## Recreation

Prove that for every acute angle $\alpha > 0$,

$$\tan\alpha + \cot\alpha \geq 2$$

# CS61B Lecture #5: Simple Pointer Manipulation

## Announcement

- **Today:** More pointer hacking.
- **Handing in labs and homework:** We'll be lenient about accepting late homework and labs for the first few. Just get it done: part of the point is getting to understand the tools involved. We will *not* accept submissions by email.

# Destructive Incrementing

*Destructive* solutions may modify objects in the original list to save time or space:

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  if (P == null)
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  else {
    P.head += n;
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/* IntList.list from HW #1 */
Q = dincrList(X, 2);
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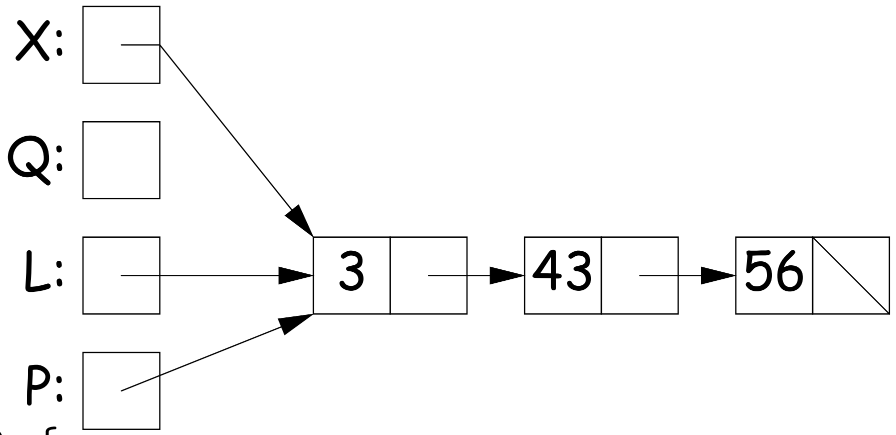

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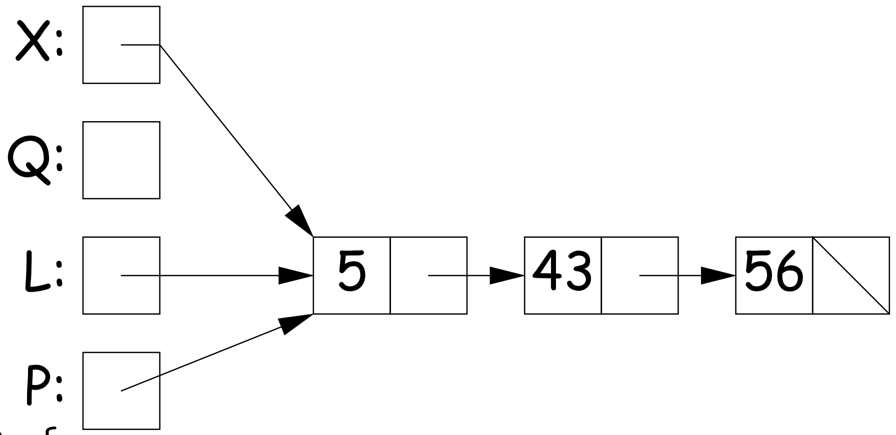

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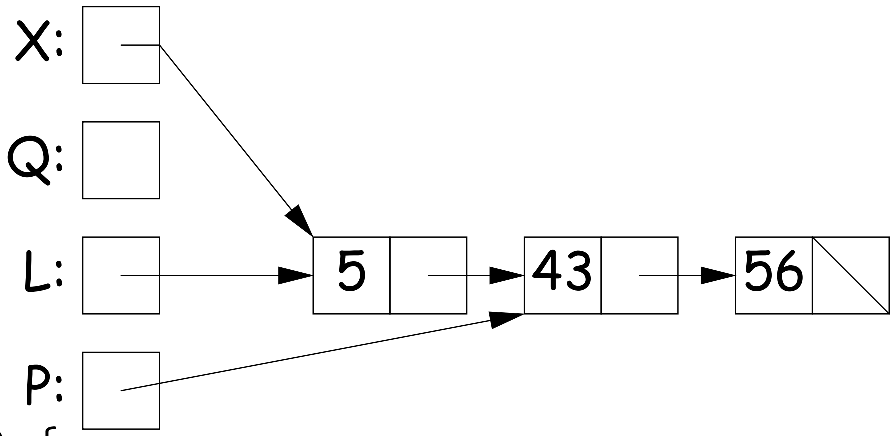

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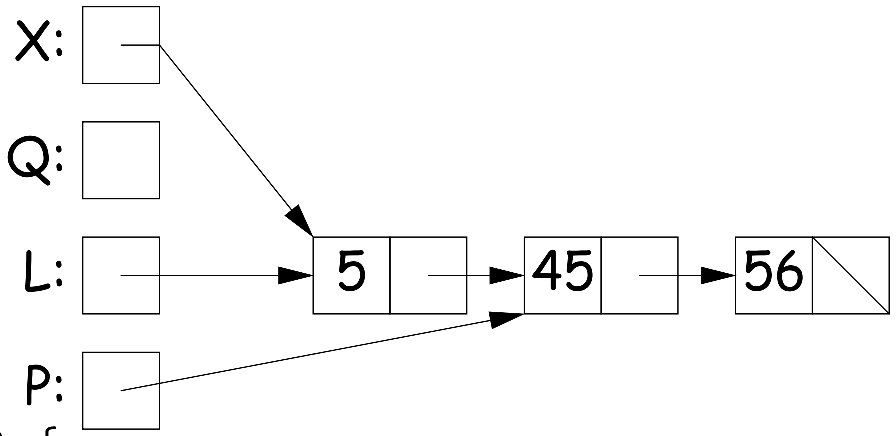

