Amdahl's Law: Parallelization Economics

CS 61c, Nov. 30, 2011
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TODAY IN CS! BERKELEY POSTDOC IMPROVES UNDERSTANDING OF MATRIX MULTIPLICATION!
Virginia Vassilevska Williams used convex optimization to tighten the known worst-case upper bound on the complexity of *n*-by-*n* mat. mult. from **$O(n^{2.374})$** to **$O(n^{2.3727})$**. A great theoretical result via a method that suggests even tighter bounds exist & can be found soon!

http://www.scottaaronson.com/blog/?p=839

Parallel Processing: Familiar Obstacles

- Many hands make light work!
  - Execute instructions simultaneously
- But parallelization is haaaarrd....
  - More workers? More overhead!
  - Shared data is hard to coordinate
  - Whine whine whine whine whine

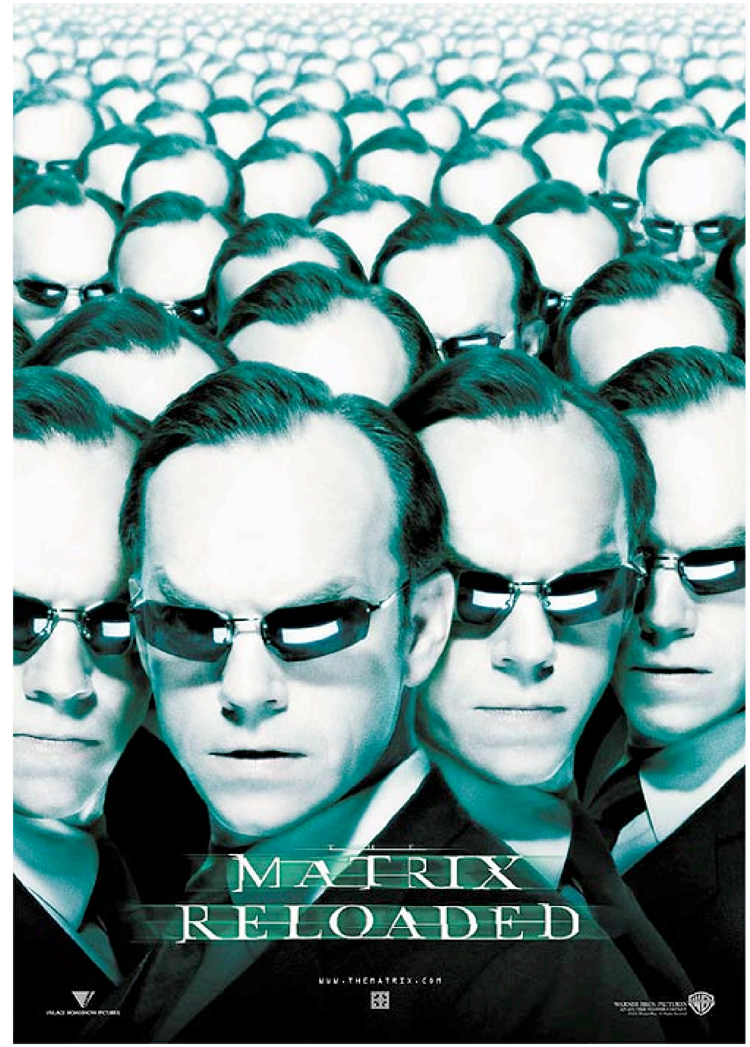
But once you *have* a parallel system...

... (after handling synchronization...

... after finding a parallel algorithm...

... after finding a memory solution...

... and after handling worker failures)...



... just add more cores forever and win!

... r-right?

Array Copying Example

```
for(i = 0; i < 100; i++) // With one core...
  y[i] = x[i];           // <-- 100 instr.
printf("DONE");          // <-- 10 instr.
```

- Takes about 110 instructions to run serially
  - Assume magical AMAT of 1 cycle
  - Assume magical cost-free 0-cycle comparator/increment
  - printf() is legacy code -- must be run serially
- **IF** we set up a successful parallelization scheme (threading?), each loop iteration could be run in parallel
  - Assume magical, no-collisions caching
  - Assume no increased work for each new thread added

Array Copying Example

```
for(i = 0; i < 100; i++)
   y[i] = x[i];
printf("DONE");          // <-- 10 instr.
```

