EE120 Fall 2014 PS #10

① $$H(z) = \frac{(1+0.9z^{-1})(1-3\sqrt{2}\,z^{-1}+9z^{-2})}{(1+0.8z^{-1})(1-0.8z^{-1})}$$

$$= \frac{\frac{1}{z}(z+0.9)\,\frac{1}{z^2}\left(z^2-3\sqrt{2}\,z+9\right)}{\frac{1}{z^2}\,(z+0.8)(z-0.8)}$$

$$z^2-3\sqrt{2}\,z+9=0 \Rightarrow z = \frac{3\sqrt{2}\pm\sqrt{18-36}}{2}$$

$$z = \frac{3\sqrt{2}}{2}(1\pm j) = 3e^{\pm j\pi/4}$$

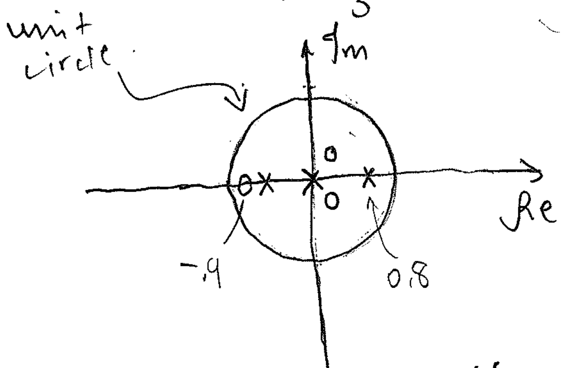
$$\therefore H(z) = \frac{(z+0.9)(z-3e^{j\pi/4})(z-3e^{-j\pi/4})}{z(z+0.8)(z-0.8)}$$

Only zeros outside unit circle are @ $3e^{\pm j\pi/4}$

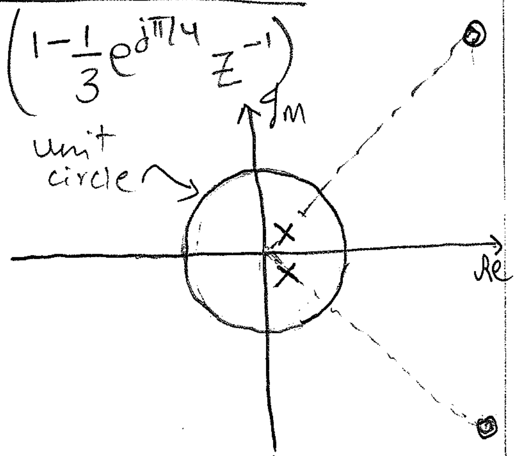
$$\therefore H(z) = \frac{(z+.9)}{z(z+.8)(z-.8)}\left[\left(z-\frac{1}{3}e^{\frac{-j\pi}{4}}\right)\left(z-\frac{1}{3}e^{\frac{j\pi}{4}}\right)\right]\left[\frac{(z-3e^{j\pi/4})(z-3e^{-j\pi/4})}{\left(z-\frac{1}{3}e^{-j\pi/4}\right)\left(z-\frac{1}{3}e^{j\pi/4}\right)}\right]$$

$$\therefore H_{min}(z) = 9 \times \frac{(1+0.9z^{-1})\left(1-\frac{1}{3}e^{-j\pi/4}z^{-1}\right)\left(1-\frac{1}{3}e^{\frac{j\pi}{4}}z^{-1}\right)}{(1+0.8z^{-1})(1+0.8z^{-1})}$$

$$H_{ap}(z) = \frac{\left(z^{-1}-\frac{1}{3}e^{-j\pi/4}\right)\left(z^{-1}-\frac{1}{3}e^{j\pi/4}\right)}{\left(1-\frac{1}{3}e^{-j\pi/4}z^{-1}\right)\left(1-\frac{1}{3}e^{j\pi/4}z^{-1}\right)}$$



Hmin(z)