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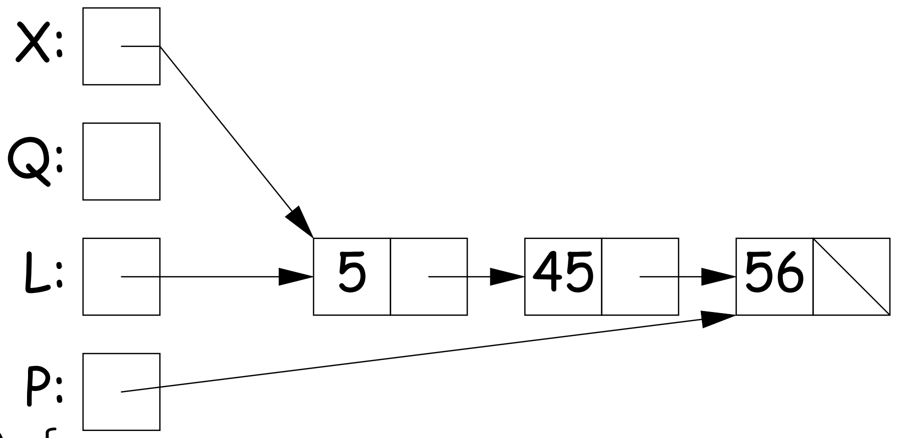

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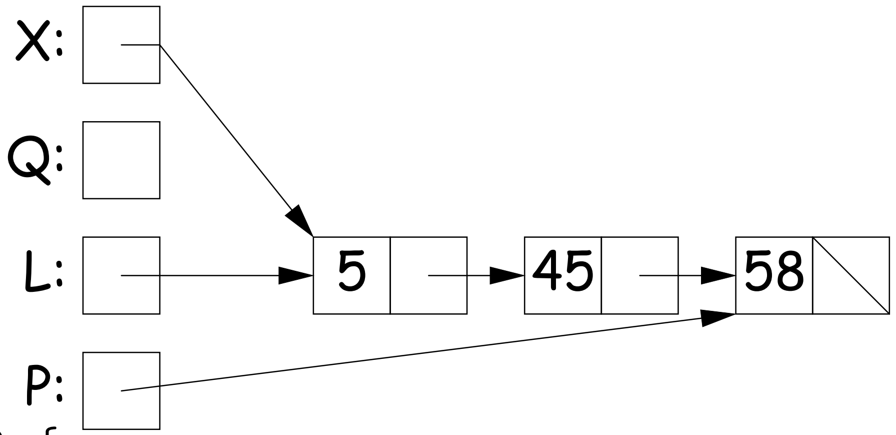

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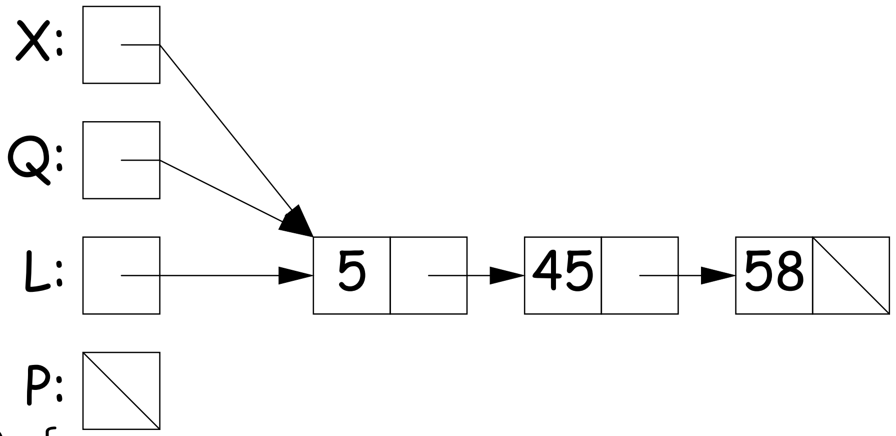

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## Another Example: Non-destructive List Deletion

If L is the list [2, 1, 2, 9, 2], we want `removeAll(L,2)` to be the new list [1, 9].

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/** The list resulting from removing all instances of X from L
 *  non-destructively. */
static IntList removeAll(IntList L, int x) {
  if (L == null)
     return /*( null with all x's removed )*/;
  else if (L.head == x)
     return /*( L with all x's removed (L != null) )*/;
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     return /*( L with all x's removed (L != null, L.head!=x) )*/;
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  else
     return new IntList(L.head, removeAll(L.tail, x));
}
```

## Aside: How to Write a Loop (in Theory)

- Try to *give* a description of how things look on *any arbitrary iteration* of the loop.
- This description is known as a *loop invariant,* because it is true from one iteration to the next.
- The loop body then must
  - Start from any situation consistent with the invariant;
  - Make progress in such a way as to make the invariant true again.

