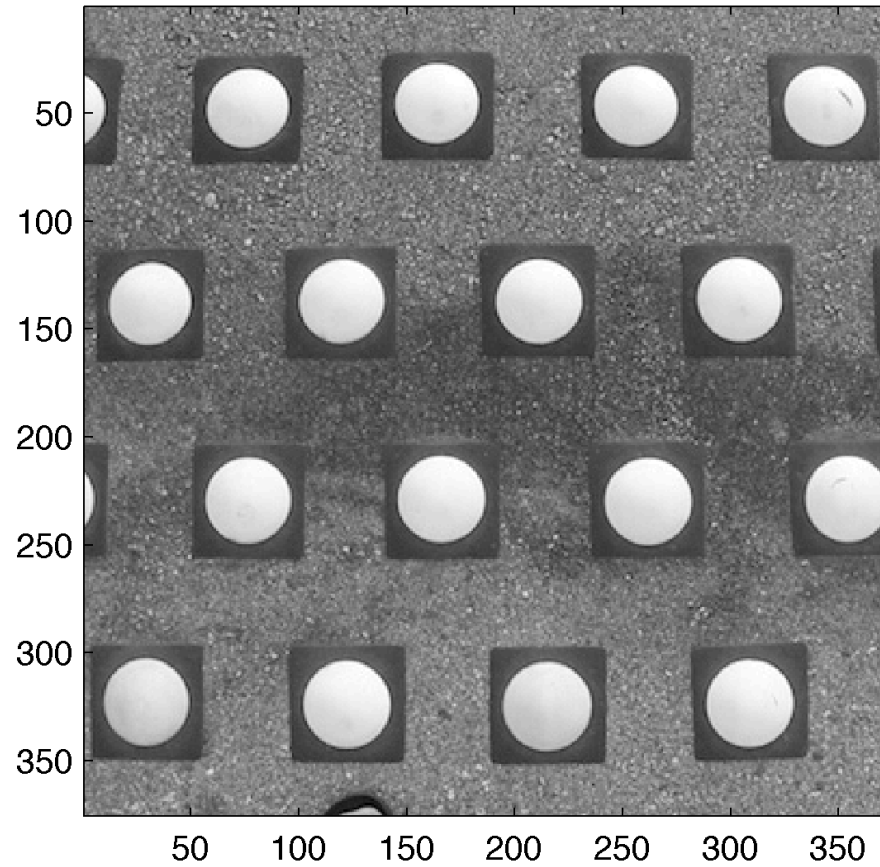


Comparing Horizontal and Vertical Second Derivatives

```
>> conv2(stop_im, ddx, 'same')
```

```
>> conv2(stop_im, ddy, 'same')
```

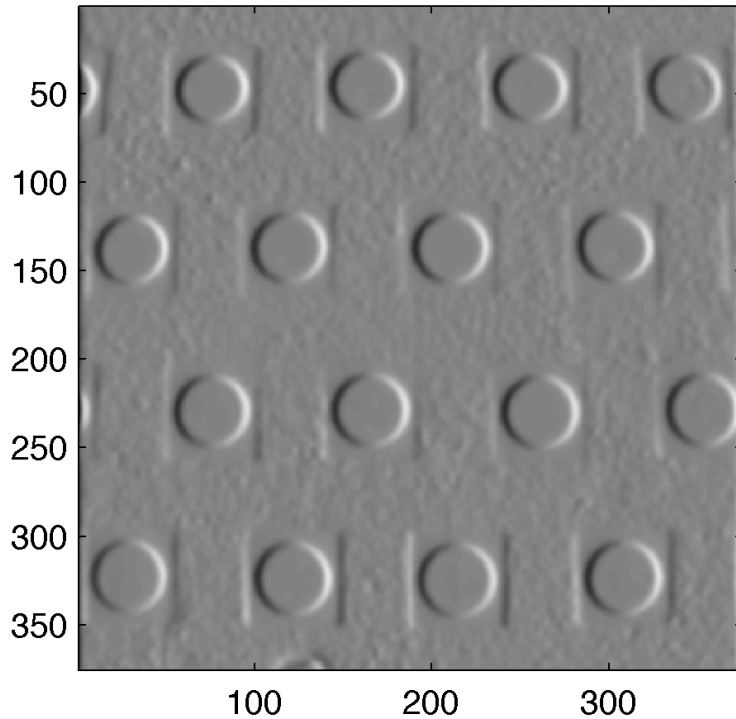
$$\frac{\partial^2}{\partial x^2} h(x, y) * f(x, y) \text{ vs. } \frac{\partial^2}{\partial y^2} h(x, y) * f(x, y)$$



