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UC Berkeley CS61C : Machine Structures

Lecture 23 – Representations of Combinational Logic Circuits

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Android Brain on Robots! ⇒

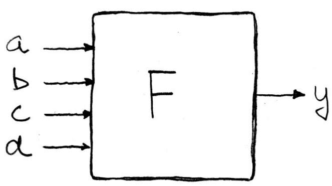
"Half the weight of some robots is due to on-board computers and the batteries needed to power them. This lightweight robot uses an Android phone as the brain, with the phone's gyroscope and camera as sensors, with cloud help!"

www.technologyreview.com/business/38953/page1/

Review

- State elements are used to:
  - Build memories
  - Control the flow of information between other state elements and combinational logic
- D-flip-flops used to build registers
- Clocks tell us when D-flip-flops change
  - Setup and Hold times important
- We pipeline long-delay CL for faster clock
- Finite State Machines extremely useful
  - Represent states and transitions

Truth Tables

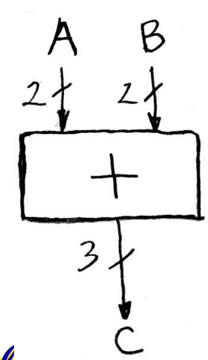


a	b	c	d	y
0	0	0	0	F(0,0,0,0)
0	0	0	1	F(0,0,0,1)
0	0	1	0	F(0,0,1,0)
0	0	1	1	F(0,0,1,1)
0	1	0	0	F(0,1,0,0)
0	1	0	1	F(0,1,0,1)
0	1	1	0	F(0,1,1,0)
0	1	1	1	F(0,1,1,1)
1	0	0	0	F(1,0,0,0)
1	0	0	1	F(1,0,0,1)
1	0	1	0	F(1,0,1,0)
1	0	1	1	F(1,0,1,1)
1	1	0	0	F(1,1,0,0)
1	1	0	1	F(1,1,0,1)
1	1	1	0	F(1,1,1,0)
1	1	1	1	F(1,1,1,1)

TT Example #1: 1 iff one (not both) a,b=1

a	b	y
0	0	0
0	1	1
1	0	1
1	1	0

TT Example #2: 2-bit adder



A	B	C
a_1a_0	b_1b_0	$c_2c_1c_0$
00	00	000
00	01	001
00	10	010
00	11	011
01	00	001
01	01	010
01	10	011
01	11	100
10	00	010
10	01	011
10	10	100
10	11	101
11	00	011
11	01	100
11	10	101
11	11	110

How Many Rows?

TT Example #3: 32-bit unsigned adder

A	B	C
000 ... 0	000 ... 0	000 ... 00
000 ... 0	000 ... 1	000 ... 01
.	.	.
.	.	.
.	.	.
111 ... 1	111 ... 1	111 ... 10

How Many Rows?

TT Example #4: 3-input majority circuit

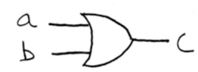
a	b	c	y
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

Logic Gates (1/2)

AND

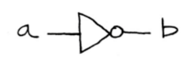
ab	c
00	0
01	0
10	0
11	1

OR



ab	c
00	0
01	1
10	1
11	1

NOT

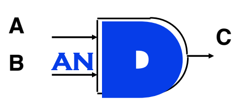


a	b
0	1
1	0

`And` vs. `Or` review – Dan's mnemonic

AND Gate

Symbol

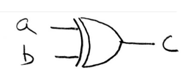


Definition

A	B	C
0	0	0
0	1	0
1	0	0
1	1	1

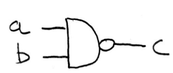
Logic Gates (2/2)

XOR



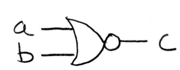
ab	c
00	0
01	1
10	1
11	0

NAND



ab	c
00	1
01	1
10	1
11	0

NOR



ab	c
00	1
01	0
10	0
11	0

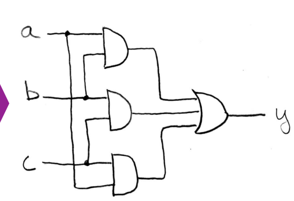
2-input gates extend to n-inputs

- **N-input XOR is the only one which isn't so obvious**
- **It's simple: XOR is a 1 iff the # of 1s at its input is odd ⇒**

a	b	c	y
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

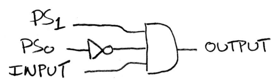
Truth Table ⇒ Gates (e.g., majority circ.)

a	b	c	y
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

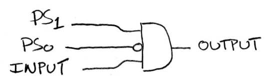


Truth Table ⇒ Gates (e.g., FSM circ.)

