CS 70 FALL 2006 — DISCUSSION #6

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1. Administrivia

(1) Course Information
- The 7^{th} homework is due October 13^{th} at 4pm in 283 Soda Hall.
- HW#4 is graded and ready to hand back today
- See the GSI after section to pick up your mid-term if you haven't already

2. Eulerian Graphs

Theorem 1. *An undirected (respectively directed) graph has an Eulerian tour iff every non-isolated vertex has even degree (respectively iff every vertex has equal in- and out-degree).*

Exercise 1. What is an easy procedure of adding edges to use to make an initial binary tree to have an Eulerian Tour?

Exercise 2. For a general tree, at least how many edges do you need to add to the tree to make it have an Eulerian Tour? Can you always create an Eulerian Tour by adding edges to a tree?

3. Hypercubes

Theorem 2. *A graph is good (an expander graph) if in order to separate n vertices from the rest of the graph you must cut n edges (for $n < \frac{1}{2}|V|$).*

Theorem 3. *In H_n (the n-hypercube) to isolate any set S of vertices $|S| \leq 2^{n-1}$ you need to cut $|E_S| \geq |S|$ edges.*

Exercise 3. If you have two hypercubes of the same size, how many edges do you need to add between them in order to make the graph good (an expander graph)?

Exercise 4. When can a hypercube also have an Eulerian Tour?

4. Counting

Exercise 5. Joe is forming a soccer team of 11 players. He wants to pick his team from his friends: David, Lorenzo, Ben, Christos, Umesh, Peter, Satish, James, Juliet, Shahar and Luca. How many choices does Joe have? What if he can additionally have Mike and Alistair on the team?

Exercise 6. What if Joe is now also choosing the numbers (from 1 to 11) of the players. How does this change things?

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