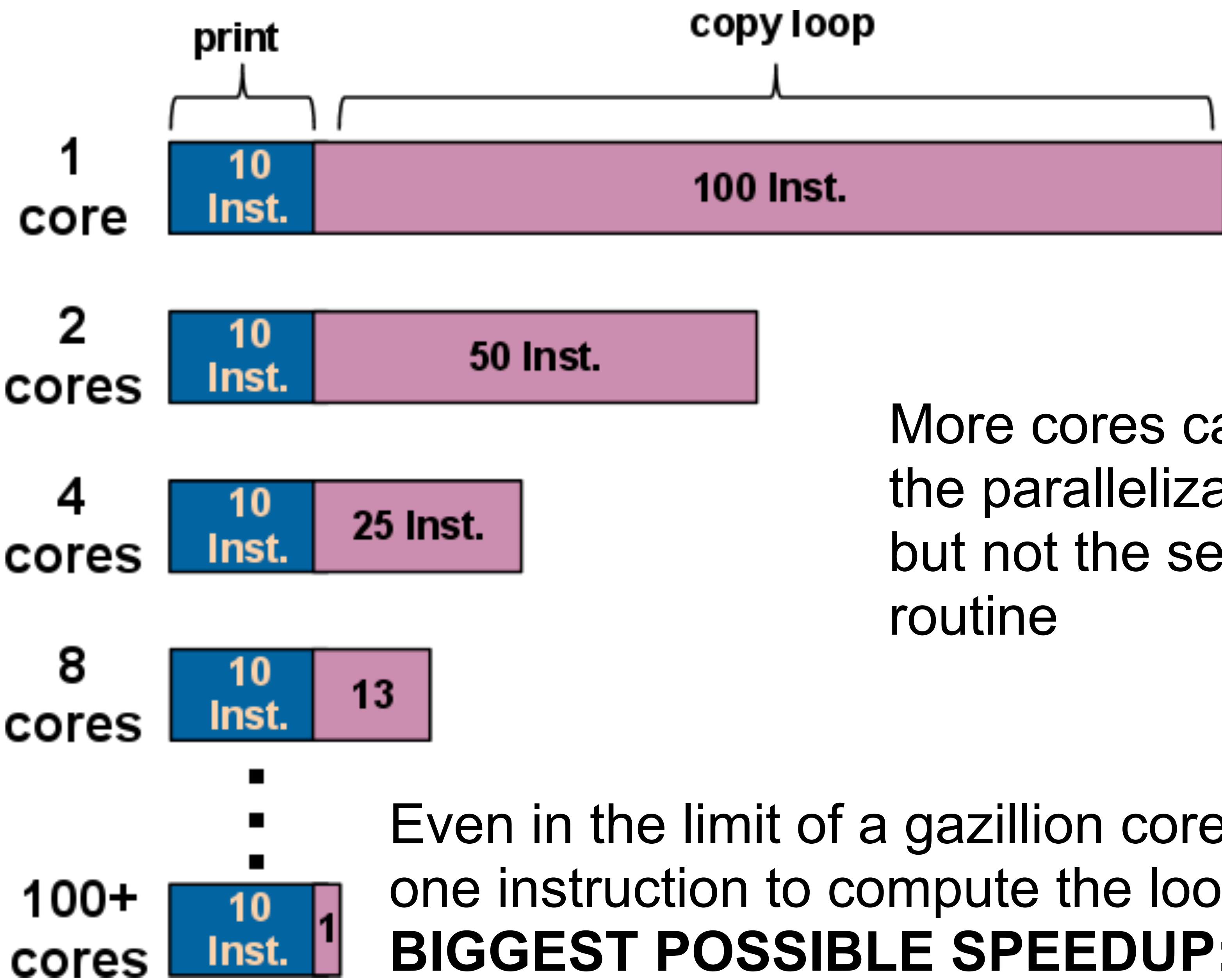
One core takes 110 instructions...

With this many cores...	... loop takes...	... printing takes...	... totaling...	... for a speedup of:
2	50 instr.	10 instr.	60 instr.	1.83x
4	25 instr.	10 instr.	35 instr.	3.14x
8	13 instr.	10 instr.	23 instr.	4.78x

2 to 4: 1.71x

4 to 8: 1.52x

Array Copying, Graphically



More cores can speed up the parallelizable copy loop, but not the serial-only print routine

Even in the limit of a gazillion cores, need at least one instruction to compute the loop.
BIGGEST POSSIBLE SPEEDUP: 110/11 = 10x