PS	Input	NS	Output
00	0	00	0
00	1	01	0
01	0	00	0
01	1	10	0
10	0	00	0
10	1	00	1

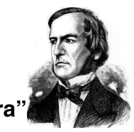


or equivalently…



Boolean Algebra

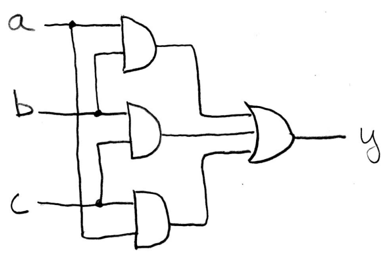
- George Boole, 19th Century mathematician
- Developed a mathematical system (algebra) involving logic
  - later known as "Boolean Algebra"



- Primitive functions: AND, OR and NOT
- The power of BA is there's a one-to-one correspondence between circuits made up of AND, OR and NOT gates and equations in BA

\+ means OR, • means AND, $\overline{x}$ means NOT

Boolean Algebra (e.g., for majority fun.)

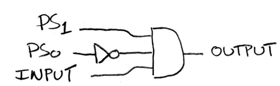


$y = a \cdot b + a \cdot c + b \cdot c$

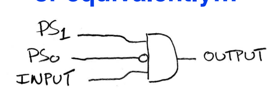
$y = ab + ac + bc$

Boolean Algebra (e.g., for FSM)

PS	Input	NS	Output
00	0	00	0
00	1	01	0
01	0	00	0
01	1	10	0
10	0	00	0
10	1	00	1

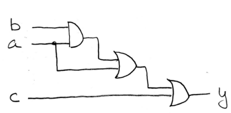


or equivalently…



$y = PS_1 \cdot \overline{PS_0} \cdot INPUT$

BA: Circuit & Algebraic Simplification



original circuit

$\downarrow$

$y = ((ab) + a) + c$ — equation derived from original circuit

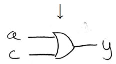
$\downarrow$

algebraic simplification

$$
\begin{aligned}
&= ab + a + c \\
&= a(b+1) + c \\
&= a(1) + c \\
&= a + c
\end{aligned}
$$

$\downarrow$

BA also great for circuit verification Circ X = Circ Y? use BA to prove!



simplified circuit

Laws of Boolean Algebra

$x \cdot \overline{x} = 0$	$x + \overline{x} = 1$	complementarity
$x \cdot 0 = 0$	$x + 1 = 1$	laws of 0's and 1's
$x \cdot 1 = x$	$x + 0 = x$	identities
$x \cdot x = x$	$x + x = x$	idempotent law
$x \cdot y = y \cdot x$	$x + y = y + x$	commutativity
$(xy)z = x(yz)$	$(x+y)+z = x+(y+z)$	associativity
$x(y+z) = xy + xz$	$x + yz = (x+y)(x+z)$	distribution
$xy + x = x$	$(x+y)x = x$	uniting theorem
$\overline{x}y + x = x + y$	$(\overline{x}+y)x = xy$	uniting theorem v.2
$\overline{x \cdot y} = \overline{x} + \overline{y}$	$\overline{x+y} = \overline{x} \cdot \overline{y}$	DeMorgan's Law

Boolean Algebraic Simplification Example

$$\begin{aligned} y &= ab + a + c \\ &= a(b+1) + c && \textit{distribution, identity} \\ &= a(1) + c && \textit{law of 1's} \\ &= a + c && \textit{identity} \end{aligned}$$

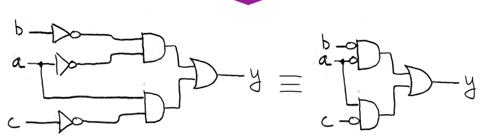
Canonical forms (1/2)

	abc	y
$\bar{a}\cdot\bar{b}\cdot\bar{c}$	000	1
$\bar{a}\cdot\bar{b}\cdot c$	001	1
	010	0
	011	0
$a\cdot\bar{b}\cdot\bar{c}$	100	1
	101	0
$a\cdot b\cdot\bar{c}$	110	1
	111	0

Sum-of-products (ORs of ANDs)

Canonical forms (2/2)

$$\begin{aligned} y &= \bar{a}\bar{b}\bar{c} + \bar{a}\bar{b}c + a\bar{b}\bar{c} + ab\bar{c} \\ &= \bar{a}\bar{b}(\bar{c}+c) + a\bar{c}(\bar{b}+b) && \textit{distribution} \\ &= \bar{a}\bar{b}(1) + a\bar{c}(1) && \textit{complementarity} \\ &= \bar{a}\bar{b} + a\bar{c} && \textit{identity} \end{aligned}$$



Peer Instruction

1) $(a+b)\cdot(\bar{a}+b) = b$
2) N-input gates can be thought of cascaded 2-input gates. I.e., (a Δ bc Δ d Δ e) = a Δ (bc Δ (d Δ e)) where Δ is one of AND, OR, XOR, NAND
3) You can use NOR(s) with clever wiring to simulate AND, OR, & NOT

	123
a:	FFF
a:	FFT
b:	FTF
b:	FTT
c:	TFF
d:	TFT
e:	TTF
e:	TTT

"And In conclusion…"

- Pipeline big-delay CL for faster clock
- Finite State Machines extremely useful
  - You'll see them again in 150, 152 & 164
- Use this table and techniques we learned to transform from 1 to another