```
while (condition) {
    // Invariant true here
    loop body
    // Invariant again true here
}
// Invariant true and condition false.
```

- So if (*invariant* and not *condition*) is enough to insure we've got the answer, we're done!

## Iterative Non-destructive List Deletion

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Here, $\mathcal{I}$ is the *loop invariant:*

Result is all elements of $L_0$ not equal to x up to and not including L, and last points to the last element of result, if any. We use $L_0$ here to mean "the original sequence of int values in L."

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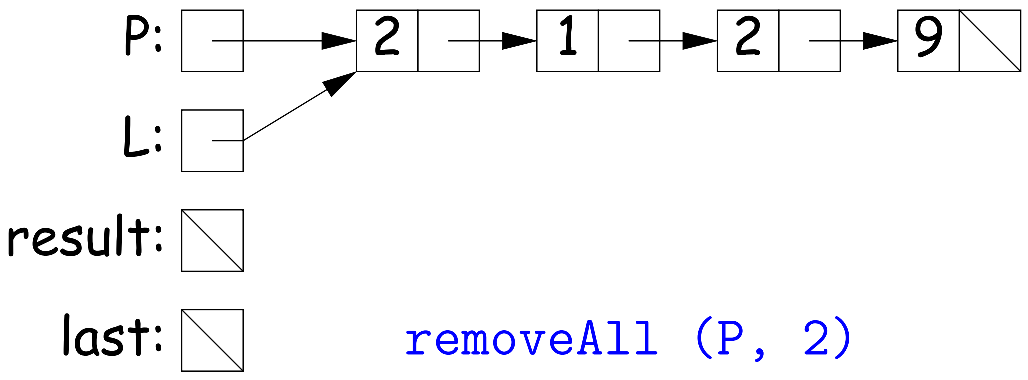