Amdahl's Law

$$f(N) = \frac{1}{(1 - P) + \frac{P}{N}}$$

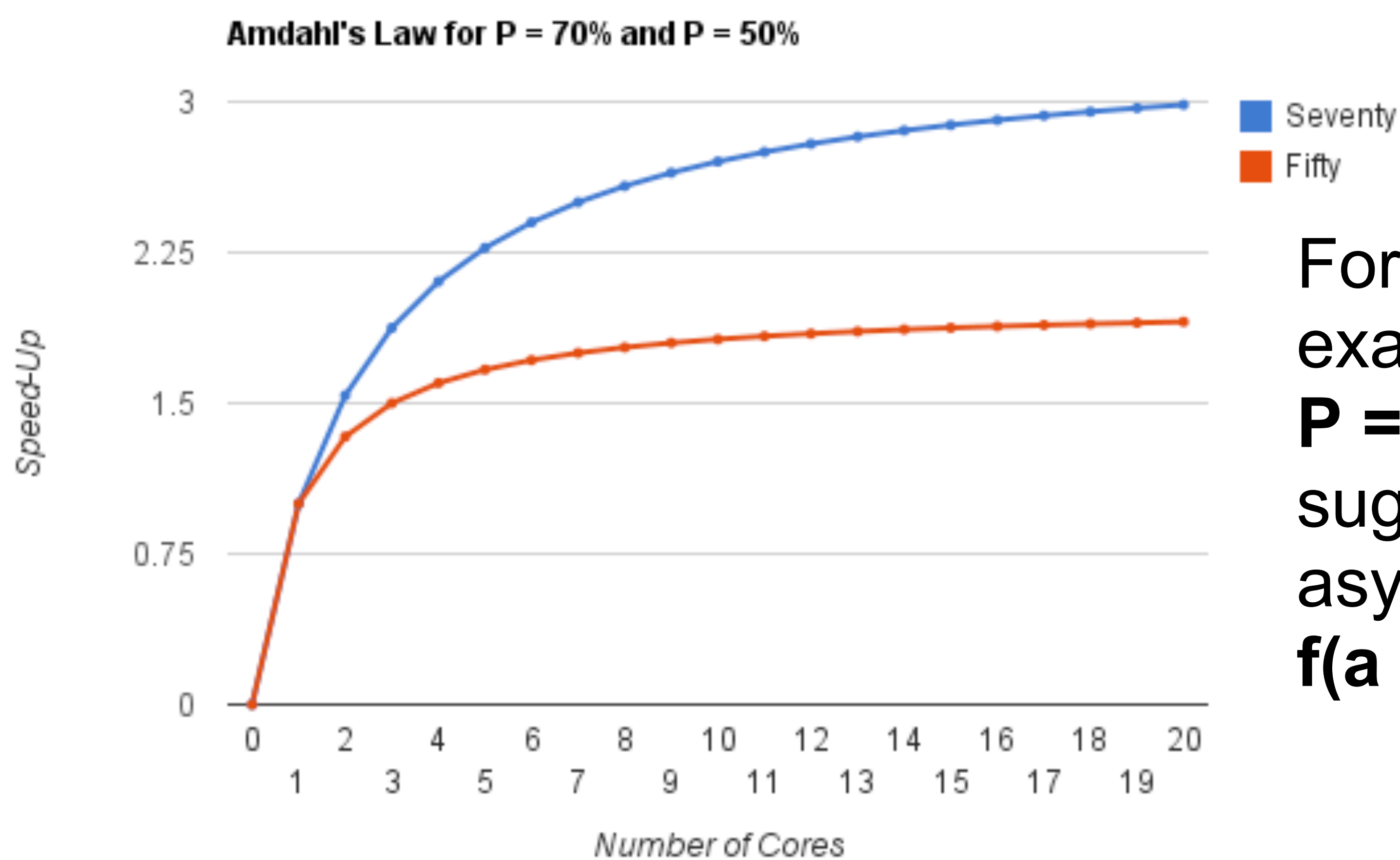
□**P** := "Percentage" of code which is parallelizable
N := Number of cores used
f(N) := Amount of speedup code gains using N cores

Suggests a maximum possible speedup:

$$\lim_{N \to \infty} f(N) = \lim_{N \to \infty} \frac{1}{(1 - P) + \frac{P}{N}} = \frac{1}{1 - P}$$

Amdahl's Law

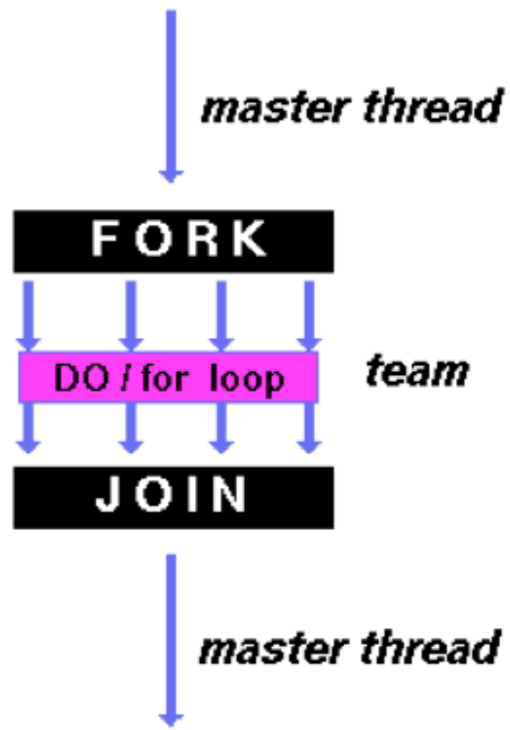
$$f(N) = \frac{1}{(1 - P) + \frac{P}{N}}$$



For our copying example,
P = 100/110 = 10/11
suggesting an asymptote of
f(a gazillion) = 11

Amdahl's Law's Assumptions

- **No contention for shared resources!**
  - All threads have equal access to caches, memory, IO, etc.
- **No per-thread overhead!**
  - Adding more threads to the parallel sections doesn't add more work for the serial section
- **No Pipelining!**
  - Some apps can send partial solutions off to one parallel thread at a time



