PROBLEMS

1.1 Find the even and odd components of each of the following signals:

(a) $x(t) = \cos(t) + \sin(t) + \sin(t)\cos(t)$

(b) $x(t) = 1 + t + 3t^2 + 5t^3 + 9t^4$

(c) $x(t) = 1 + t\cos(t) + t^2\sin(t) + t^3\sin(t)\cos(t)$

(d) $x(t) = (1 + t^3)\cos^3(10t)$

1.2 Determine whether the following signals are periodic. If they are periodic, find the fundamental period.

(a) $x(t) = (\cos(2\pi t))^2$

(b) $x(t) = \Sigma_{k=-5}^{5} w(t - 2k)$ for $w(t)$ depicted in Fig. P1.2b.

(c) $x(t) = \Sigma_{k=-\infty}^{\infty} w(t - 3k)$ for $w(t)$ depicted in Fig. P1.2b.

(d) $x[n] = (-1)^n$

(e) $x[n] = (-1)^{n^2}$

(f) $x[n]$ depicted in Fig. P1.2f.

(g) $x(t)$ depicted in Fig. P1.2g.

(h) $x[n] = \cos(2n)$

(i) $x[n] = \cos(2\pi n)$

1.3 The sinusoidal signal

$$x(t) = 3\cos(200t + \pi/6)$$

is passed through a square-law device defined by the input–output relation

$$y(t) = x^2(t)$$

Using the trigonometric identity

$$\cos^2\theta = \tfrac{1}{2}(\cos 2\theta + 1)$$

show that the output $y(t)$ consists of a dc component and a sinusoidal component.

(a) Specify the dc component.

(b) Specify the amplitude and fundamental frequency of the sinusoidal component in the output $y(t)$.

1.4 Categorize each of the following signals as an energy or power signal, and find the energy or power of the signal.

(a) $x(t) = \begin{cases} t, & 0 \le t \le 1 \\ 2 - t, & 1 \le t \le 2 \\ 0, & \text{otherwise} \end{cases}$

(b) $x[n] = \begin{cases} n, & 0 \le n \le 5 \\ 10 - n, & 5 \le n \le 10 \\ 0, & \text{otherwise} \end{cases}$

(c) $x(t) = 5\cos(\pi t) + \sin(5\pi t), \quad -\infty < t < \infty$

(d) $x(t) = \begin{cases} 5\cos(\pi t), & -1 \le t \le 1 \\ 0, & \text{otherwise} \end{cases}$

(e) $x(t) = \begin{cases} 5\cos(\pi t), & -0.5 \le t \le 0.5 \\ 0, & \text{otherwise} \end{cases}$

(f) $x[n] = \begin{cases} \sin(\pi n), & -4 \le n \le 4 \\ 0, & \text{otherwise} \end{cases}$

(g) $x[n] = \begin{cases} \cos(\pi n), & -4 \le n \le 4 \\ 0, & \text{otherwise} \end{cases}$

(h) $x[n] = \begin{cases} \cos(\pi n), & n \ge 0 \\ 0, & \text{otherwise} \end{cases}$

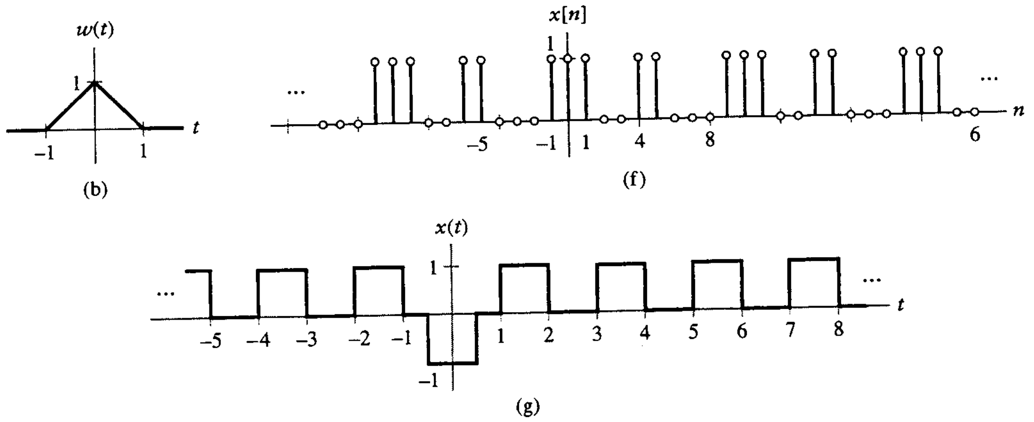


FIGURE P1.2

1.5 Consider the sinusoidal signal

$$x(t) = A\cos(\omega t + \phi)$$

Determine the average power of $x(t)$.

