Notes 15 largely plagiarized by %khc

1 AM Single Sideband (AM-SSB)

AM-SSB is more bandwidth-efficient than AM-DSB methods, consuming only the bandwidth required to send the message. There are two methods of generating SSB signals: one using a high pass or low pass filter, and the other using a Hilbert transformer.

2 Hilbert Transform

As previously derived in notes 07, $\mathcal{F}[\operatorname{sgn} t] = \frac{2}{j\omega}$. So by duality:

$$\mathcal{F}[\frac{2}{jt}] = 2\pi\operatorname{sgn}(-\omega)$$

$$\mathcal{F}[\frac{1}{\pi t}] = -j\operatorname{sgn}(\omega)$$

A filter with impulse response $\frac{1}{\pi t}$ is called a Hilbert transformer. As a brief review of the symmetry properties, note that since the impulse response is real, the frequency response exhibits conjugate symmetry, and has an even magnitude response and an odd phase response.
Exercise Verify this.

One interesting thing to note what a Hilbert transformer does to a sinusoidal input. In particular, consider $\cos\omega_c t$ as input, as in Figure 1. The area of the impulse at ω_c changes from π to $-j\pi$, while the area of the impulse at $-\omega_c$ changes from π to $j\pi$. This is the transform of $\sin\omega_c t$. Since the cosine became a sine, the Hilbert transformer is sometimes called a 90° phase shift network.
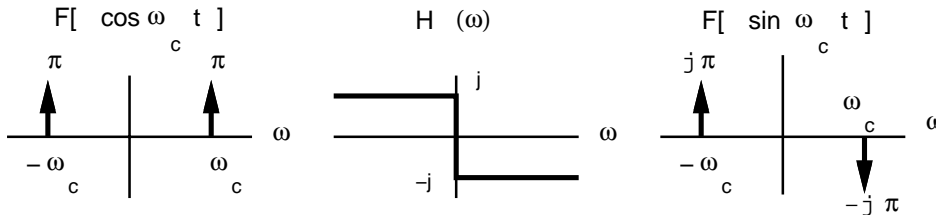

Figure 1: Cosine meets Hilbert transformer and becomes sine.

Exercise Show that the Hilbert transformer mutates $\sin\omega_c t$ into $-\cos\omega_c t$.

3 AM-SSB: Filtering Method

In the filtering method, we utilize the same setup as for AM-DSB-SC. However, instead of sending out the AM-DSB-SC signal, we can pass that signal through a high pass filter to generate an upper sideband (USB) signal, as in Figure 2. Similarly, we can pass the signal through a low pass filter to generate a lower sideband (LSB) signal.

The disadvantage of the filter method is that very sharp filters are required, "sharp" being defined as a very fast transition from the passband region (the range of frequencies which are passed by the filter) to the stopband region (the range of frequencies which are rejected by the filter).
Exercise Verify that using an LPF instead of an HPF in Figure 2 will give you LSB.

4   AM-SSB: Hilbert Transform Method

If we subtract the Hilbert transformed message modulated by a sine from the original message modulated by a cosine, as in Figure 3, we end up with AM-SSB USB. The time domain analysis of this is quite messy, but in the frequency domain, things are much simpler, as illustrated in Figure 4.

Note that if the negative sign on the summer is changed to a positive sign, AM-SSB LSB is generated.
Exercise Verify this.

5   AM-SSB Demodulation with Phase Lock

AM-SSB cannot be demodulated by the asynchronous demodulator. However, we can still use the synchronous demodulator in Figure 5 to recover the signal.

If the carrier and the oscillator are perfectly synchronized, we have phase lock, so then $y(t) = \cos \omega_c t$. We then end up with the frequency domain waveforms in Figure 6.