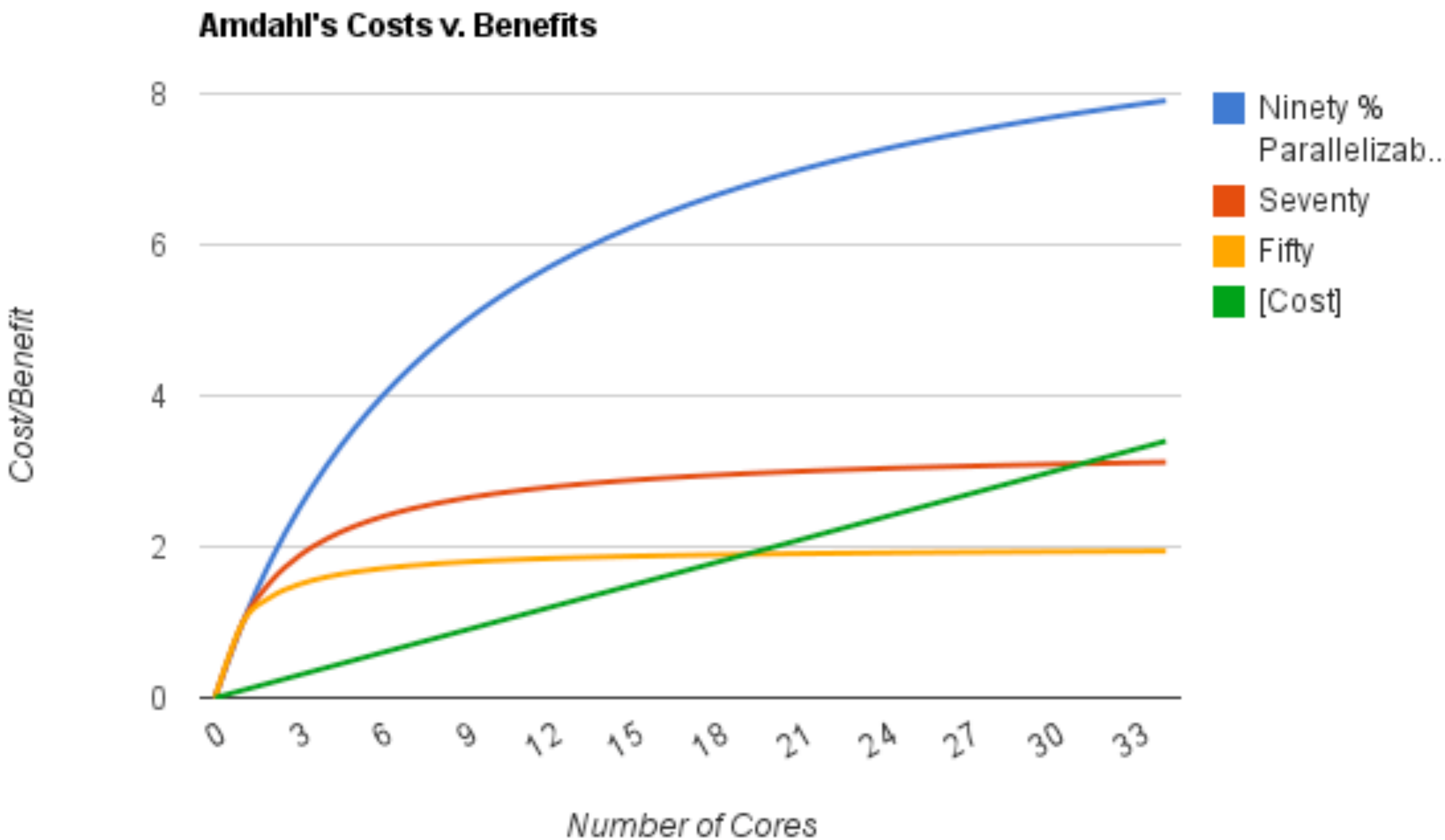
(Also, let's just round off **quantization**, too!)

Amdahl: TO THE CLOUD

- Hourly computer rental
  - Speedup of 2x?
    - Twice the revenue!
    - Same rental fee!
- "Elastic" cluster size
  - Pay $x for 1 core?
  - Via virtualization: Pay $*k*x for *k* cores!
- Hardware price points
  - m1.small, $0.085/hr
    - 1x ~1.2 GHz
    - 1.7 GB RAM
  - c1.xlarge, $0.68/hr
    - 8x ~3 GHz
    - 7 GB RAM