1.6 The angular frequency Ω of the sinusoidal signal

$$x[n] = A\cos(\Omega n + \phi)$$

satisfies the condition for $x[n]$ to be periodic. Determine the average power of $x[n]$.

1.7 The raised-cosine pulse $x(t)$ shown in Fig. P1.7 is defined as

$$x(t) = \begin{cases} \frac{1}{2}[\cos(\omega t) + 1], & -\pi/\omega \le t \le \pi/\omega \\ 0, & \text{otherwise} \end{cases}$$

Determine the total energy of $x(t)$.

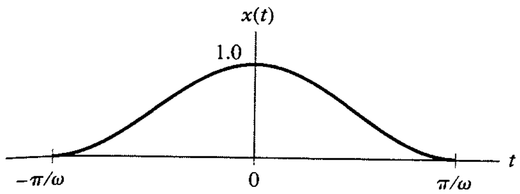


FIGURE P1.7

1.8 The trapezoidal pulse $x(t)$ shown in Fig. P1.8 is defined by

$$x(t) = \begin{cases} 5 - t, & 4 \le t \le 5 \\ 1, & -4 \le t \le 4 \\ t + 5 & -5 \le t \le -4 \\ 0, & \text{otherwise} \end{cases}$$

Determine the total energy of $x(t)$.

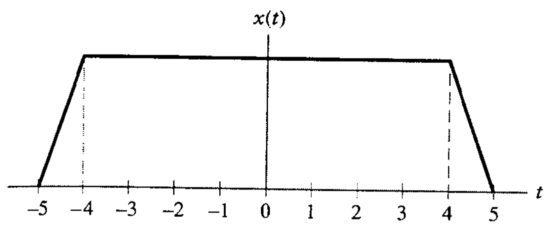


FIGURE P1.8

1.9 The trapezoidal pulse $x(t)$ of Fig. P1.8 is applied to a differentiator, defined by

$$y(t) = \frac{d}{dt}x(t)$$

(a) Determine the resulting output $y(t)$ of the differentiator.

(b) Determine the total energy of $y(t)$.

1.10 A rectangular pulse $x(t)$ is defined by

$$x(t) = \begin{cases} A, & 0 \le t \le T \\ 0, & \text{otherwise} \end{cases}$$

The pulse $x(t)$ is applied to an integrator defined by

$$y(t) = \int_0^t x(\tau)\,d\tau$$

Find the total energy of the output $y(t)$.

1.11 The trapezoidal pulse $x(t)$ of Fig. P1.8 is time scaled, producing

$$y(t) = x(at)$$

Sketch $y(t)$ for (a) $a = 5$ and (b) $a = 0.2$.

1.12 A triangular pulse signal $x(t)$ is depicted in Fig. P1.12. Sketch each of the following signals derived from $x(t)$:

(a) $x(3t)$

(b) $x(3t + 2)$

(c) $x(-2t - 1)$

(d) $x(2(t + 2))$

(e) $x(2(t - 2))$

(f) $x(3t) + x(3t + 2)$

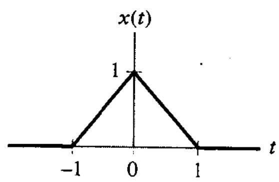


FIGURE P1.12

1.13 Sketch the trapezoidal pulse $y(t)$ that is related to that of Fig. P1.8 as follows:

$$y(t) = x(10t - 5)$$

1.14 Let $x(t)$ and $y(t)$ be given in Figs. P1.14(a) and (b), respectively. Carefully sketch the following signals:

(a) $x(t)y(t - 1)$

(b) $x(t - 1)y(-t)$

(c) $x(t + 1)y(t - 2)$

(d) $x(t)y(-1 - t)$

(e) $x(t)y(2 - t)$

(f) $x(2t)y(\frac{1}{2}t + 1)$

(g) $x(4 - t)y(t)$

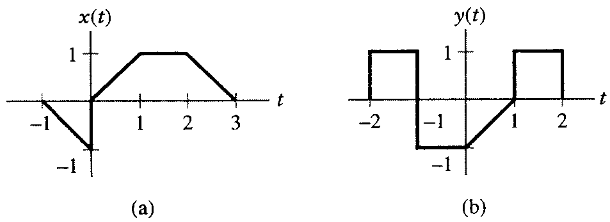


FIGURE P1.14

1.15 Figure P1.15(a) shows a staircase-like signal $x(t)$ that may be viewed as the superposition of four rectangular pulses. Starting with the rectangular pulse $g(t)$ shown in Fig. P1.15(b), construct this waveform, and express $x(t)$ in terms of $g(t)$.

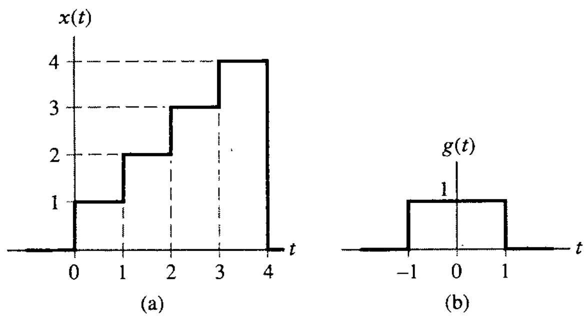


FIGURE P1.15

1.16 Sketch the waveforms of the following signals:

(a) $x(t) = u(t) - u(t-2)$

(b) $x(t) = u(t+1) - 2u(t) + u(t-1)$

(c) $x(t) = -u(t+3) + 2u(t+1) - 2u(t-1) + u(t-3)$

(d) $y(t) = r(t+1) - r(t) + r(t-2)$

(e) $y(t) = r(t+2) - r(t+1) - r(t-1) + r(t-2)$

1.17 Figure P1.17(a) shows a pulse $x(t)$ that may be viewed as the superposition of three rectangular pulses. Starting with the rectangular pulse $g(t)$ of Fig. P1.17(b), construct this waveform, and express $x(t)$ in terms of $g(t)$.

