

1. (15 pts) All-pass filter (Lec 22)

A discrete time causal LTI system has transfer function: $H(z) = \frac{(1+0.9z^{-1})(1-3\sqrt{2}z^{-1}+9z^{-2})}{1-0.64z^{-2}}$
 Find the minimum phase system $H_{min}(z)$ and an all-pass system $H_{ap}(z)$ such that $H(z) = H_{min}(z)H_{ap}(z)$ and plot the respective pole-zero diagrams.

2. (15 pts) Z transform review

At year $n = 0$, Bob has an annual salary of $s[0] = \$100,000$. Bob's salary increases by 5% per year. He can invest with a return of 7%.

- a. Starting at year $n = 0$ Bob invests 5% of his gross salary annually. How much does Bob have at $n = 40$?
- b. Instead of starting at $n = 0$, starting at year $n = 10$, Bob invests $x\%$ of his gross salary. What x is needed to obtain the same result as in part a. at $n = 40$?

3. (15 pts) Steady State Error (Lec 22, 23)

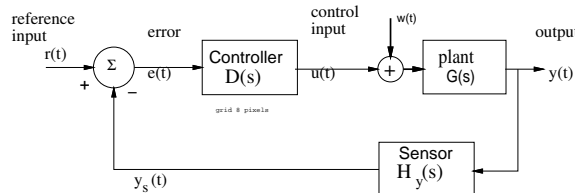
For the system below, let $D(s) = 1$, $G(s) = \frac{10(s+5)}{s(s+2)}$, and $H_y(s) = \frac{s}{s+6}$. Assume $w(t) = 0$.

- a) What is the system type?
- b) What input waveform $r(t)$ would yield a constant error? (e.g. step, ramp, parabola, or ?)
- c) Assuming stability, what is the steady state error for a unit input of $r(t)$ from b)?

4. (25 pts) PID control

For the system below, let $D(s) = k_p + \frac{k_I}{s} + k_D s$, $H_y(s) = 1$, and $G(s) = \frac{1}{s^2 + 2\zeta\omega_n s + \omega_n^2}$.

- a) With $r(t) = 0$, determine the trend of $e(t)$ as $t \rightarrow \infty$ with respect to step disturbance input $W(s)$.
 - b) With $w(t) = 0$, determine the trend of $e(t)$ as $t \rightarrow \infty$ with respect to step and ramp reference inputs $R(s)$.
- Consider an elevator control system, where $G(s)$ represents the transfer function for the motor control system.
- c) A reference command to move between floors can be given by $r(t) = t[u(t) - u(t - 10)] + 10u(t - 10)$. What type of controller $D(s)$ should be used, and why?
 - d) 10 people get on the elevator at $t = 0$. What type of controller $D(s)$ should be used, and why?



5. (30 pts) Compensation using All-pass filter (Lec.22)

This problem is intended to be more open-ended, and you will need to use several signal processing tools to answer the question. For this problem a skeleton iPython notebook `PythonPS10-Prob5-Question.ipynb` will be provided on the class web page, and data files. You are given a channel $H(z)$ (with input $x[n]$ and output $y[n]$) which has 2 minimum-phase zeros (at unknown location) and 2 poles at $z = 0.95$. The file `sig1-filt.wav` has been processed by this channel, and you are asked to try to recover the original signal $x[n]$ by applying a filter to $y[n]$.

- a. You are given an input-output pair of signals `sig2-orig.wav` and `sig2-filt.wav` for this channel. From this information, estimate the zero locations and $H(z)$. Describe the method used, e.g. with appropriate plots. (Hint `ipython np.fft.fft()`).
- b. Calculate an approximate inverse filter $H_c(z)$ such that $|H(e^{j\omega})H_c(e^{j\omega})| = 1$.
- c. Find the LDE corresponding to $H_c(z)$.
- d. Apply the LDE for $H_c(z)$ to `sig1-filt.wav` and plot the input signal $y[n]$, and output signal $c[n]$ in the range from $n = 80000 \dots 84000$.
- e. Compare audio quality for filtered and recovered playback. Be sure to check the scale of your operations so that signals are in the range ± 32767 to avoid clipping.