Chapter 4

Digital communication

A digital signal is a discrete-time binary signal m : *Integers* $\to$ *Bin* $= \{0, 1\}$. To transmit such a signal it must first be transformed into a baseband analog signal. The baseband signal is then transmitted as such or modulated using techniques of Chapter 3. Many schemes can be modeled as shown in figure 4.1.

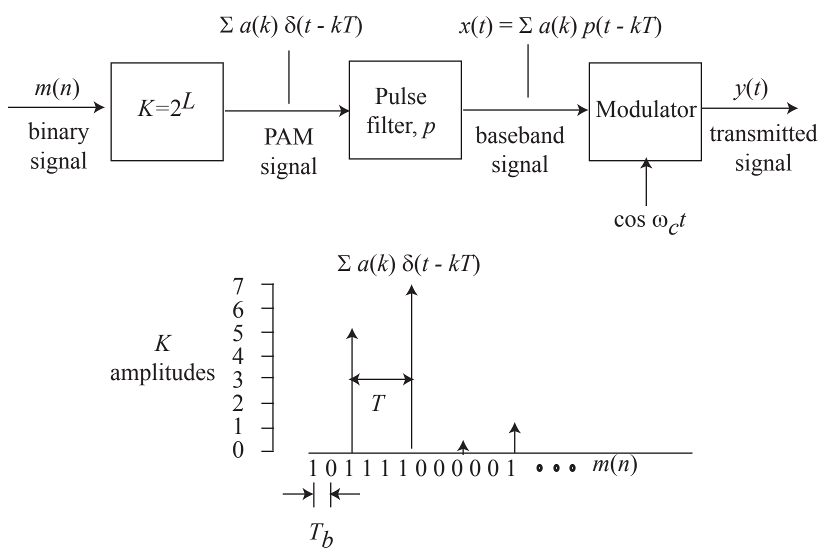

Figure 4.1: The binary signal m is converted into an impulse train and then into an analog baseband signal x which is then modulated.

The binary sequence m is divided into L-bit blocks or symbols. Each of the $K = 2^L$ symbols is mapped into an amplitude. (The figure shows the case $L = 3$.) If the time between two bits is T_b sec (the bit rate is $R_b = T_b^{-1}$ bits/sec), the time between two symbols is $T = LT_b$. The symbol or baud rate is $R = T^{-1} = L^{-1}R_b$ baud/sec. The resulting symbol sequence $\{a(k)\}$ modulates a train of pulses of the same shape p. This can be represented as a convolution of $\sum_n a(n)\delta(t - nT)$ and the impulse response p. The resulting analog signal

$$\forall t, \quad x(t) = \sum_k a(k)p(t - kT)$$

is called a PAM (pulse amplitude modulated) signal. It can be transmitted directly (as in a baseband modem) or it can be used to modulate a carrier following one of the schemes discussed in the last chapter.

Binary signaling

Here $L = 1$ so there are only two symbols,-1 and +1 representing 1 and 0. Take the pulse shape to be a constant, $p(t) = 1, 0 < t < T$. So if $a(k) \in \{-1, +1\}$ is the kth symbol, the baseband signal during the kth symbol time is

$$\forall t, \quad x(t) = a(k), \quad (k-1)T \leq t < kT,$$

as shown on the top in figure 4.2 for the case where $\{a_k\}$ alternates between -1 and +1.

