Professor Fearing EECS120/Problem Set 2 v 1.0 Fall 2014

Due at 4 pm, Fri. Sep 19 in HW box under stairs (1st floor Cory)

Note: $comb(t) = \sum_{n=-\infty}^{\infty} \delta(t-n)$ and $\Pi(t) \;=\; u(t+\frac{1}{2}) - u(t-\frac{1}{2})$.

1. (16 pts) An echo can be modelled as a system with input $x(t)$ and output $y(t)$ such that $y(t) = x(t) + \alpha y(t-T)$, where each echo is delayed by T and scaled by α.
a) Find the impulse response for this systems, assuming $x(t) = 0, y(t) = 0$ for $t < 0$.
b) Show that the system is stable if $0 < \alpha < 1$ and unstable if $\alpha > 1$.
c) Find an LTI inverse $g(t)$ such that $y(t) * g(t) = x(t)$.

2. (12 pts) An LTI system with input $x(t)$ and output $y(t)$ is described by the LDE:

$$\frac{d^2y(t)}{dt^2} + 300\frac{dy(t)}{dt} + 2 \times 10^4 y(t) = 10^3 \frac{dx(t)}{dt} \tag{1}$$

a. Draw a block diagram realization of this system using multiply by constant, summation blocks, and the minimum number of integration blocks.
b. What is the response of the system described by the LDE in eq.(**??**) to a complex exponential input $x(t) = e^{j\omega t}$? (Hint: $e^{j\omega t}$ is an eigenfunction).

3. (16 pts) An LTI system with input $x[n]$ and output $y[n]$ is described by the LDE:

$$y[n] + 20y[n-1] + 1700y[n-2] = x[n] + 20x[n-1] \tag{2}$$

a. Draw a block diagram realization of this system using multiply by constant, summation blocks, and the minimum number of delay blocks.
b. What is the response of the system described by the LDE in eq.(**??**) to a complex exponential input $x[n] = e^{j\omega n}$? (Hint: $e^{j\omega n}$ is an eigenfunction).

4. (24 pts) Calculate the Fourier Series for the following signals. That is, find the complex scaling coefficients a_k and fundamental frequency $\omega_o = \frac{2\pi}{T_o}$. Sketch the line spectrum (a_k vs. ω) for each signal. Note that $\delta(at) = \frac{1}{a}\delta(t)$.
a. $\Pi(t/8) * comb(0.1t)$
b. $\Pi(4t) * comb(0.1t)$
c. $\Pi(t/2) * \Pi(t-1) * comb(0.1t)$

5. (16 pts) Calculate the Discrete Fourier Series for the following signals. Find complex scaling coefficients a_k for each signal.
a. $x[n] = \delta[n-1] + \delta[n-2]$ with $N = 8$.
b. $x[n] = (-1)^n$ with $N = 32$.

6. (16 pts) Compute the Fourier Series ($x(t) = \sum_k a_k e^{jk\omega_o t}$) for the following periodic functions and then determine the time average power in the signal. Let $h(t) = e^{-120\pi t}u(t)$.
a) $x_1(t) = \cos(120\pi t)$ b) $x_2(t) = |\cos(120\pi t)|$
c) $x_3(t) = x_1(t) * h(t)$ d) $x_4(t) = x_2(t) * h(t)$