Hap(z)

② a) salary $s[n] = 100000\,(1.05)^n u[n]$ $\qquad s[-1]=0$

investment $b[n] = (.05\,s[n] + 1.07\,b[n-1])u[n]$ $\qquad b[-1]=0,$

$$= 5000\,(1.05)^n u[n] + 1.07\,b[n-1]u[n]$$

$$B(z) = 5000\,\frac{1}{1-1.05z^{-1}} + 1.07\left[\cancel{b[-1]}^{\,0} + z^{-1}B(z)\right]$$ note unstable (a good thing for \$)

$$B(z) = \frac{5000}{(1-1.05z^{-1})(1-1.07z^{-1})}$$

$$= 5000\left[\frac{-52.5}{1-1.05z^{-1}} + \frac{53.5}{1-1.07z^{-1}}\right]$$

$$b[n] = 5000(-52.5)(1.05)^n u[n] + 5000(53.5)(1.07)^n u[n]$$

$$b[40] \approx 2{,}157{,}670$$

b) Same as if starting salary was $100{,}000\,(1.05)^{10}$ and evaluating salary at $n=30$:

$$s[n] = \underbrace{100{,}000\,(1.05)^{10}}_{\beta}\,(1.05)^n u[n]$$

$$b[n] = (x\,s[n] + 1.07\,b[n-1])\,u[n]$$

$$= \beta x\,(1.05)^n u[n] + 1.07\,b[n-1]u[n]$$

$$B(z) = \beta x\cdot\frac{1}{1-1.05z^{-1}} + 1.07\left(\cancel{b[-1]}^{\,0} + z^{-1}B(z)\right)$$

$$= \frac{\beta x}{(1-1.05z^{-1})(1-1.07z^{-1})}$$

$$= \frac{-52.5\beta x}{1-1.05z^{-1}} + \frac{53.5\beta x}{1-1.07z^{-1}}$$

$$b[n] = -52.5\beta x\,(1.05)^n u[n] + 53.5\beta x\,(1.07)^n u[n]$$

$$b[30] = -52.5\cdot 100{,}000\,(1.05)^{10}(1.05)^{30}\cdot x + 53.5\cdot 100{,}000\,(1.05)^{10}\cdot(1.07)^{30}x$$

$$\approx 29{,}377{,}212\,x = 2{,}157{,}670$$

$$x \approx 7.3\%$$

note $(1.05)^{40} = 7.04$

$(1.07)^{40} = 15.0$

$(1.073)^{30} = 8.28$

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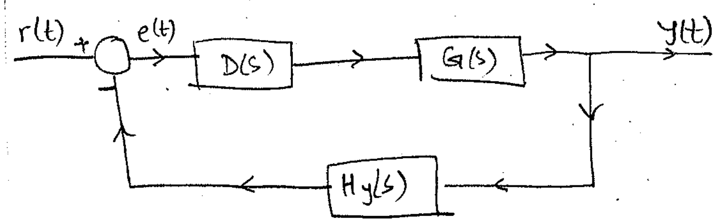
③



we set $w(t) = 0$ for steady-state.

$D(s) = 1$; $G(s) = \frac{10(s+5)}{s(s+2)}$; $H_y(s) = \frac{8}{s+6}$

$E(s) = R(s) - E(s)D(s)G(s)H_y(s)$.

$\therefore E(s) = \frac{R(s)}{1 + D(s)G(s)H_y(s)} = \frac{R(s)}{1 + \frac{10(s+5)}{(s+2)(s+6)}}$

a) 0 poles @ $s=0$ for $DGH = \frac{10(s+5)}{(s+2)(s+6)}$

$\therefore$ $\boxed{\text{Type } 0}$

b) $\lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s) = \frac{\lim_{s\to 0} sR(s)}{1 + \frac{(10)(5)}{(2)(6)}}$

$\therefore \lim_{t\to\infty} e(t) = \frac{\lim_{t\to\infty} r(t)}{1 + 50/12}$

For this expression to be constant, we need $\lim_{t\to\infty} r(t) =$ constant. So, $r(t)$ is not a ramp or a parabola. So, $r(t) \to$ step

Check: set $r(t) =$ ramp $\to R(s) = 1/s^2$.