1.18 Let $x[n]$ and $y[n]$ be given in Figs. P1.18(a) and (b), respectively. Carefully sketch the following signals:

(a) $x[2n]$

(b) $x[3n-1]$

(c) $y[1-n]$

(d) $y[2-2n]$

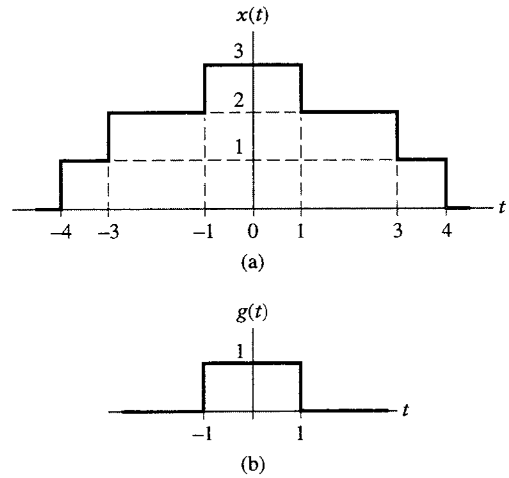


FIGURE P1.17

(e) $x[n-2] + y[n+2]$

(f) $x[2n] + y[n-4]$

(g) $x[n+2]y[n-2]$

(h) $x[3-n]y[n]$

(i) $x[-n]y[-n]$

(j) $x[n]y[-2-n]$

(k) $x[n+2]y[6-n]$

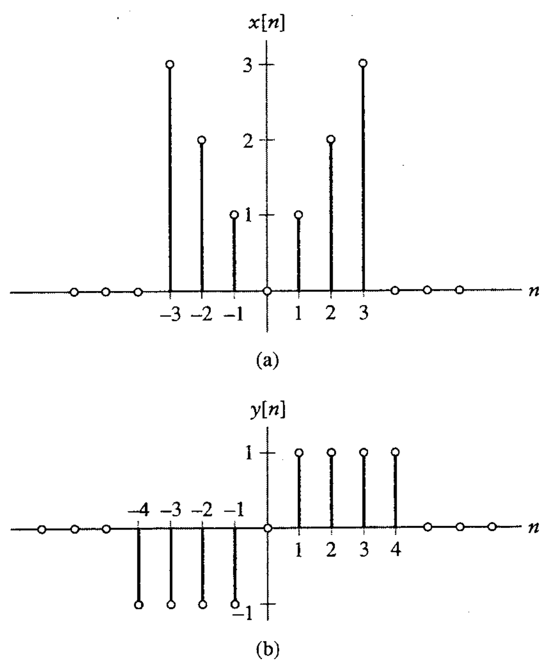


FIGURE P1.18

1.19 Consider the sinusoidal signal

$$x[n] = 10\cos\left(\frac{4\pi}{31}n + \frac{\pi}{5}\right)$$

Determine the fundamental period of $x(n)$.

1.20 The sinusoidal signal $x[n]$ has fundamental period $N = 10$ samples. Determine the smallest angular frequency Ω for which $x[n]$ is periodic.

1.21 Determine whether the following signals are periodic. If they are periodic, find the fundamental period.

(a) $x[n] = \cos(\frac{8}{15}\pi n)$
(b) $x[n] = \cos(\frac{7}{15}\pi n)$
(c) $x(t) = \cos(2t) + \sin(3t)$
(d) $x(t) = \sum_{k=-\infty}^{\infty}(-1)^k\delta(t - 2k)$
(e) $x[n] = \sum_{k=-\infty}^{\infty}\{\delta[n - 3k] + \delta[n - k^2]\}$
(f) $x(t) = \cos(t)u(t)$
(g) $x(t) = v(t) + v(-t)$, where $v(t) = \cos(t)u(t)$
(h) $x(t) = v(t) + v(-t)$, where $v(t) = \sin(t)u(t)$
(i) $x[n] = \cos(\frac{1}{5}\pi n)\sin(\frac{1}{3}\pi n)$

1.22 A complex sinusoidal signal $x(t)$ has the following components:

$$\mathrm{Re}\{x(t)\} = x_R(t) = A\cos(\omega t + \phi)$$
$$\mathrm{Im}\{x(t)\} = x_I(t) = A\sin(\omega t + \phi)$$

The amplitude of $x(t)$ is defined by the square root of $x_R^2(t) + x_I^2(t)$. Show that this amplitude equals A, independent of the phase angle ϕ.

1.23 Consider the complex-valued exponential signal

$$x(t) = Ae^{\alpha t + j\omega t}, \qquad \alpha > 0$$

Evaluate the real and imaginary components of $x(t)$.

1.24 Consider the continuous-time signal

$$x(t) = \begin{cases} t/T + 0.5, & -T/2 \le t \le T/2 \\ 1, & t \ge T/2 \\ 0, & t < -T/2 \end{cases}$$

which is applied to a differentiator. Show that the output of the differentiator approaches the unit impulse $\delta(t)$ as T approaches zero.

1.25 In this problem, we explore what happens when a unit impulse is applied to a differentiator. Consider a triangular pulse $x(t)$ of duration T and amplitude $2T$, as depicted in Fig. P1.25. The area under the pulse is unity. Hence as the duration T approaches zero, the triangular pulse approaches a unit impulse.

(a) Suppose the triangular pulse $x(t)$ is applied to a differentiator. Determine the output $y(t)$ of the differentiator.

(b) What happens to the differentiator output $y(t)$ as T approaches zero? Use the definition of a unit impulse $\delta(t)$ to express your answer.

(c) What is the total area under the differentiator output $y(t)$ for all T? Justify your answer.

Based on your findings in parts (a) to (c), describe in succinct terms the result of differentiating a unit impulse.

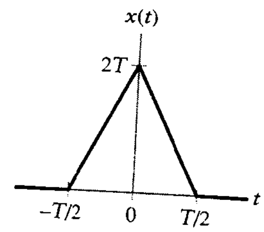


FIGURE P1.25

1.26 The derivative of impulse function $\delta(t)$ is referred to as a *doublet*. It is denoted by $\delta'(t)$. Show that $\delta'(t)$ satisfies the sifting property

$$\int_{-\infty}^{\infty}\delta'(t - t_0)\,f(t)\,dt = -f'(t_0)$$

where

$$f'(t_0) = \left.\frac{d}{dt}f(t)\right|_{t=t_0}$$

Assume that the function $f(t)$ has a continuous derivative at time $t = t_0$.