6   AM-SSB Demodulation without Phase Lock

What happens if there is no phase lock though? Assume that the oscillator in the synchronous demodulator of Figure 5 is out of phase from the carrier by θ. So

$$
\begin{aligned}
y(t) &= \cos(\omega_c t + \theta) \\
&= \cos \omega_c t \cos \theta - \sin \omega_c t \sin \theta
\end{aligned}
$$

In the frequency domain, we end up with the waveforms in Figure 7. The signals at baseband are what we are interested in, since those are the output of the LPF. Note that there are no values of θ for which the message can completely degrade to zero. If the imaginary part of the output of the LPF is gone, we still have the real part. If the real part of the LPF output is gone, we can still recover the message by Hilbert transforming the imaginary part.
Exercise Verify this.

7   Quadrature Amplitude Modulation (QAM)

There is another way of using the same amount of bandwidth as AM-SSB without the hassle of using a Hilbert transform.

Let's consider sending the message $x(t) - jy(t)$. We'll modulate it with $e^{j\omega_c t}$ and take the real part before sending it off.

$$
\begin{aligned}
\mathcal{R}e\left[(x(t) - jy(t))e^{j\omega_c t}\right] &= \mathcal{R}e\left[(x(t) - jy(t))(\cos \omega_c t + j \sin \omega_c t\right] \\
&= \mathcal{R}e\left[x(t) \cos \omega_c t + jx(t) \sin \omega_c t + jy(t) \cos \omega_c t + y(t) \sin \omega_c t\right] \\
&= x(t) \cos \omega_c t + y(t) \sin \omega_c t
\end{aligned}
$$

This suggests that if we realize this equation, we could possibly get a workable modulation scheme.
Exercise Verify that the QAM setup in Figure 8 works; that is, if we send $x(t)$ and $y(t)$, we can recover both signals. Verify that this setup is as bandwidth-efficient as AM-SSB.
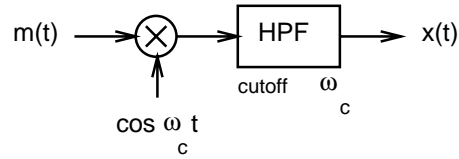
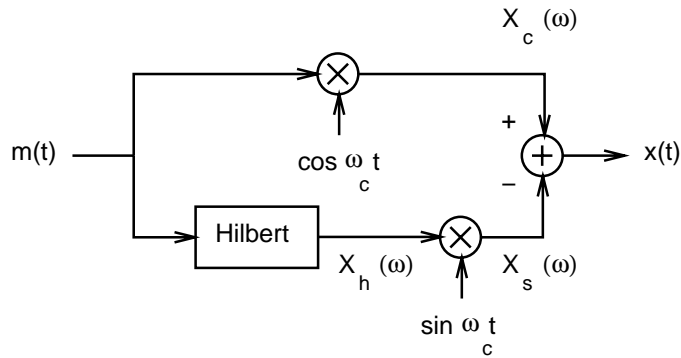Figure 2: AM-SSB USB transmitter using filter method.

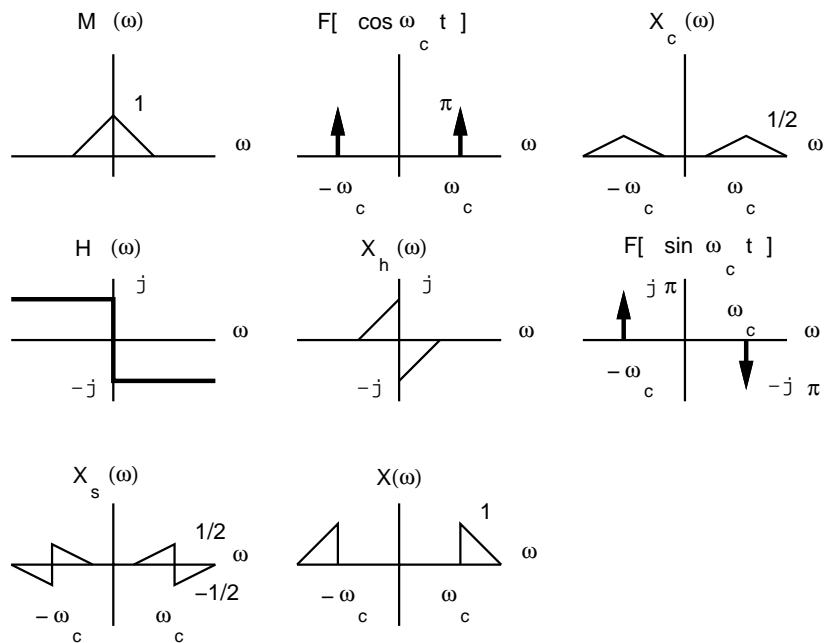Figure 3: AM-SSB USB transmitter using Hilbert transform method.

