Problem Set 13

1. Power Law Distributions

   (a) Show that the power law has infinite infinite mean iff $a \leq 2$.

   (b) Show that the power law has infinite variance iff $a \leq 3$.

2. A paradox in conditional probability?
   Here is some on-time arrival data for two airlines, A and B, into the airports of Los Angeles and Chicago. (Predictably, both airlines perform better in LA, which is subject to less flight congestion and less bad weather.)

	Airline A		Airline B	
	# flights	# on time	#flights	# on time
Los Angeles	600	534	200	188
Chicago	250	176	900	685

   (a) Which of the two airlines has a better chance of arriving on time into Los Angeles? What about Chicago?

   (b) Which of the two airlines has a better chance of arriving on time overall?

   (c) Do the results of parts (a) and (b) surprise you? Explain the apparent paradox, and interpret it in terms of conditional probabilities.

3. Cardinality
   Recall that a real number which is not a rational number is called irrational. Are the irrationals countably infinite or uncountably infinite? Prove your answer. (You may use the fact that the reals are uncountable).

4. Bijections
   Show that the set of real numbers $\{x : 0 < x \leq 1\}$ has the same cardinality as the set of real numbers $\{x : x \geq 1\}$.

5. Undecidability
   We say that two programs are equivalent if they give the same output on every input. Prove that it is impossible to write a computer program that takes as input two pieces of code $code_1$ and $code_2$ and tests whether the two programs are equivalent.