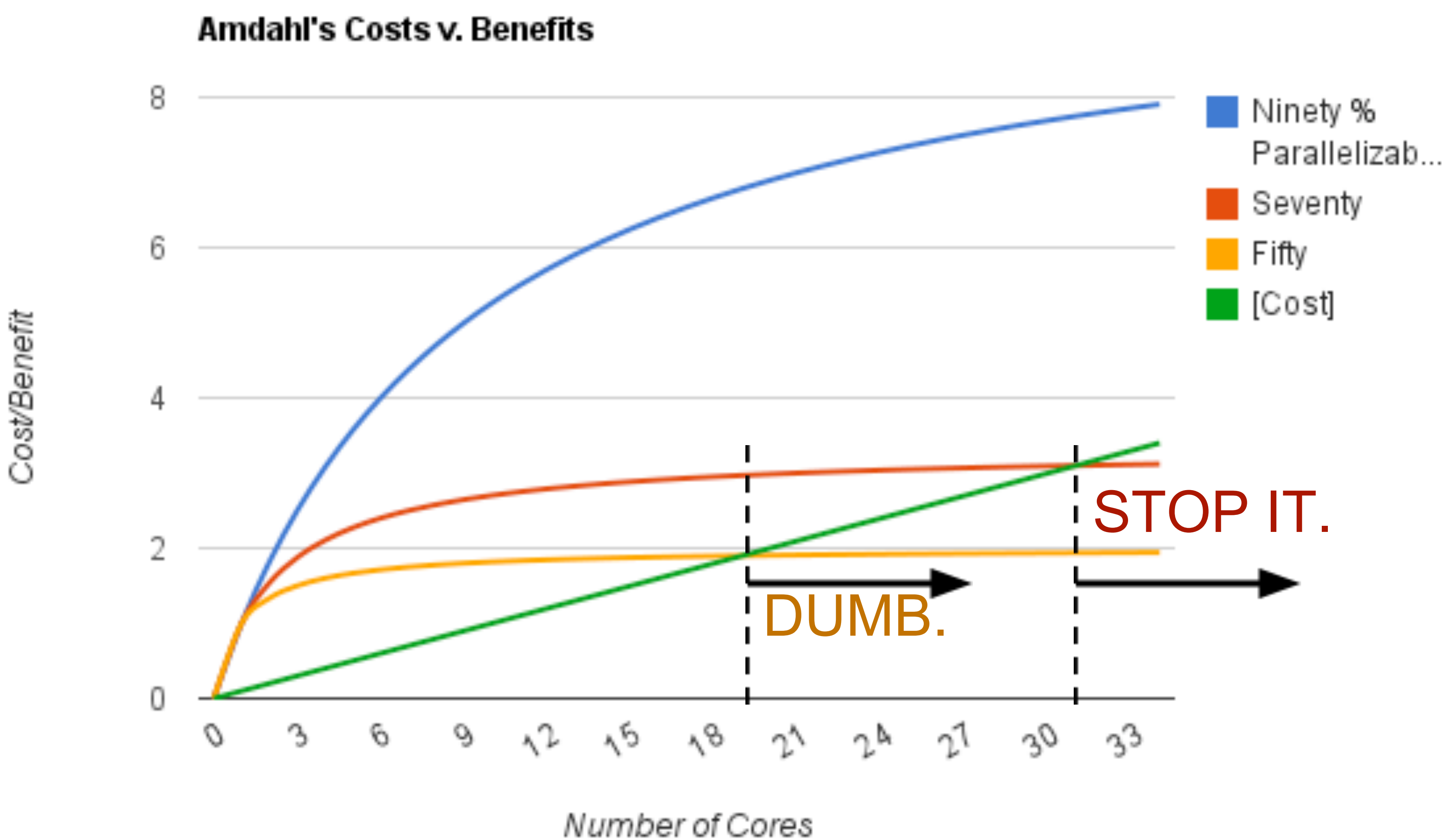
(Most of these cost structures also hold even if you build your own rig -- more cores? Higher power bill!)