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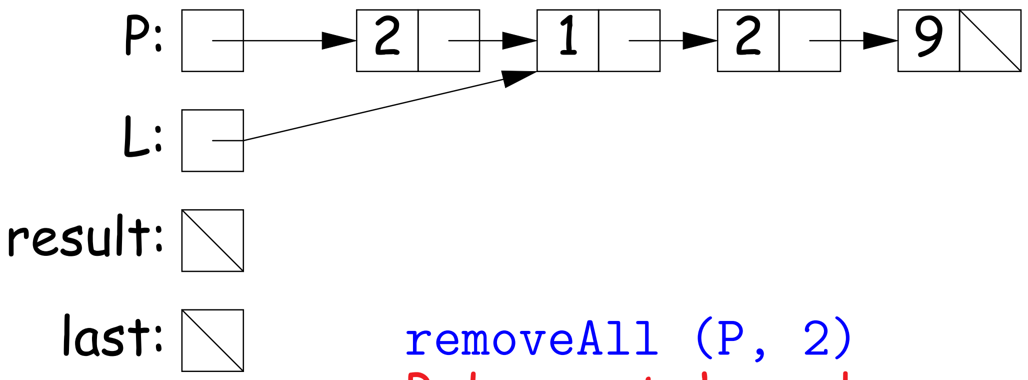

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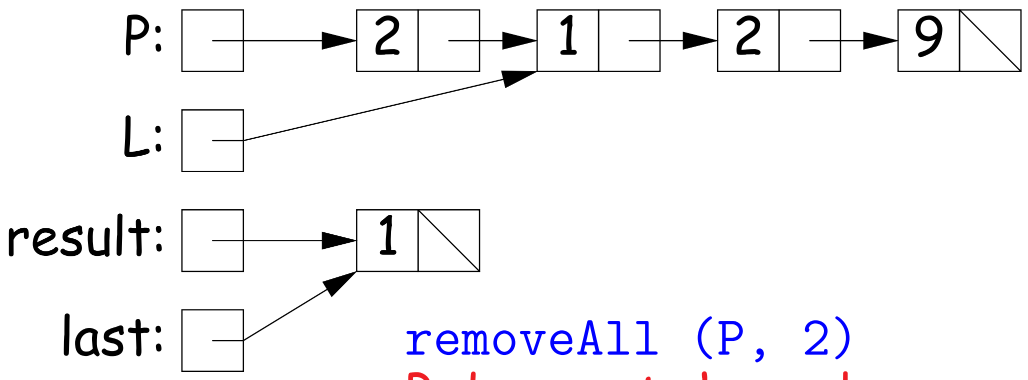

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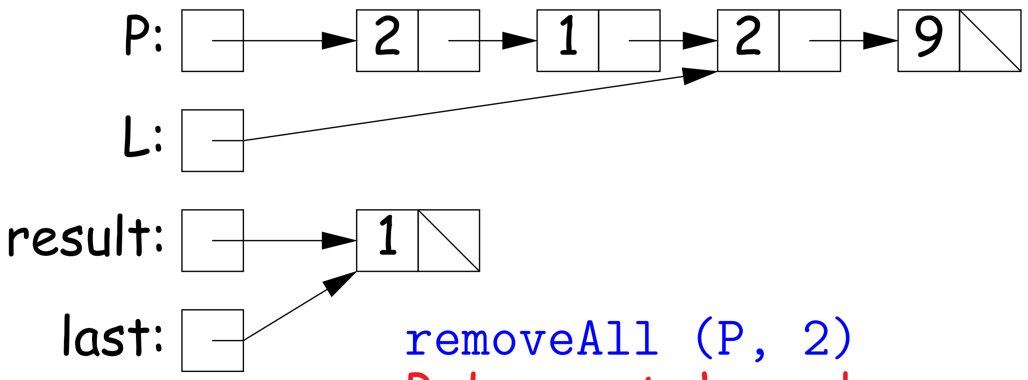

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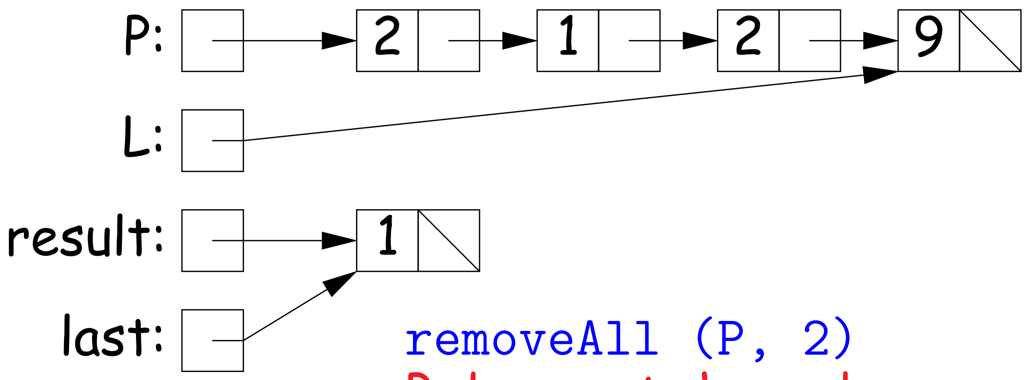

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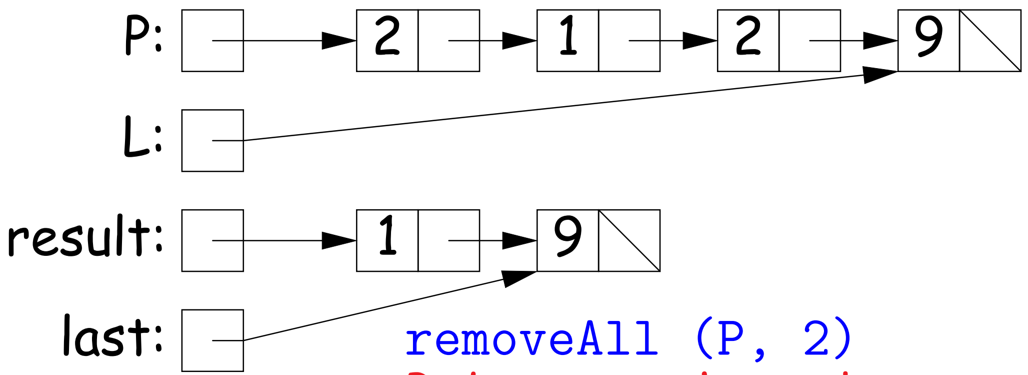

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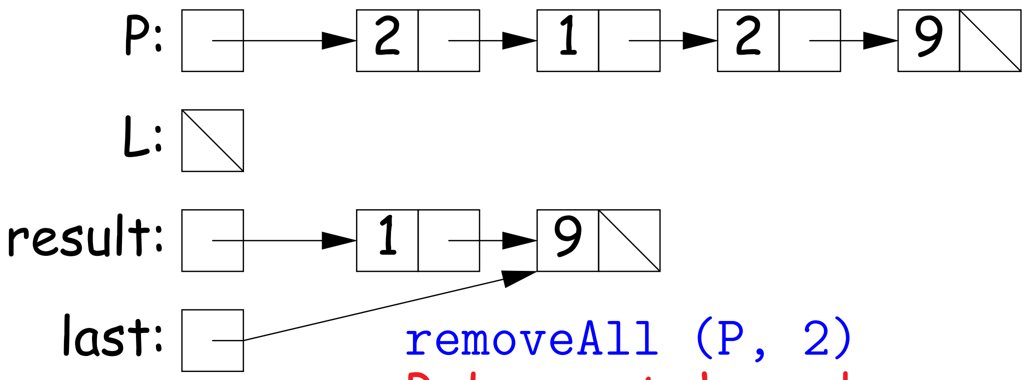

