Professor Fearing EECS120/Problem Set 8 v 1.0 Fall 2014

Due at 4 pm, Fri. Nov. 14. MT 2, Wed. Nov. 19 in class

0. (0 pts) Warmup exercises

OW problems 9.5, 9.10, 9.15, 11.3, 11.20 (answers in back of book). These do not need to be turned in.

1. (10 pts) Region of convergence, pole/zero diagram OW 9.2)

For each part below, $y(t)$ is the output for an LTI system with impulse response $h(t)$ and input $x(t)$. Show the pole and zero locations, and the region of convergence in the $\sigma - j\omega$ plane for each $Y(s)$.

i. $x(t) = e^{-2t}u(t), h(t) = e^{+2t}u(t)$

ii. $x(t) = e^{-4t}u(t), h(t) = \cos(4\pi t)u(t)$.

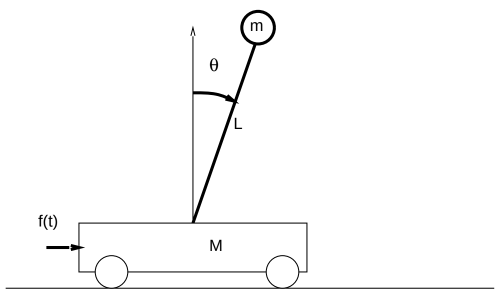

Figure 1: Broom balancer.

2. (25 pts) (OW 9.5, 9.7)

The differential equation for the broom balancing system (Fig. 1) is given by

$$\frac{1}{3}(4M + m)L\ddot{\theta}(t) = (m + M)g\theta(t) - f(t)$$

under the assumption that $|\theta| << 1$ and the approximation $sin\,\theta = \theta$, and $f(t)$ is the force applied to move the cart.

a. Find the Laplace transform relation for the broom balancing system including both the zero state response and the zero input response.

b. Suppose you can measure $\theta(t)$. You want to balance the broom by choosing $f(t)$ in feedback form as $f(t) = \alpha L\theta(t)$. Will this scheme result in balancing the broom, i.e. so that $\theta(t) \to 0$ as $t \to \infty$, for any small initial condition $\theta(0-)$ and $\dot{\theta}(0-)$? Explain why or why not.

c. Suppose you can measure $\theta(t)$ and $\dot{\theta}(t)$. You want to balance the broom by choosing f in feedback form as $f(t) = \alpha L\theta(t) + \beta L\dot{\theta}(t)$ For what values of α and β will this scheme result in balancing the broom, i.e. so that $\theta(t) \to 0$ as $t \to \infty$, for any small initial condition $\theta(0-)$ and $\dot{\theta}(0-)$.

3. (25 pts) Gain and Phase Margin (OW 6.5, 11.5)

A closed loop controller has transfer function $\frac{K(s)H(s)}{1+K(s)H(s)}$ where $K(s)H(s) = \frac{100}{(s+1)^2(s+10)}$, where $Y = KHE$, with error $E(s) = R(s) - Y(s)$.

a. Draw a block diagram for the feedback system.

b. Sketch the log magnitude-phase diagram for $K(s)H(s)$.

c. Find the frequency for which the phase of $K(j\omega)H(j\omega)$ is $(2n+1)\pi$, and the gain margin at this frequency.

d. Find the frequency for which the magnitude of $K(j\omega)H(j\omega)$ is 1, and the phase margin at this frequency.

e. What is the largest time delay which could be introduced in the feedback path between E and K without the system going unstable?

4. (20 pts) Frequency and Phase response using graphical techniques. (OW 9.4)

For each transfer function below (all are causal and at least marginally stable):

a. sketch the pole-zero diagram.

b. sketch the magnitude and phase response of $H(j\omega)$ on linear-linear scale, using the approximate methods discussed in class and the lecture notes.

c. sketch the impulse response of the system.

$H_1(s) = \frac{1}{(s+1)^2}$ $\quad$ $H_2(s) = \frac{s+2}{s+1}$ $\quad$ $H_3(s) = \frac{s-1}{(s+10)(s+1)}$ $\quad$ $H_4(s) = \frac{s}{s^2+2s+1+4\pi^2}$

5. (20 pts) (OW 9.4)

Consider the frequency response of a real, stable system shown below.

a) Explain why the number of poles must equal the number of zeros.

b) Explain why the poles and zeros must be either on the real axis or appear as complex conjugates.

c) Sketch a pole-zero diagram for a stable system (using a minimum number of poles and zeros) which would have the given frequency (magnitude) response. (The topology of pole-zero locations for this problem is more important than precise locations).

d) Sketch the phase response for this pole-zero diagram. Is the phase reponse unique?