Amdahl: Costs and Benefits



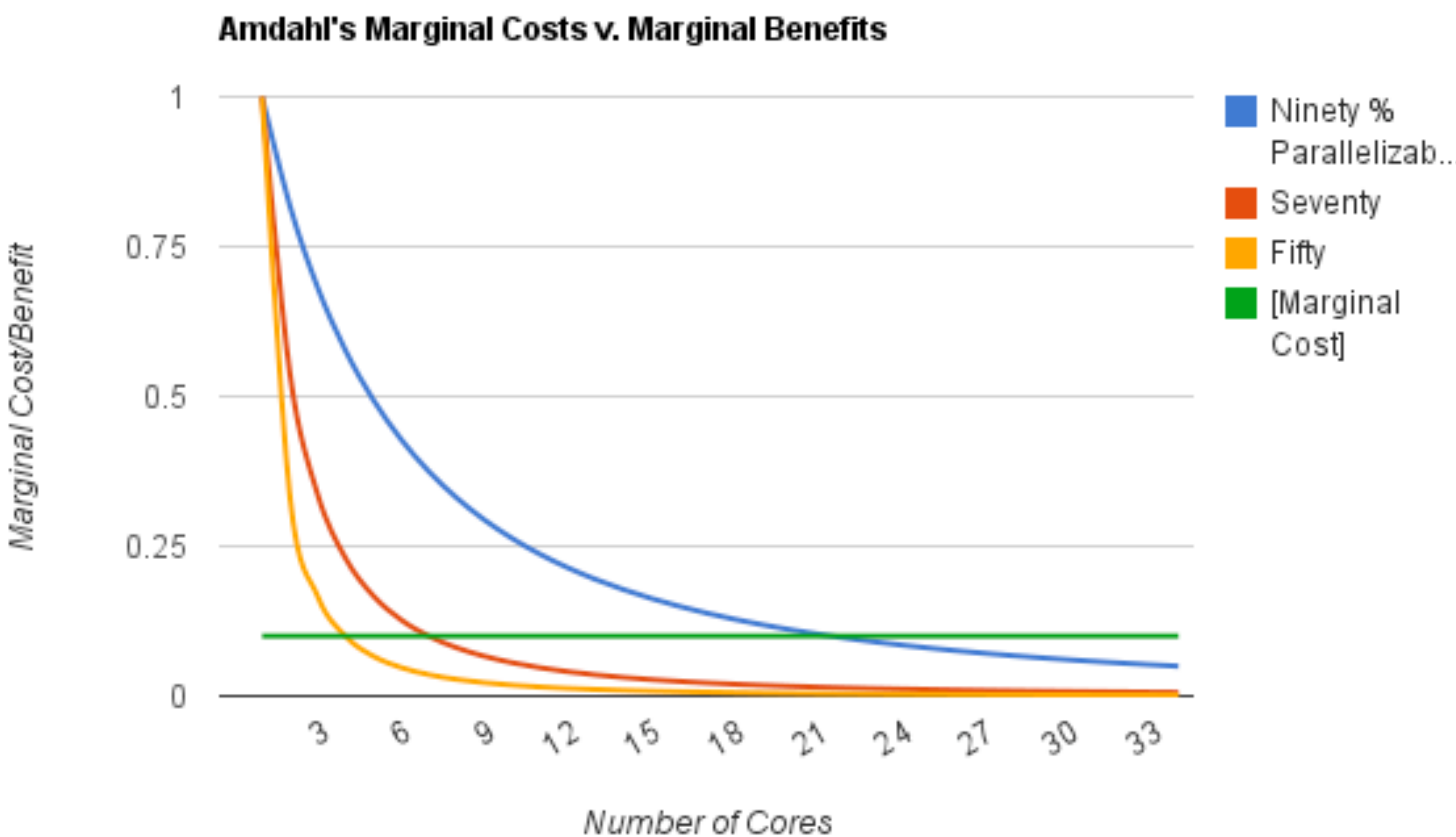
- Benefits of more cores rise as Amdahl's Law
  - f(N) speedup? f(N) more customers served!
- Costs of more cores rises *linearly* in N
  - Steeper slope = cheap customers, pricey cores, or both.

Amdahl: Costs and Benefits



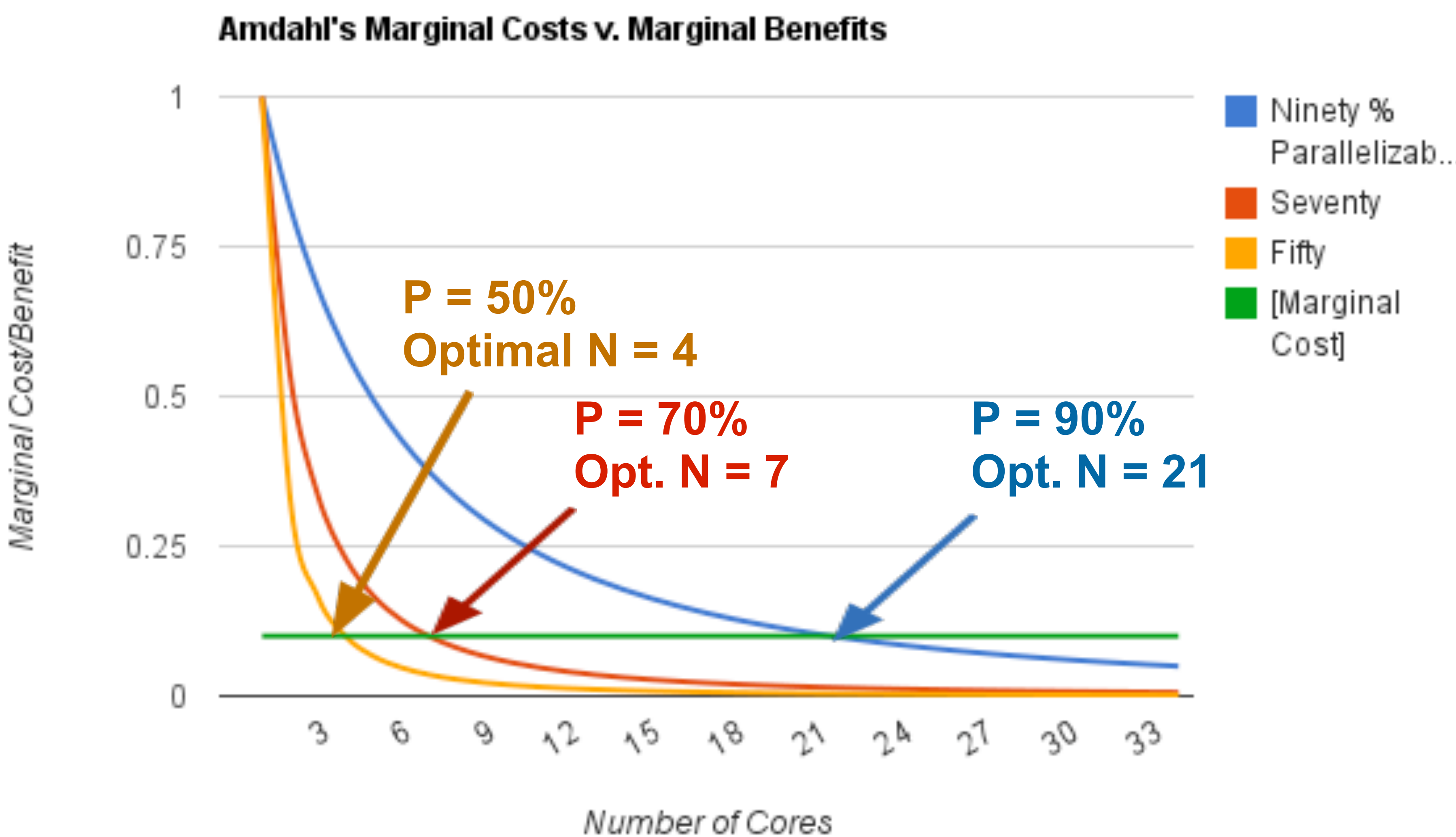
- Profit = Benefits - Costs; should *at least* be positive
  - Clear bounds on N for P = 50% and P = 70%
  - Note that both are quite asymptotic by that point anyway
- Insufficient to just have *positive* profit -- want the *maximum!*