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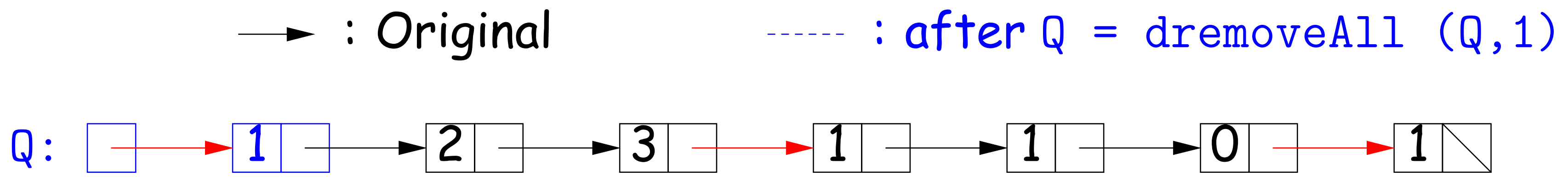

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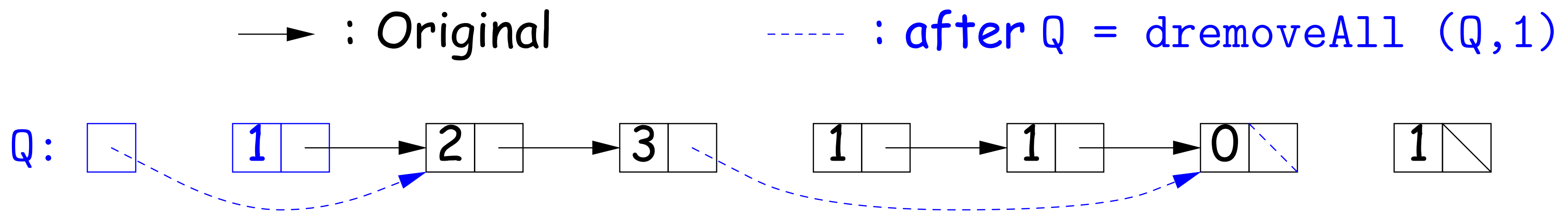

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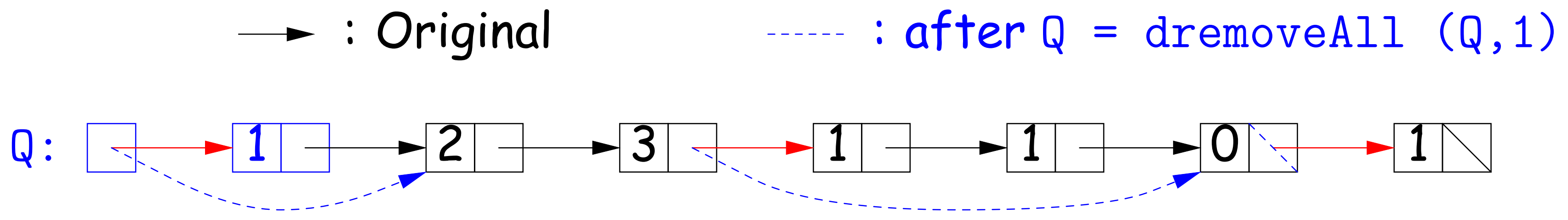

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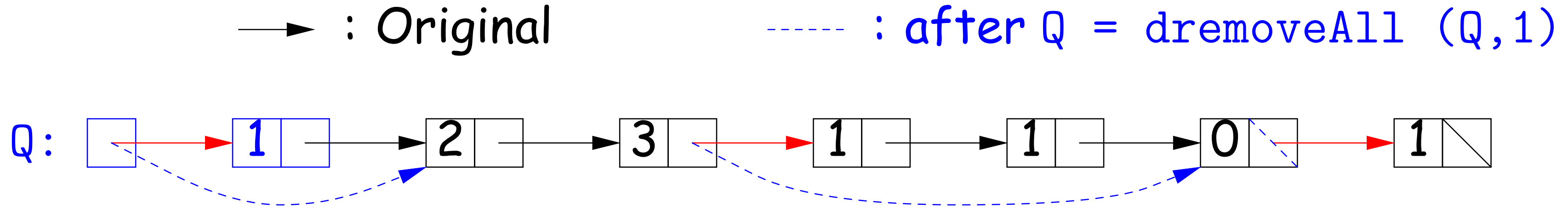

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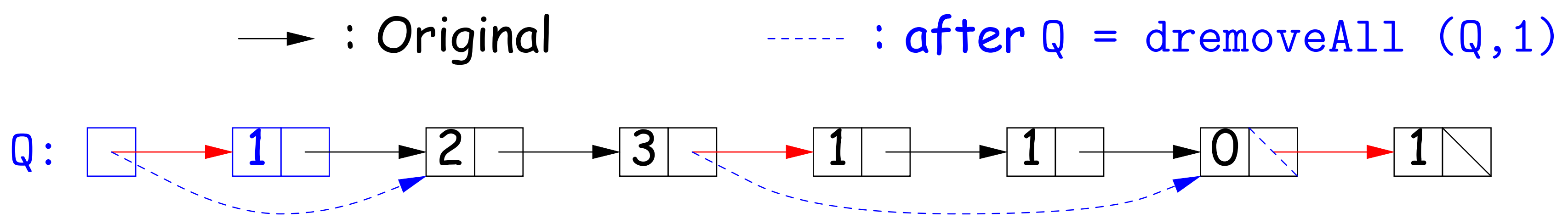

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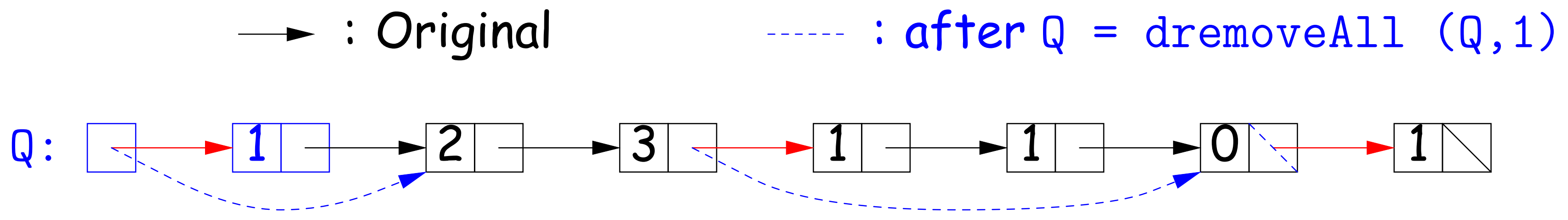

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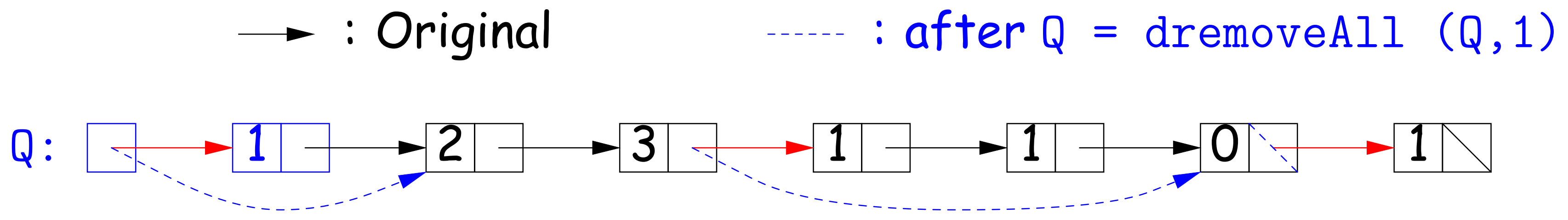
```