1.27 A system consists of several subsystems connected as shown in Fig. P1.27. Find the operator H relating $x(t)$ to $y(t)$ for the subsystem operators given by:

$H_1: y_1(t) = x_1(t)x_1(t - 1)$
$H_2: y_2(t) = |x_2(t)|$
$H_3: y_3(t) = 1 + 2x_3(t)$
$H_4: y_4(t) = \cos(x_4(t))$

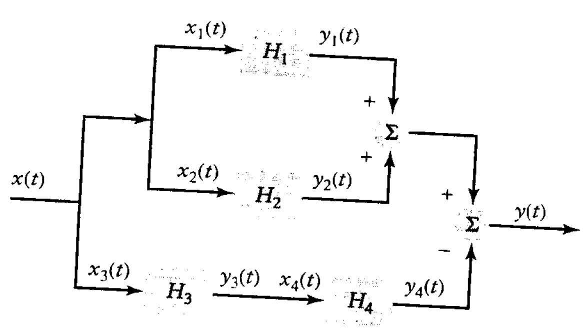
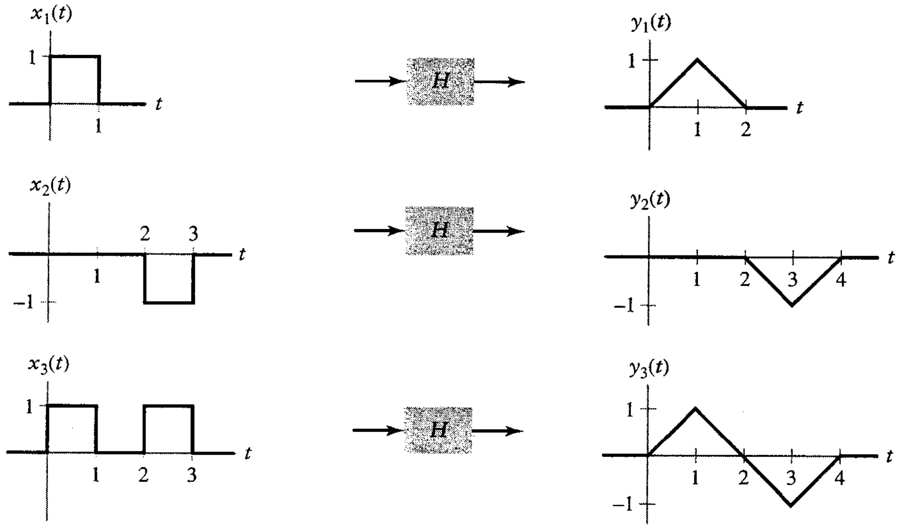
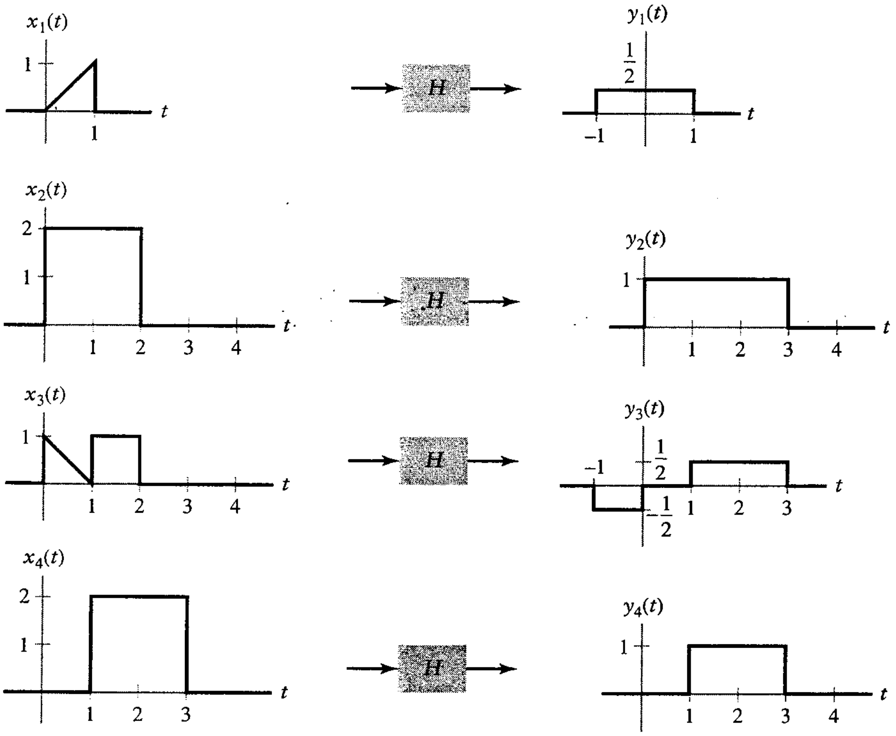