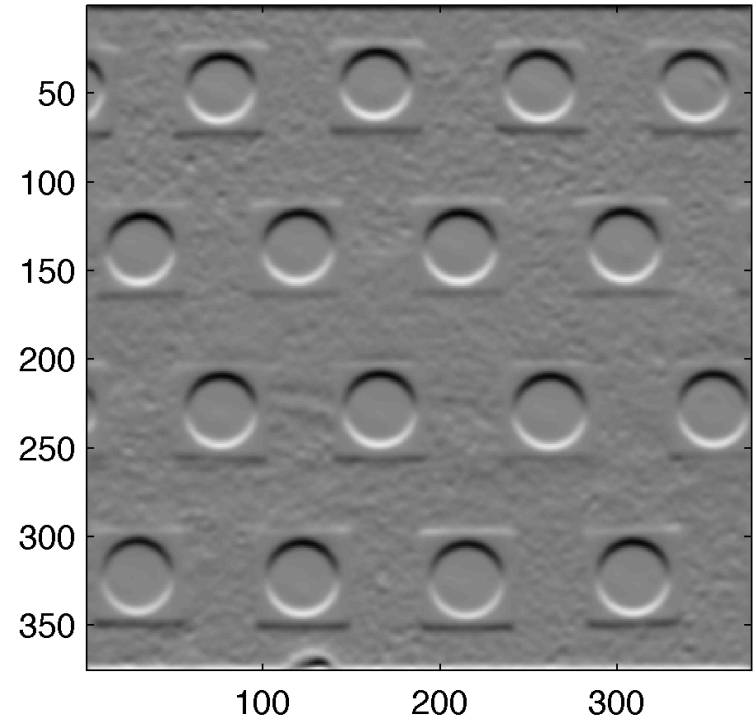
Detecting Botts Dots

```
>> botts=imread('botts.jpg');
```

dx Filtered Image



dy Filtered Image



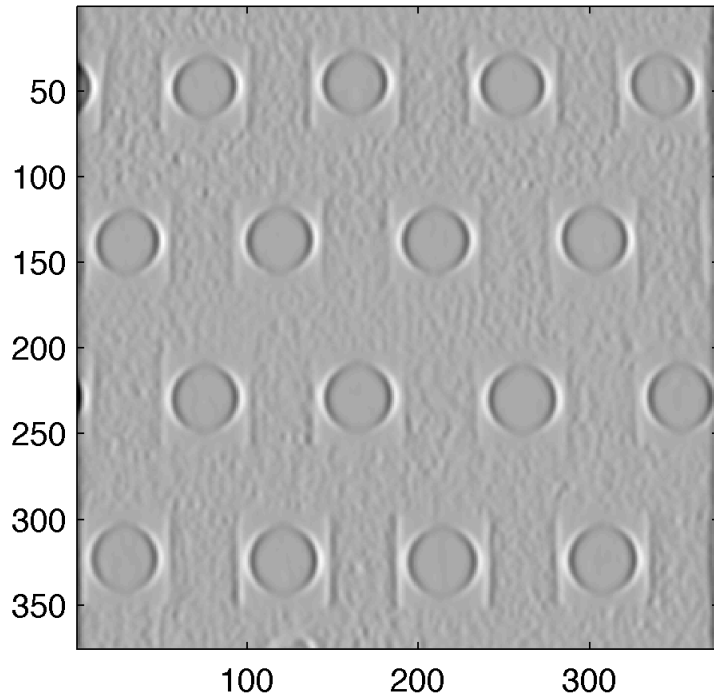
Horizontal and Vertical Derivative Filtering

```
>> conv2(botts,dx,'same')
```

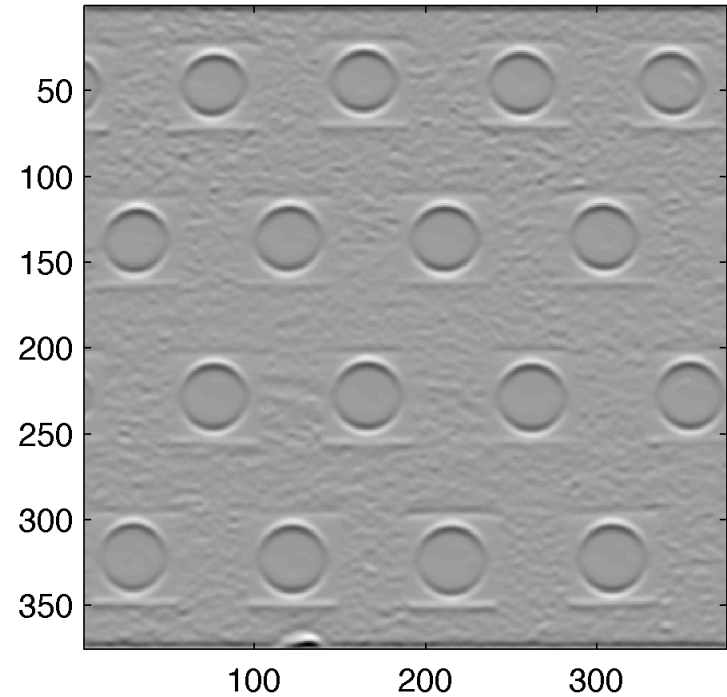
```
>> conv2(botts,dy,'same')
```

$$\frac{\partial}{\partial x} h(x, y) * f(x, y) \text{ vs. } \frac{\partial}{\partial y} h(x, y) * f(x, y)$$

