Due at 4 pm, Fri. Oct. 31 in HW box under stairs (1st floor Cory)

1. (30 pts) DFT (Lec. 11,12,13 DFT H.O.)

Consider the signal flow diagram shown in Figure 1. For each window $w(t)$, signal $x(t)$, and sampling combination below, sketch $x(t), x_w(t), x_\delta(t), x'(t)$ and their magnitude spectra. Also sketch magnitude and phase for $X[k]$ (derived from $X'(j\omega)$).

i. Let $w(t) = \Pi(\frac{t}{T_o})$, $T_o = 8T_s$, $T_s = 1/3$ sec, $x(t) = \cos(3\pi t/2)$.

ii. Let $w(t) = \Pi(\frac{2t}{T_o})$, $T_o = 8T_s$, $T_s = 1/3$ sec, $x(t) = \cos(3\pi t/2)$.

iii. Let $w(t) = \Pi(\frac{2(t-T_o/4)}{T_o})$, $T_o = 8T_s$, $T_s = 1/3$ sec, $x(t) = \cos(3\pi t/2)$.

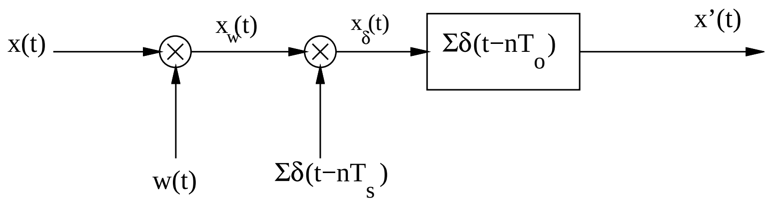


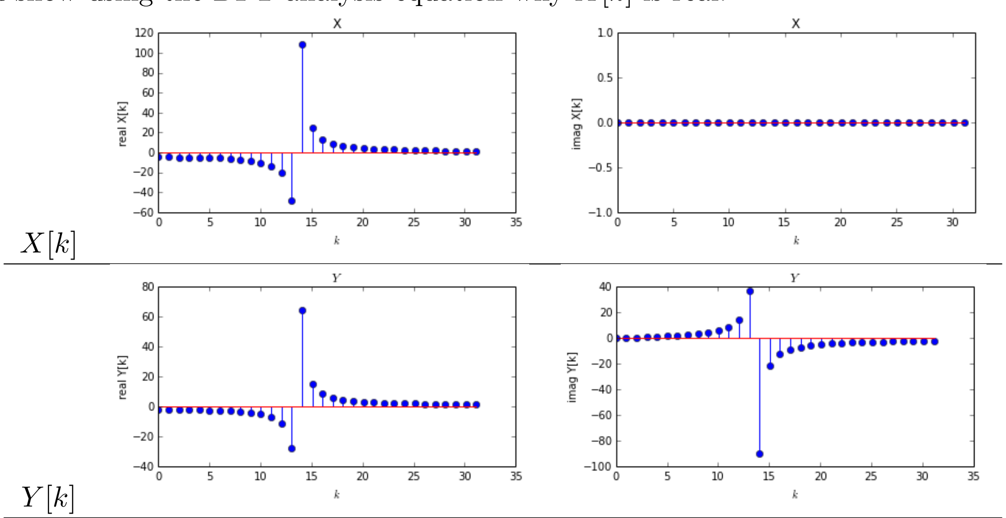
Fig. 1. DFT equivalent block diagram.

2. (20 pts) DFT (Lec. 11,12, DFT H.O.)

The DFTs of the signals $x[n] = \cos(\omega_o(\frac{nT_o}{N} - \tau))$ and $y[n] = \cos(\omega_o \frac{nT_o}{N})$ are calculated, with $\omega_o = 2\pi 13.7$, $n = 0...255$, $T_o = 1$ sec, and $N = 256$, as shown below for samples $X[0]...X[31]$, and $Y[0]...Y[31]$.

a) Using reasoning as in problem 1 above, explain the differences between the DFT of $x[n]$ and $X(j\omega)$, the FT of $x(t) = \cos(\omega_o t)$.

b) $Y[k]$ is complex. A time shift τ was used to make $X[k]$ pure real. Determine this value of τ, and show using the DFT analysis equation why $X[k]$ is real.



3. (30 pts) Lec14 OW Ch. 7

(Refer to OW Fig. 7.37). The procedure for interpolation or upsampling by an integer factor N can be thought of as a cascade of two operations. The first operation, involving system A, corresponds to inserting N-1 zero-sequence values between each sequence value of x[n], such that:
$x_p[n] = x_d[n/N]$ for $n = 0, \pm N, \pm 2N, ...$ and 0 otherwise. For exact bandlimited interpolation, $H(e^{\jmath\omega})$ is an ideal low-pass filter.

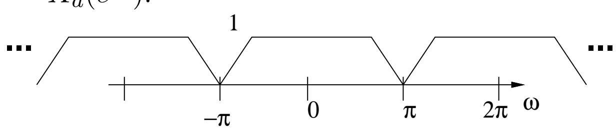
a. Determine whether or not system A is linear.

b. Determine whether or not system A is time invariant.

c. for $X_d(e^{\jmath\omega})$ as shown below, and with $N = 3$, sketch $X_p(e^{\jmath\omega})$.

d. For $N = 3$, $X_d(e^{\jmath\omega})$ as shown below, and $H(e^{j\omega})$ appropriately chosen for exact bandlimited interpolation, sketch $X(e^{j\omega})$.

$X_d(e^{\jmath\omega})$:



4. (20 pts) Upsampling, Lec 14, Ch. 7

Download `PS6-upsample.ipynb` and `music.wav` from the class web page.

The bandlimited sound sample $x[n]$ has been down sampled to 8820 Hz. Upsample back to 44.1 kHz, and use an appropriate DFT interpolation filter to create $y[n]$ by filling in missing samples.

a) Plot the magnitude of the DFT for x and y, and specify the interpolation filter $H[k]$.

b) Plot $x[8000 : 8200]$ and $y[40000 : 41000]$.

c) Save and listen to the upsampled and interpolated signal. How does it compare to the original signal?