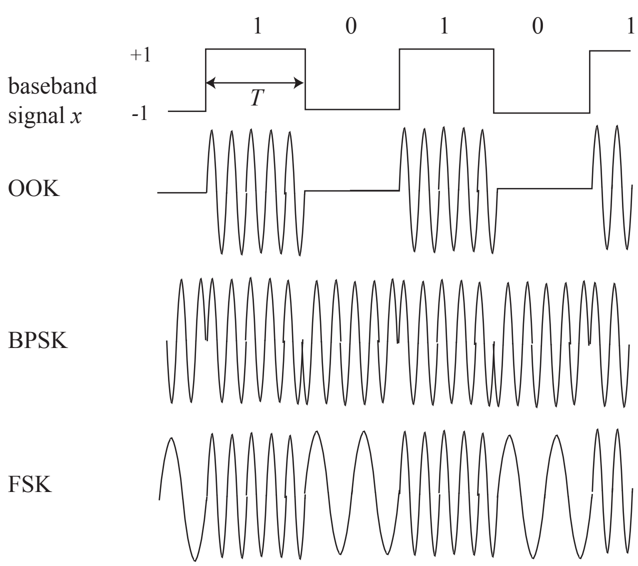

Figure 4.2: The binary signal $1010\cdots$, the baseband signal x, and the waveform produced using OOK, BPSK and FSK.

OOK In *on-off* keying or OOK, the modulation scheme is AM, so the transmitted bandpass signal is

$$\forall t, \quad y(t) = [1 + x(t)] \cos \omega_c t,$$

as shown. OOK is used in optical communication: the laser is turned on for the duration of a bit time T to signal '1' and turned off for the same amount of time to signal '0'.

BPSK In *binary phase-shift* keying or BPSK, phase modulation is used, so the transmitted bandpass signal is

$$\forall t, \quad y(t) = \cos(\omega_c t + \beta x(t)).$$

In figure 4.2, $\beta = \pi/2$. So $x(t) = 1$ is transmitted as $\cos(\omega_c t + \pi/2) = -\sin\omega_c t$ and $x(t) = -1$ is transmitted as $\cos(\omega_c t - \pi/2) = \sin\omega_c t$. BPSK is used to transfer data over coaxial cable.

FSK In *frequency shift* keying or FSK, frequency modulation is used so the transmitted bandpass signal is

$$\forall t, \quad y(t) = \cos(\omega_c + x(t)\omega_0)t.$$

So $x(t) = 1$ is transmitted as a sinusoid of frequency $\omega_c + \omega_0$, and $x(t) = -1$ is transmitted as a sinusoid of frequency $\omega_0 - \omega_0$.

Multilevel signaling

Each block of L bits is mapped into $K = 2^L$ complex amplitudes, $R_l e^{j\theta_l}, l = 1, \cdots, 2^L$, arranged symmetrically in the complex plane. The arrangement is called a *constellation*. The pulse shape is again a constant, $p(t) = 1, 0 < t < T$. So if $a(k) \in \{R_l e^{j\theta_l}\}$ is the kth symbol, the complex baseband signal during the kth sumbol time is

$$\forall t, \quad x(t) = a(k), \quad , (k-1)T \le t < kT.$$

QAM A 16-level *quadrature amplitude* modulation or QAM constellation is shown on the left in figure 4.3.

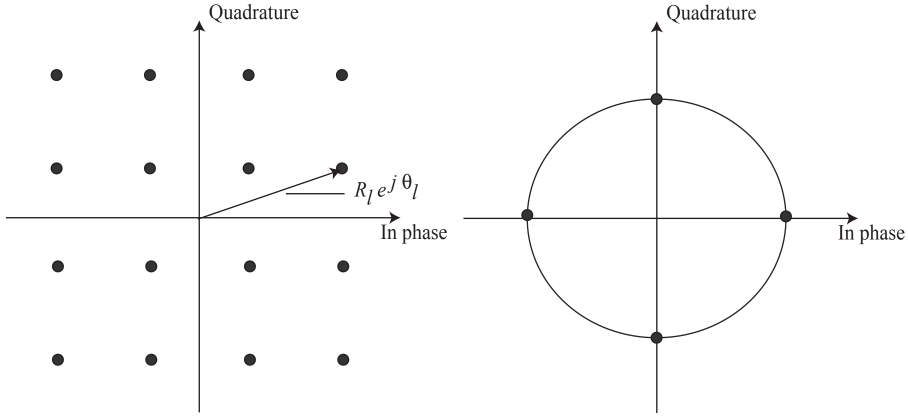

Figure 4.3: 16-QAM constellation on the left, QPSK constellation on the right.