Amdahl: Marginal Costs and Benefits



- Take the first derivative of both benefit and cost
- Find the point *right before* adding one more machine *marginally* costs more than it *marginally* benefits

Amdahl: Marginal Costs and Benefits



- Optimal N can occur quite a bit before asymptote kicks in
- If marginal cost rises a little, can cause Opt. N to drop a lot
- Bigger Opt. N --> More Speedup --> More Profit!

Sum-of-Squares Example

```
s = 0;
for(i = 0; i < 100; i++)
   s += x[i]**2;    // 2 Inst per loop
```

- Each iteration depends on the result of the iteration before!
- As written, unparallelizable:
  - P = **0 %**
  - max f(N) = 1/(1-P) = 1x speedup, *max.*
  - Have to run all 200 instructions serially!
  - DOOOOOOM!

Sum-of-Squares: One Good Idea

```
s = 0;
for(i = 0; i < 100; i++)
   y[i] = x[i]**2;  // square
for(i = 0; i < 100; i++)
   s += y[i];    // accumulate
```

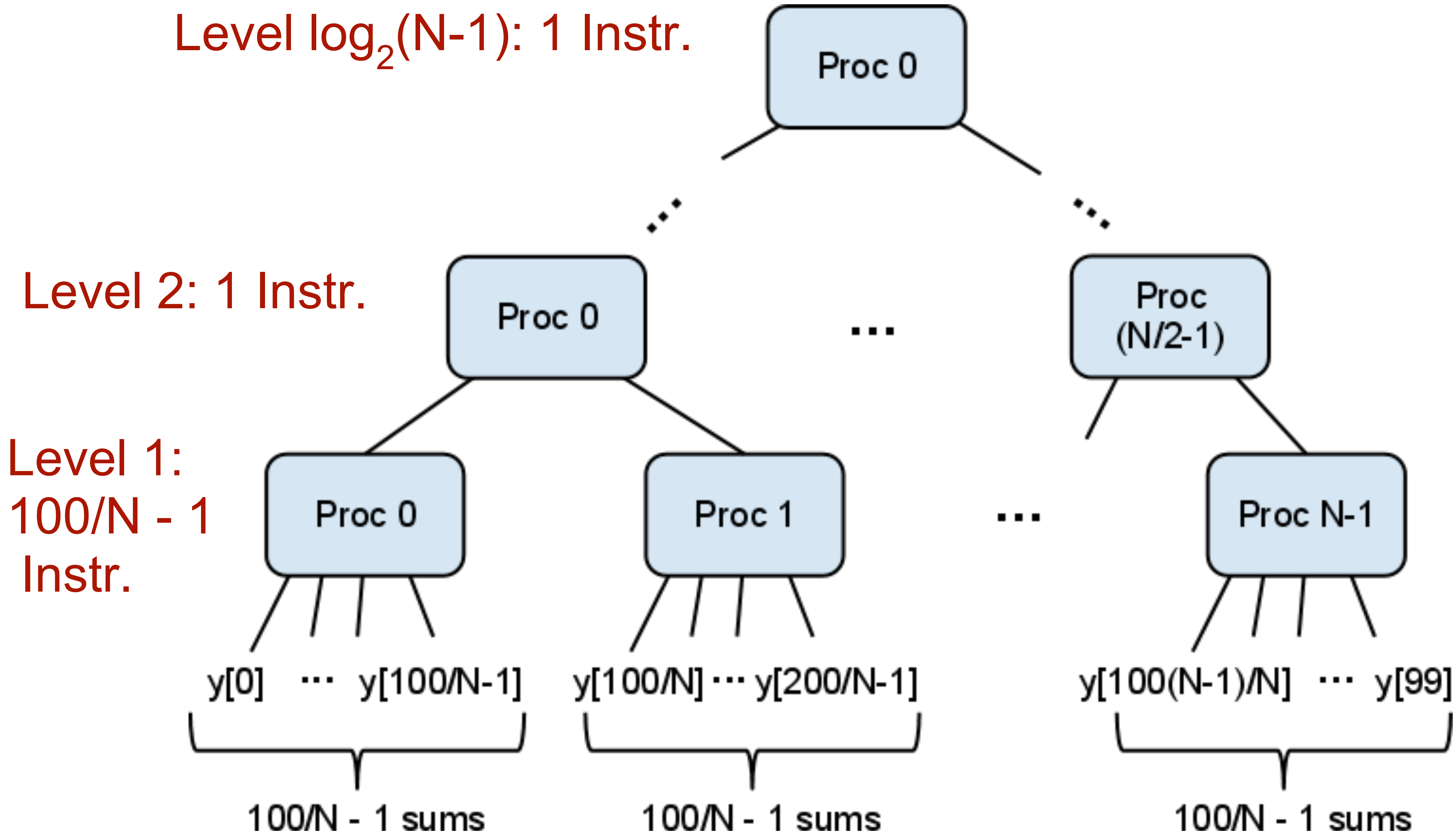
- Good idea: Break the loop into 2!
  - First square, then sum
  - Use more memory to save time
- First loop now parallelizable:
  - P = **50 %**
  - max f(N) = 1/(1-P) = 2x speedup, *max.*
  - Even 2x speedup requires a gazillion cores (a gazilllion dollars).
  - dooooooooom.