Then,

$\lim_{t\to\infty} e(t) = \left(\lim_{s\to 0} \frac{1}{s}\right) \frac{1}{1 + 50/12} \to \infty$.

c) Let $\lim_{t\to\infty} r(t) = A$. Then,

$\lim_{t\to\infty} e(t) = \frac{A}{1 + \frac{50}{12}} = \frac{12}{62} A = \frac{6}{31} A$.

④ a) $E(s) = R(s) - Y(s)H_y(s) = -Y(s) + R(s) = R(s) - DGE(s)$

$Y(s) = G(s)\cdot\left[W(s) + E(s)D(s)\right]$

so $-E(s) = G(s)W(s) + G(s)E(s)D(s)$

$$E(s) = \frac{-G(s)W(s)}{1+G(s)D(s)} = \frac{\frac{-1}{s^2+2\zeta\omega_n s+\omega_n^2}\cdot\frac{1}{s}}{1+\frac{K_P+\frac{K_I}{s}+K_D s}{s^2+2\zeta\omega_n s+\omega_n^2}}$$ ← from step

$$\lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s) = \lim_{s\to 0}\frac{-s}{(K_P s+K_I+K_D s^2)+(s^3+2\zeta\omega_n s^2+\omega_n^2 s)} = \boxed{0}$$

b) $E(s) = R(s) - Y(s)$

$Y(s) = G(s)\left[E(s)D(s) + \cancel{W(s)}\right]$ 0

so $E(s) = R(s) - G(s)E(s)D(s)$

$$E(s) = \frac{R(s)}{1+G(s)D(s)}$$

When $R(s) = \frac{1}{s}$: $$E(s) = \frac{1/s}{1+\frac{K_P+\frac{K_I}{s}+K_D s}{s^2+2\zeta\omega_n s+\omega_n^2}}$$ ← step

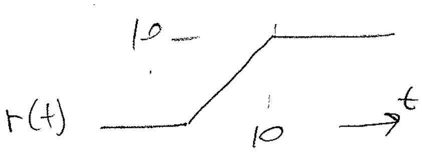
$$\lim_{s\to 0} sE(s) = \lim_{s\to 0}\frac{s^3+2\zeta\omega_n s^2+\omega_n^2 s}{s^3+2\zeta\omega_n s^2+\omega_n^2 s+K_P s+K_I+K_D s^2} = 0$$

$$\lim_{t\to\infty} e(t) = 0$$

when $R(s) = \frac{1}{s^2}$: $$\lim_{s\to 0} sE(s) = \lim_{s\to 0}\frac{s^2+2\zeta\omega_n s+\omega_n^2}{s^3+2\zeta\omega_n s^2+\omega_n^2 s+K_P s+K_I+K_D s^2}$$ ← Ramp

$$= \frac{\omega_n^2}{K_I}$$

$$\lim_{t\to\infty} e(t) = \frac{\omega_n^2}{K_I}$$



c) We want the system to have $\lim_{t\to\infty} e(t) = 0$ for reference step, and minimal error for a (temporary) reference ramp. From part b, we see that this can be achieved with $D(s) = \frac{K_I}{s}$ with large K_I. An integrator will work well for the controller, assuming stability.

d) We would like to reject a step disturbance so that $\lim_{t\to\infty} e(t) = 0$ when $w(t) = u(t)$. From part a, we see that this can be achieved when $K_I \neq 0$. When $D(s) = \frac{K_I}{s}$, we can asymptotically reject a step disturbance. Also, this controller has the benefit of asymptotically tracking a reference step.