Professor Fearing EECS120/Problem Set 1 v 1.01 Fall 2014

Due at 4 pm, Fri. Sep. 12 in HW box under stairs (1st floor Cory)

Reading: O&W Ch 1, Ch2.

Note: $\Pi(t) = u(t+\frac{1}{2}) - u(t-\frac{1}{2})$, and $r(t) = tu(t)$ where $u(t)$ is the unit step.

1. (15 pts) What are signals and systems? For each example below, identify the input x, system $H\{\}$, and output y. Which of (x, H, y) are known *a priori*, which would need to be calculated or designed? *Example: NASDAQ market manipulation. Answer: Input: buy and sell orders. System: dealer network plus investor behavior plus order book. Output: share price. System known, output directly measured, design input to control price.*

a) radiosurgery b) medical magnetic resonance imaging
c) reflection seismology d) a digital camera
e) Facebook user experiments (6/14)

2. (18 pts) For $f(t) = [u(t) - r(t-1)]u(2-t)$, sketch:

a) $f(t)$ b) $f(\frac{t}{2})$
c) $f(\frac{3}{2}t+1)$ d) $f(-\frac{3}{2}t+1)$
e) $f(3[-\frac{1}{2}t+1])$ f) $f(t)\Pi(t-1)$

3. (10 pts) Show that the following functions have the two properties of the unit impulse, i.e. #1: $\delta(t-t_o) = 0$ for t not equal t_o and #2: $\int_{-\infty}^{\infty} \delta(t)dt = 1$

a) $\delta_1(t) = \lim_{a\to\infty} \frac{a}{2} e^{-a|t|}$

b) $\delta_2(t) = \lim_{a\to 0} \frac{1}{2a}\Pi(\frac{t-a}{2a})$

4. (12 pts) Complex review. Given $z = x + jy = re^{j\theta}$. Derive the following relations:

a. $zz^* = r^2$ b. $\frac{z}{z^*} = e^{j2\theta}$
c. $(z_1 z_2)^* = z_1^* z_2^*$ d. $(\frac{z_1}{z_2})^* = \frac{z_1^*}{z_2^*}$

5. (8 pts) Express the following as the sum of an odd and an even function, and sketch the functions: a) $u(t)$ b) $\cos(2\pi t)u(t)$

6. (20 pts) For each of the following impulse responses, determine whether the system is BIBO stable. If the system is not BIBO stable, find a bounded input $x(t)$ or $x[n]$ which gives an unbounded output, and show that the output is unbounded for this input.

a. $h(t) = e^t u(t)$ b. $h(t) = (t-1)^2 e^{(1-t)} u(t)$
c. $h[n] = u[n-4]$ d. $h[n] = \cos[2\pi n]u[n]$
e. $h(t) = \sum_{n=-\infty}^{\infty} \delta(t-2n)$

7. (18 pts) Compute and sketch the output $y(t) = x(t) * h(t)$ for the following input and impulse response pairs:

a. $x(t) = \Pi(t-1)$ $h(t) = r(t)$
b. $x(t) = e^{-t}u(t)$ $h(t) = \Pi(t-\frac{1}{2})$
c. $x(t) = \sum_{n=-\infty}^{\infty} \delta(t-\frac{1}{2}-n)$ $h(t) = \Pi(t)\sin(2\pi t)$