FIGURE P1.27



(a)



(b)

FIGURE P1.39

1.28 The systems given below have input $x(t)$ or $x[n]$ and output $y(t)$ or $y[n]$, respectively. Determine whether each of them is (i) memoryless, (ii) stable, (iii) causal, (iv) linear, and (v) time invariant.

(a) $y(t) = \cos(x(t))$

(b) $y[n] = 2x[n]u[n]$

(c) $y[n] = \log_{10}(|x[n]|)$

(d) $y(t) = \int_{-\infty}^{t/2} x(\tau)\, d\tau$

(e) $y[n] = \sum_{k=-\infty}^{n} x[k+2]$

(f) $y(t) = \frac{d}{dt}x(t)$

(g) $y[n] = \cos(2\pi x[n+1]) + x[n]$

(h) $y(t) = \frac{d}{dt}\{e^{-t}x(t)\}$

(i) $y(t) = x(2-t)$

(j) $y[n] = x[n]\sum_{k=-\infty}^{\infty} \delta[n-2k]$

(k) $y(t) = x(t/2)$

(l) $y[n] = 2x[2^n]$

1.29 The output of a discrete-time system is related to its input $x[n]$ as follows:

$$y[n] = a_0x[n] + a_1x[n-1] + a_2x[n-2] + a_3x[n-3]$$

Let the operator S^k denote a system that shifts the input $x[n]$ by k time units to produce $x[n-k]$. Formulate the operator H for the system relating $y[n]$ to $x[n]$. Hence develop a block diagram representation for H, using (a) cascade implementation and (b) parallel implementation.

1.30 Show that the system described in Problem 1.29 is BIBO stable for all a_0, a_1, a_2, and a_3.

1.31 How far does the memory of the discrete-time system described in Problem 1.29 extend into the past?

1.32 Is it possible for a noncausal system to possess memory? Justify your answer.

1.33 The output signal $y[n]$ of a discrete-time system is related to its input signal $x[n]$ as follows:

$$y[n] = x[n] + x[n-1] + x[n-2]$$

Let the operator S denote a system that shifts its input by one time unit.

(a) Formulate the operator H for the system relating $y[n]$ to $x[n]$.

(b) The operator H^{-1} denotes a discrete-time system that is the inverse of this system. How is H^{-1} defined?

1.34 Show that the discrete-time system described in Problem 1.29 is time invariant, independent of the coefficients a_0, a_1, a_2, and a_3.

1.35 Is it possible for a time-variant system to be linear? Justify your answer.

1.36 Show that an Nth power-law device defined by the input–output relation

$$y(t) = x^N(t), \quad N \text{ integer and } N \neq 0, 1$$

is nonlinear.

1.37 A linear time-invariant system may be causal or noncausal. Give an example for each one of these two possibilities.

1.38 Figure 1.50 shows two equivalent system configurations on condition that the system operator H is linear. Which of these two configurations is simpler to implement? Justify your answer.

1.39 A system H has its input–output pairs given. Determine whether the system could be memoryless, causal, linear, and time invariant for (a) signals depicted in Fig. P1.39(a) and (b) signals depicted in Fig. P1.39(b). For all cases, justify your answers.

1.40 A linear system H has the input–output pairs depicted in Fig. P1.40(a). Determine the following and explain your answers:

(a) Is this system causal?

(b) Is this system time invariant?

(c) Is this system memoryless?

(d) Find the output for the input depicted in Fig. P1.40(b).

1.41 A discrete-time system is both linear and time invariant. Suppose the output due to an input $x[n] = \delta[n]$ is given in Fig. P1.41(a).

(a) Find the output due to an input $x[n] = \delta[n-1]$.

(b) Find the output due to an input $x[n] = 2\delta[n] - \delta[n-2]$.

(c) Find the output due to the input depicted in Fig. P1.41(b).