Sum-of-Squares: One GREAT Idea

```
s = 0;
for(i = 0; i < 100; i++)
   y[i] = x[i]**2;
parAccum(y,100); //parallel accumulator
```

- GREAT idea: build a parallelizable accumulator
  - Sum Reduction from 10.14.11's lecture is our friend here!
- How close can we get to full parallelizability?
  - The better we build **parAccum**, the closer P gets to 100%

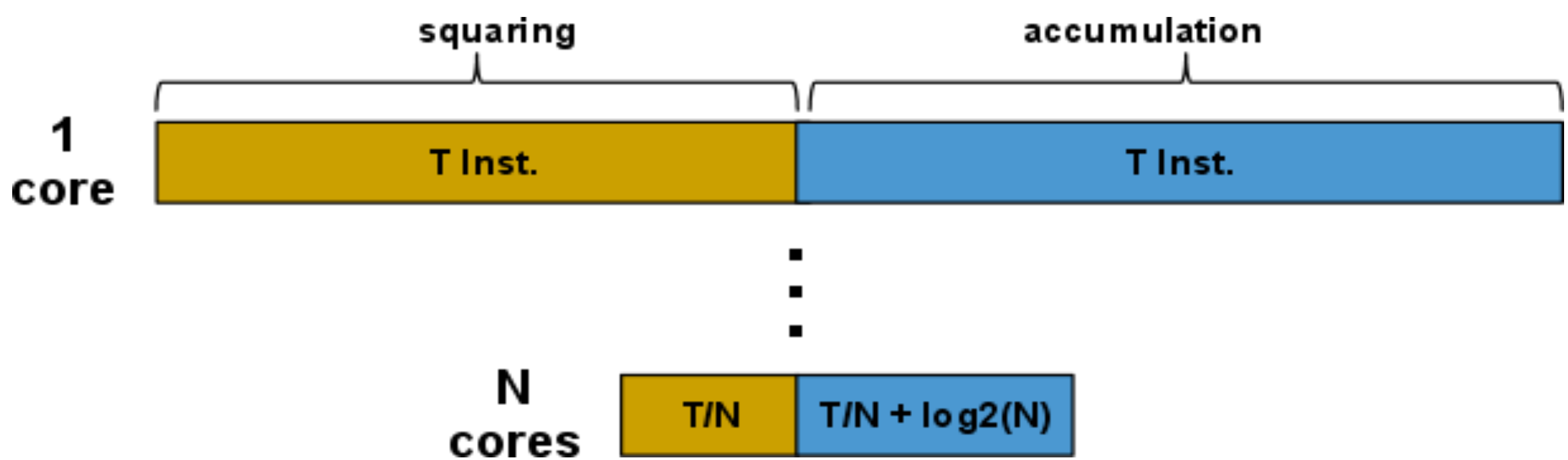
Sum-of-Squares: parAccum(y, 100)



TOTAL = 100/N + $\log_2$(N-1) - 2 steps to complete.

Sum-of-Squares: One GREAT Idea

```
s = 0;
for(i = 0; i < T; i++) // squaring loop
   y[i] = x[i]**2;
parAccum(y,T); //parallel accumulator
```



- N cores provide:
  - Linear reduction in squaring loop
  - *Almost* linear reduction in accumulation
  - For large T, smallish N, it's awful close to **P = 100%**

EC2 Usage

- Regular troughs at mid-day: Perfect for AWS!
- Peak usage: 292 instances
- Median usage: 52
- Mean usage: 81.44
- About $2,400!