Figure 4: AM-SSB USB transmitter analysis in the frequency domain.
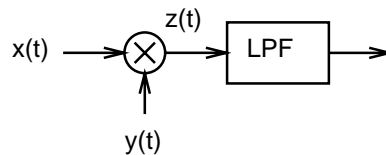

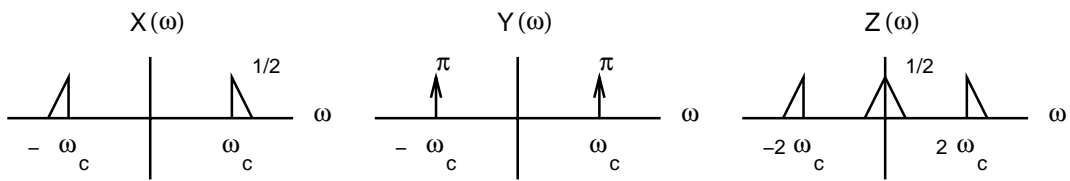Figure 5: Synchronous demodulator.

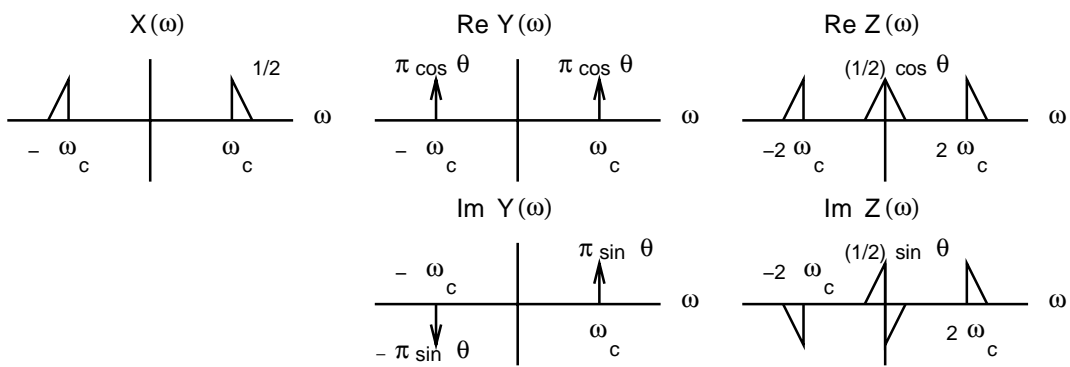Figure 6: Frequency domain analysis of synchronous demodulator in phase lock.

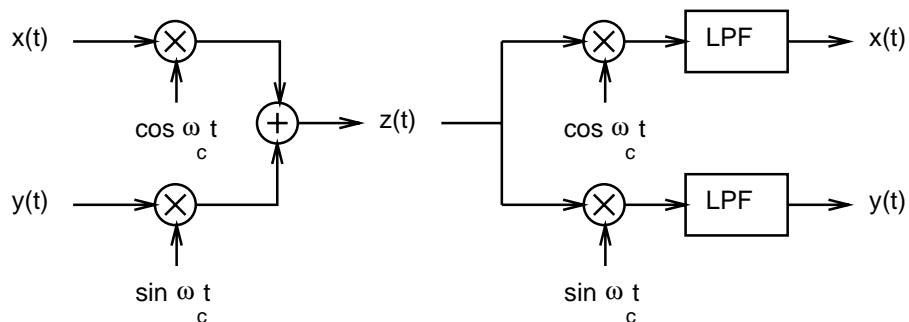Figure 7: Frequency domain analysis of synchronous demodulator without phase lock.

Figure 8: QAM transmitter and receiver.