# Destructive Deletion



```
/** The list resulting from removing all instances of X from L.
 *  The original list may be destroyed. */
static IntList dremoveAll(IntList L, int x) {
  if (L == null)
     return null;
  else if (L.head == x)
     return dremoveAll(L.tail, x);
  else {
     L.tail = dremoveAll(L.tail, x);
     return L;
  }
}
```

## Iterative Destructive Deletion

```
/** The list resulting from removing all instances of X from L.
 *  Original contents of L may be destroyed. */
static IntList dremoveAll(IntList L, int x) {
  IntList result, last;
  result = last = null;
  while (L != null) {
    IntList next = L.tail;
    if (x != L.head) {
      if (last == null)
        result = last = L;
      else
        last = last.tail = L;
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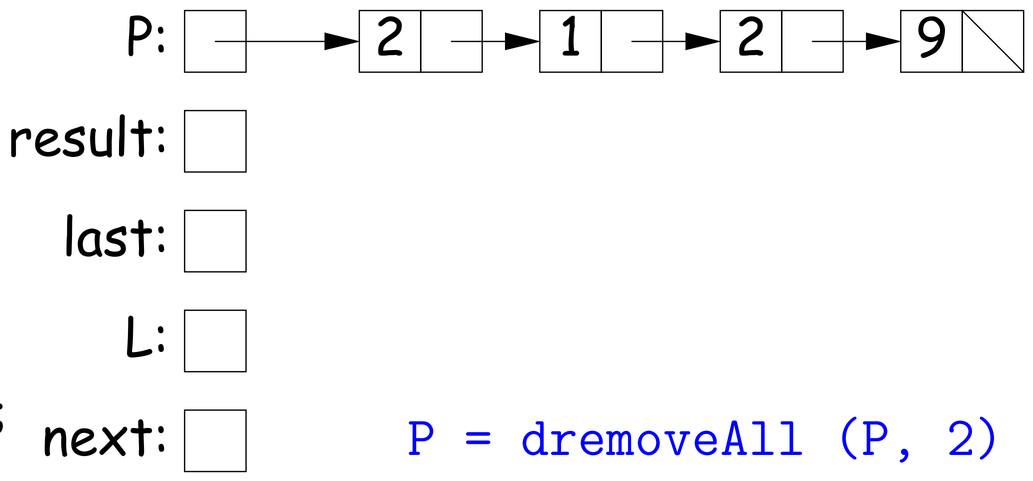

P = dremoveAll (P, 2)

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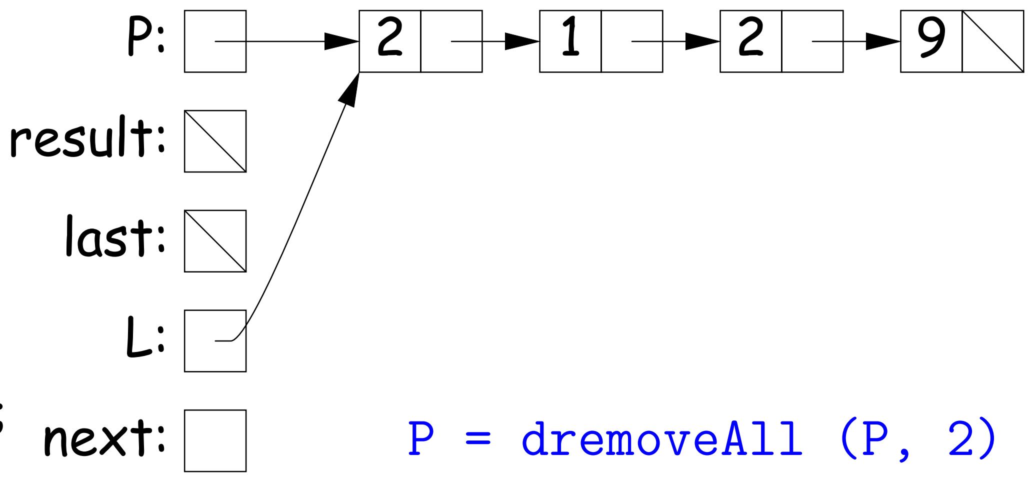

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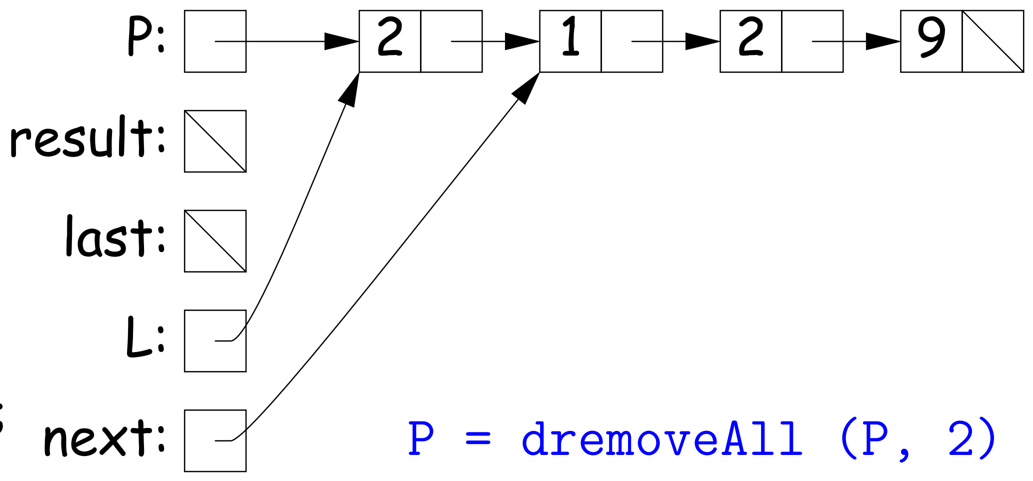

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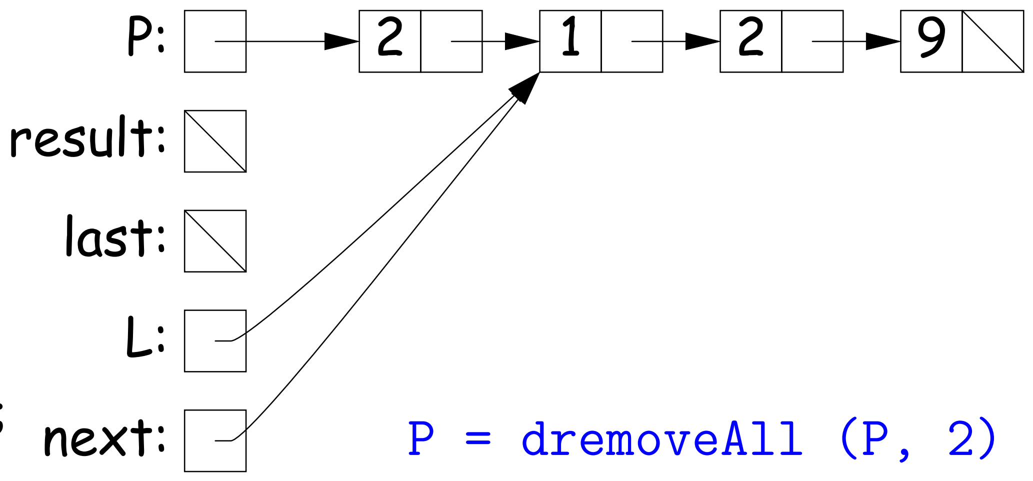

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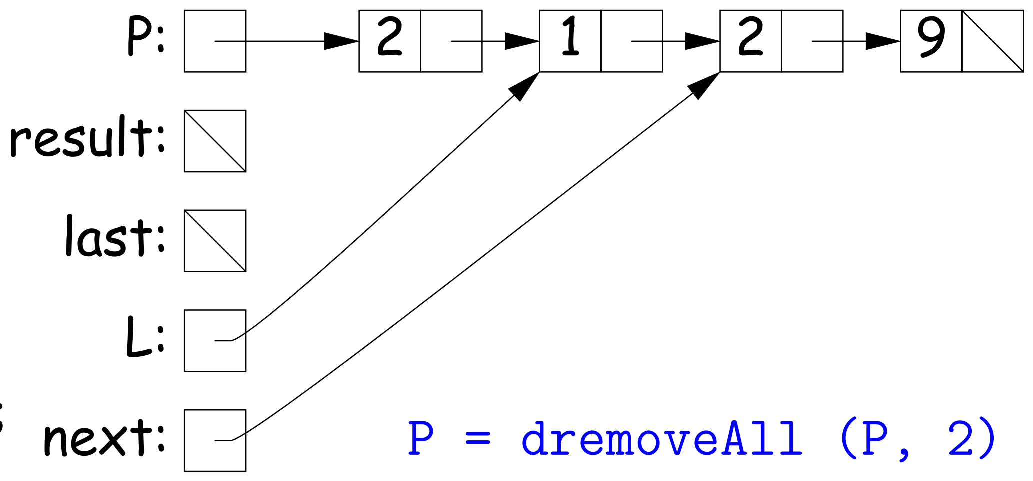

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  }
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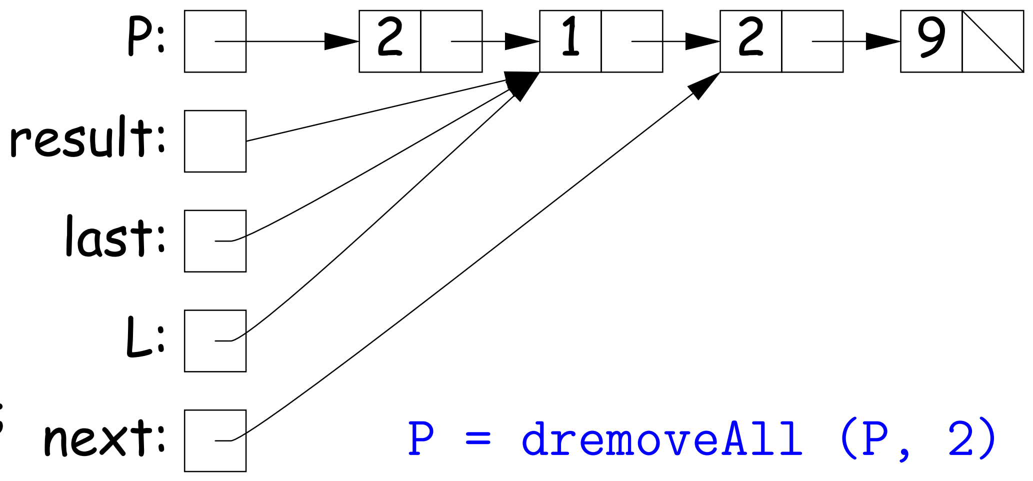