▸ Computer Experiments

1.42 Write a set of MATLAB commands for approximating the following continuous-time periodic waveforms:

(a) Square wave of amplitude 5 volts, fundamental frequency 20 Hz, and duty cycle 0.6.

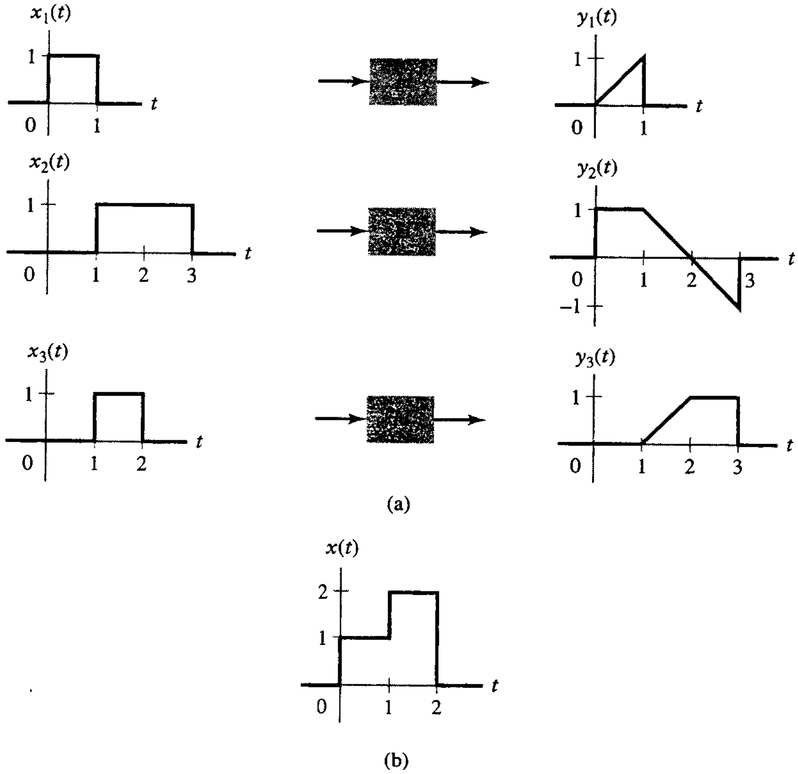


FIGURE P1.40

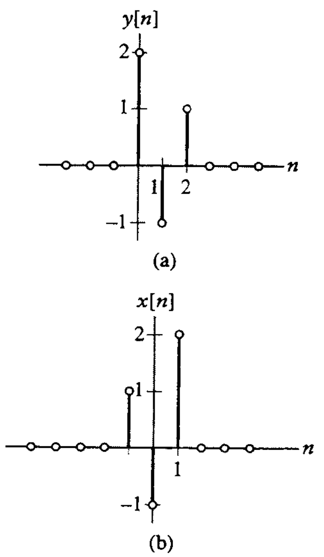


FIGURE P1.41

(b) Sawtooth wave of amplitude 5 volts, and fundamental frequency 20 Hz.

Hence plot five cycles of each of these two waveforms.

1.43 (a) The solution to a linear differential equation is given by

$$x(t) = 10e^{-t} - 5e^{-0.5t}$$

Using MATLAB, plot $x(t)$ versus t for `t = 0:0.01:5`.

(b) Repeat the problem for

$$x(t) = 10e^{-t} + 5e^{-0.5t}$$

1.44 An exponentially damped sinusoidal signal is defined by

$$x(t) = 20 \sin(2\pi \times 1000t - \pi/3) \exp(-at)$$

where the exponential parameter a is variable; it takes on the following set of values: $a = 500$, 750, 1000. Using MATLAB, investigate the effect of varying a on the signal $x(t)$ for $-2 \leq t \leq 2$ milliseconds.

1.45 A raised-cosine sequence is defined by

$$w[n] = \begin{cases} \cos(2\pi F n), & -1/2F \leq n \leq 1/2F \\ 0, & \text{otherwise} \end{cases}$$

Use MATLAB to plot $w[n]$ versus n for $F = 0.1$.

1.46 A rectangular pulse $x(t)$ is defined by

$$x(t) = \begin{cases} 10, & 0 \leq t \leq 5 \\ 0, & \text{otherwise} \end{cases}$$

Generate $x(t)$ using:

(a) A pair of time-shifted step functions.

(b) An M-file.