The QAM modulated signal is

$$\begin{aligned}\forall t, \quad y(t) &= \mathit{Re}\{x(t)e^{j\omega_c t}\} \\ &= x_c(t)\cos\omega_c t + x_s(t)\sin\omega_c t.\end{aligned}$$

So the symbol $x(t) = R_l e^{j\theta_l}$ is transmitted as

$$\begin{aligned} y(t) = \textbf{\textit{Re}}\{R_l e^{j(\omega_c t + \theta_l)}\} &= R_l \cos\theta_l \cos\omega_c t - R_l \sin\theta_l \sin\omega_c t \\ &= x_c(t)\cos\omega_c t + x_s(t)\sin\omega_c t. \end{aligned}$$

The waveform x_c is called the *in-phase* component and x_s is called the *quadrature* component.

Transmission of digital data downstream over a 6 MHz cable TV channel using 64 QAM can achieve a 28 Mbps bit rate, for a spectral efficiency of 10.76 bits/Hz. The symbol rate is 28/64 = 437.5 kilobaud/sec.

The QAM modulator is shown in figure 4.4. Note the resemblance to the AM-SSB modulator. The demodulator is similar.

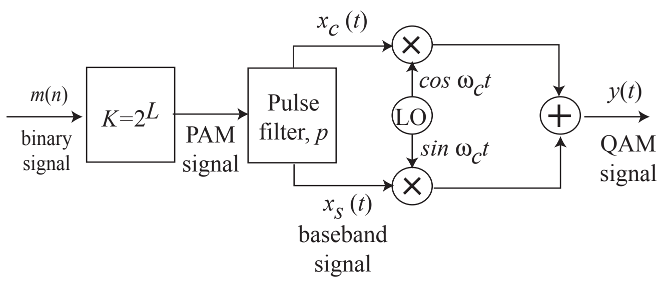

Figure 4.4: The QAM modulator. LO is the local oscillator which produces the carrier signal.

QPSK *Quadrature phase-shift* keying or K-ary phase-shift keying is similar to QAM with $K = 2^L$ levels, except that the amplitudes $\{R_l e^{j\theta_l}$ have the same magntiudes $R_l equiv 1$, say. Thus the information is contained in the phase. One possible 4-level QPSK constellation is shown in the right of figure 4.3.

Spectral efficiency

The bandpass signal y in the schemes above is centered around the carrier frequency ω_c. The bandwidth of the signal is the same as that of the baseband signal

$$forallt, \quad x(t) = \sum a(k)p(t - kT),$$

and it depends on the symbol sequence $\{a(k)\}$ and the shape of the pulse p.

Consider the case of binary signaling, with the symbol sequence alternating between +1 and -1, $a(k) = (-1)^k$. Suppose the pulse is a squarewave as in figure 4.2. So the baseband signal is periodic with period $2T$:

$$\forall t, \quad x(t) = (-1)^k, \quad (k-1)T \le t < kT.$$

This periodic signal has a Fourier series, say,

$$\forall t, \quad x(t) = \sum_{-\infty}^{\infty} P_n e^{jn\omega_0 t},$$

and a FT X,

$$\forall\omega, \quad X(\omega) = 2\pi \sum_{-\infty}^{\infty} P_n \delta(\omega - n\omega_0).$$

Here $\omega_0 = 2\pi/T$. We can define the bandwidth of x as follows. Let N be the smallest number of harmonics that contain 95% of the power,

$$\frac{\sum_{-N}^{N} |P_n|^2}{\sum_{-\infty}^{\infty} |P_n|^2} \ .$$

The 95% bandwidth of the baseband signal x is defined as $W_{95} = 2\pi \times 2N\omega_0$ Hz. This will be the bandwidth of the modulated signal.

Per unit time, this modulated signal carries $b = 1/T$ bits of information. So the **spectral efficiency** if this modulation scheme is b/W_{95} bits/sec/Hz. The spectral efficiency depends on the shape of the pulse p.