dx^2 Filtered Image



dy^2 Filtered Image



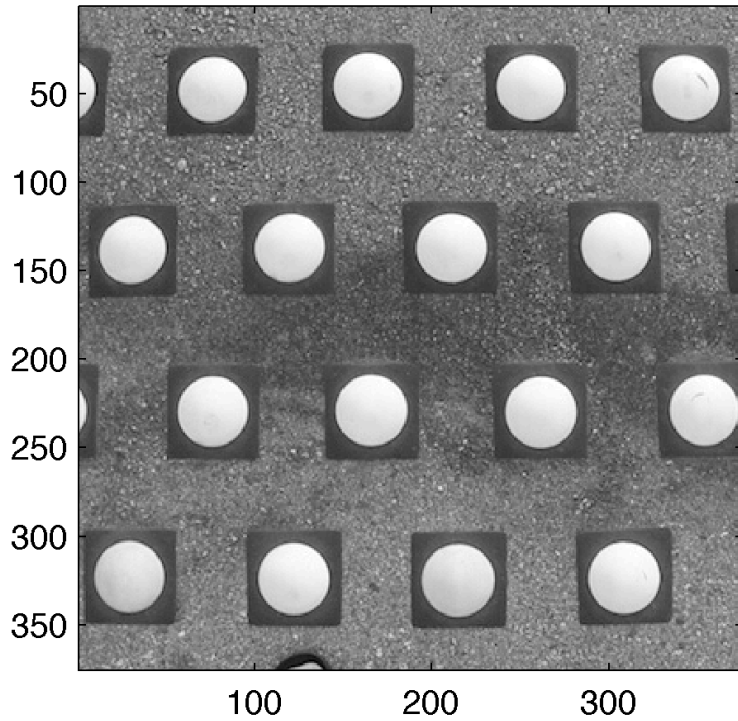
Horizontal and Vertical Second Derivative Filtering

```
>> conv2(botts, ddx, 'same')
```

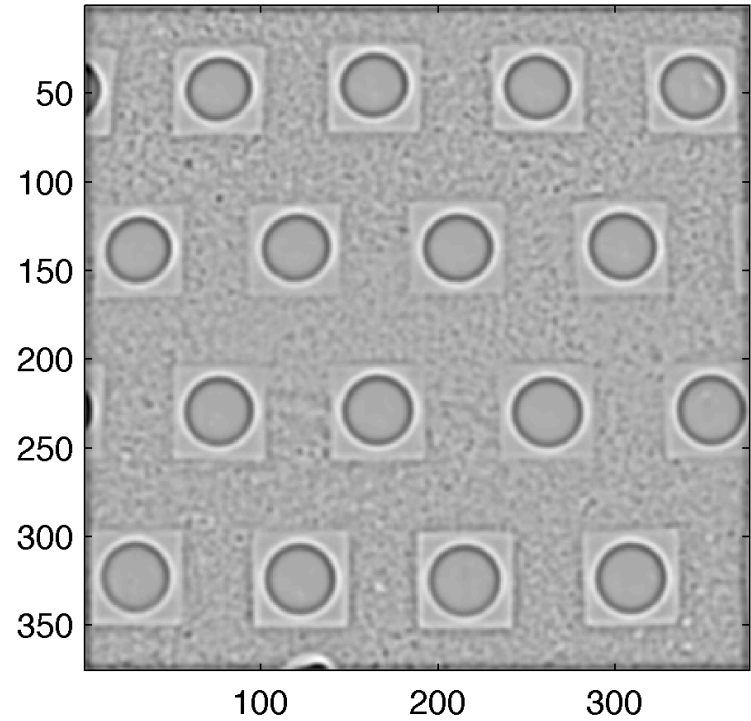
```
>> conv2(botts, ddy, 'same')
```

$$\frac{\partial^2}{\partial x^2} h(x, y) * f(x, y) \text{ vs. } \frac{\partial^2}{\partial y^2} h(x, y) * f(x, y)$$

Original Image



Laplacian Filtered Image



Original versus Laplacian

```
>> conv2(botts,lg,'same')
```

$$\nabla^2 h * f$$