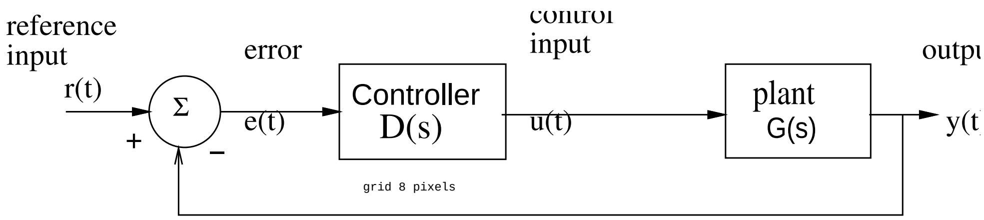

Consider the case of the unit step, $r(t) = u(t)$ and $R(s) = \frac{1}{s}$.
The error for reference input $r(t)$ is given as:

$$E(s) = \frac{1}{1 + D(s)G(s)} R(s) \tag{1}$$

Assume that $E(s)$ has no right-half plane poles. Consider the number of poles at $s = 0$ for $D(s)G(s)$.

Type 0 has zero poles at $s = 0$, Type 1 has 1 pole, etc.

1 DG has zero poles at $s = 0$

Type 0
The openloop system is:

$$D(s)G(s) = \frac{N(s)}{\Pi_i(s + \alpha_i)}$$

where $Re(\alpha_i) > 0$. Here the denominator of $D(s)G(s)$ has been factored into first order terms (with α_i possibly appearing as complex conjugates).

The error response for the closed-loop system with input $r(t) = u(t)$ is:

$$E(s) = \frac{1}{s} \cdot \frac{1}{1 + \frac{N(s)}{\Pi_i(s+\alpha_i)}} = \frac{1}{s} \cdot \frac{\Pi_i(s + \alpha_i)}{\Pi_i(s + \alpha_i) + N(s)} \tag{2}$$

Applying the final value theorem:

$$\lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s) = \frac{s}{s} \cdot \frac{\Pi_i(s + \alpha_i)}{\Pi_i(s + \alpha_i) + N(s)} \tag{3}$$

$$= \frac{\Pi_i(\alpha_i)}{\Pi_i(\alpha_i) + N(0)} \neq 0 \tag{4}$$

where it was assumed there are no closed-loop poles introduced at $s = 0$.

2 DG has one pole at $s = 0$

Type 1
The openloop system is:

$$D(s)G(s) = \frac{N(s)}{s\Pi_i(s + \alpha_i)}$$

where $Re(\alpha_i) > 0$.

The error response for the closed-loop system with input $r(t) = u(t)$ is:

$$E(s) = \frac{1}{s}\frac{1}{1 + \frac{N(s)}{s\Pi_i(s+\alpha_i)}} = \frac{1}{s} \cdot \frac{s\Pi_i(s+\alpha_i)}{s\Pi_i(s+\alpha_i) + N(s)} \tag{5}$$

Applying the final value theorem:

$$\lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s) = \lim_{s\to 0} \frac{s\Pi_i(s+\alpha_i)}{s\Pi_i(s+\alpha_i) + N(s)} \tag{6}$$

$$= \frac{0 \cdot \Pi_i(\alpha_i)}{0 \cdot \Pi_i(\alpha_i) + N(0)} = 0 \tag{7}$$

3 DG has two poles at $s = 0$

Type 2
The openloop system is:

$$D(s)G(s) = \frac{N(s)}{s^2\Pi_i(s+\alpha_i)}$$

where $Re(\alpha_i) > 0$.

The error response for the closed-loop system with input $r(t) = u(t)$ is:

$$E(s) = \frac{1}{s}\frac{1}{1 + \frac{N(s)}{s^2\Pi_i(s+\alpha_i)}} = \frac{1}{s} \cdot \frac{s^2\Pi_i(s+\alpha_i)}{s^2\Pi_i(s+\alpha_i) + N(s)} \tag{8}$$

Applying the final value theorem:

$$\lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s) = \lim_{s\to 0} \frac{s^2\Pi_i(s+\alpha_i)}{s^2\Pi_i(s+\alpha_i) + N(s)} \tag{9}$$

$$= \frac{0 \cdot \Pi_i(\alpha_i)}{0 \cdot \Pi_i(\alpha_i) + N(0)} = 0 \tag{10}$$

where it was assumed there are no closed-loop poles introduced at $s = 0$. Unless $N(s)$ has a zero at $s = 0$, the ROC will include $s = j\omega$, and hence the final value theorem is valid to apply.

The Type 2 system will also have zero steady state error for the step input.