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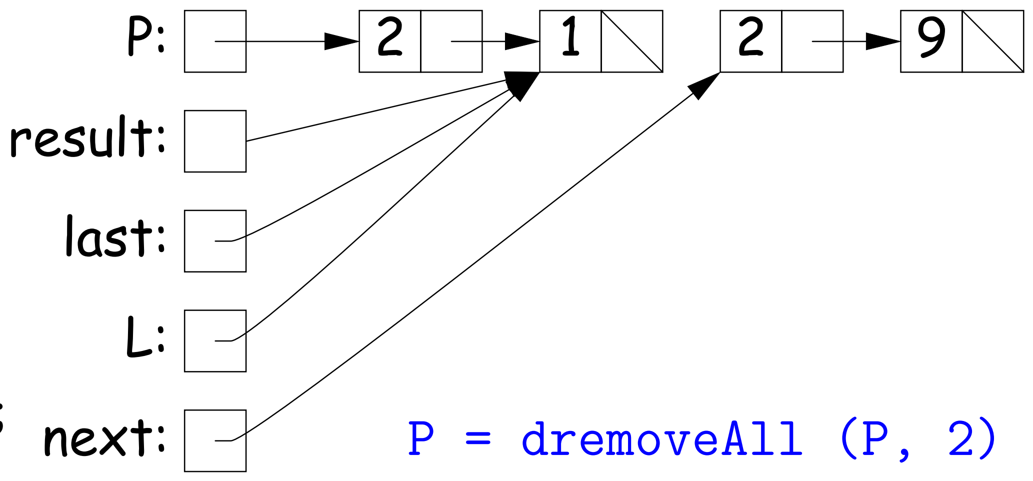

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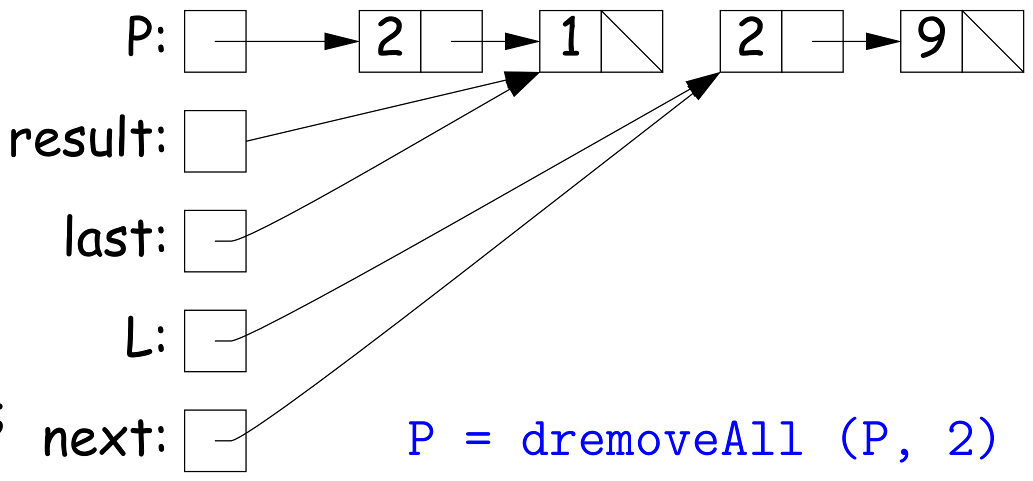

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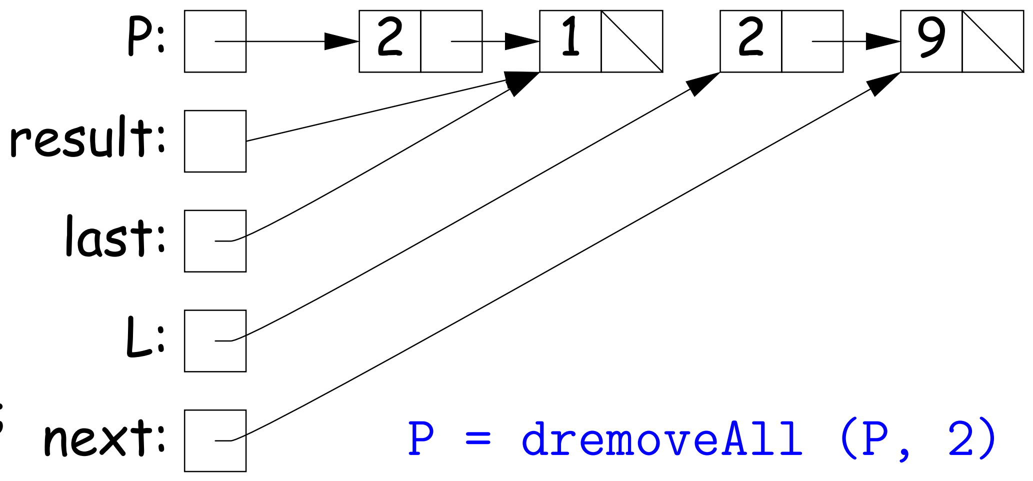

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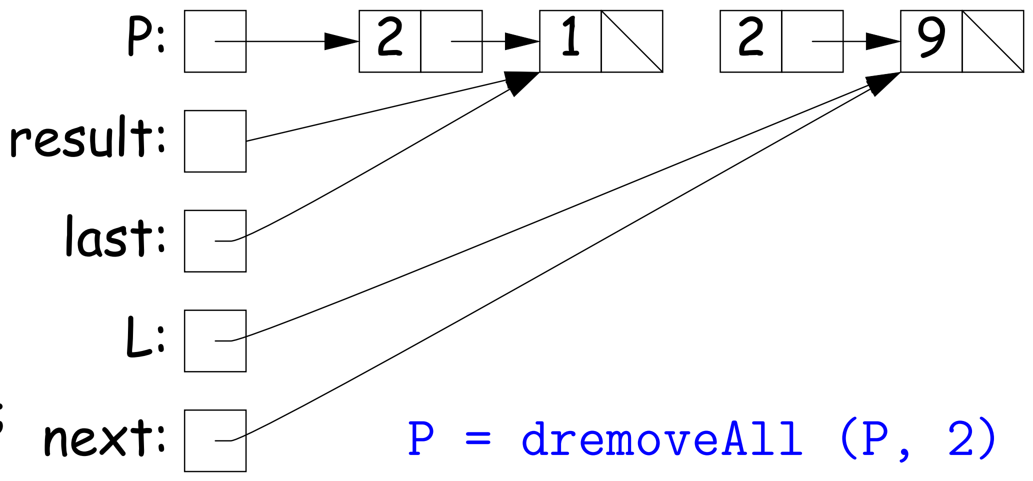

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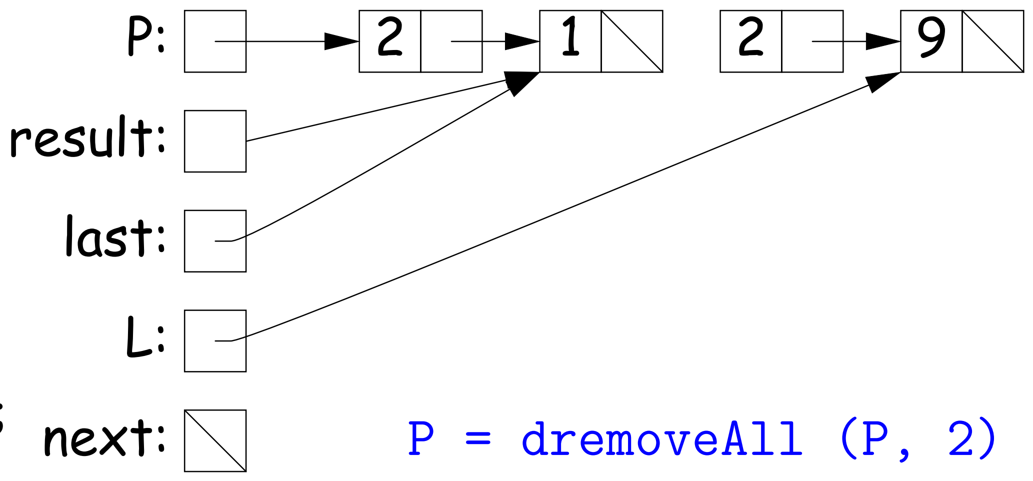

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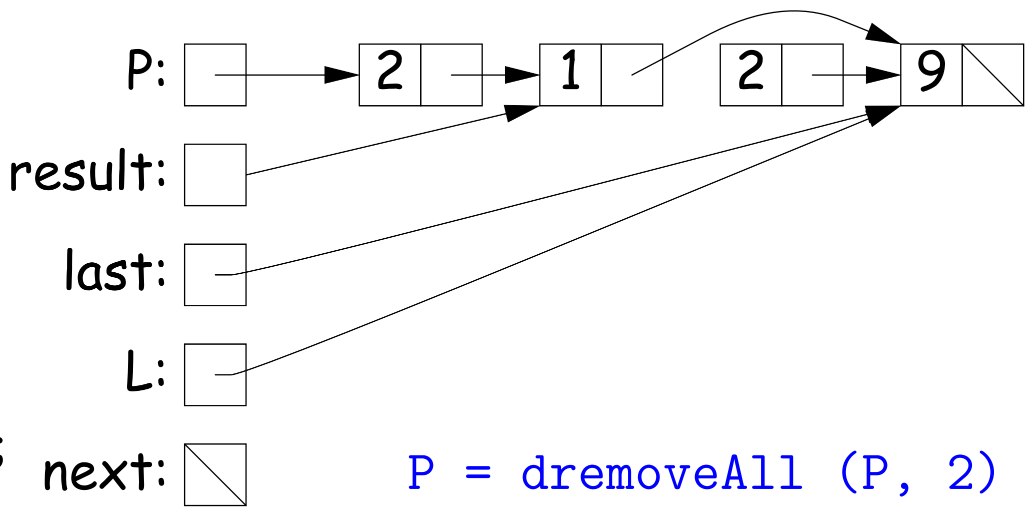

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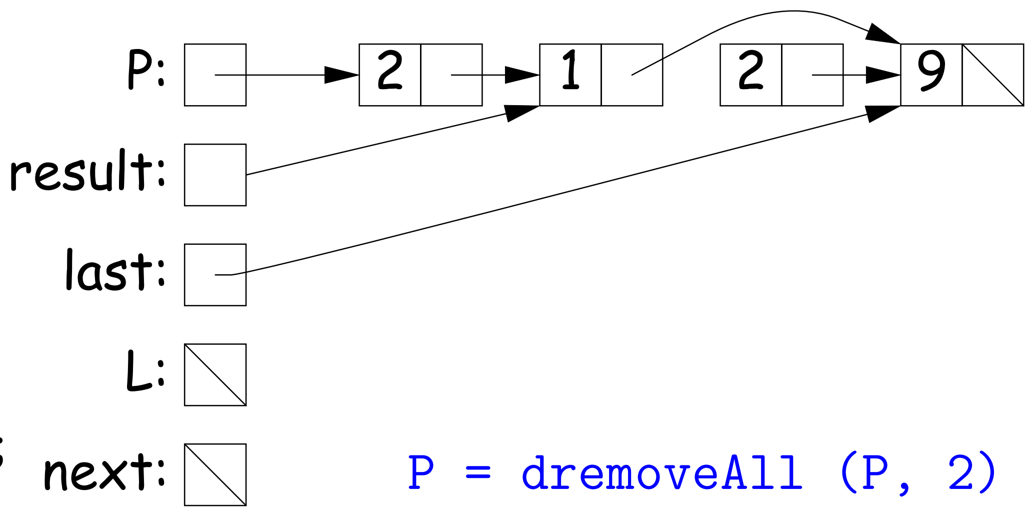

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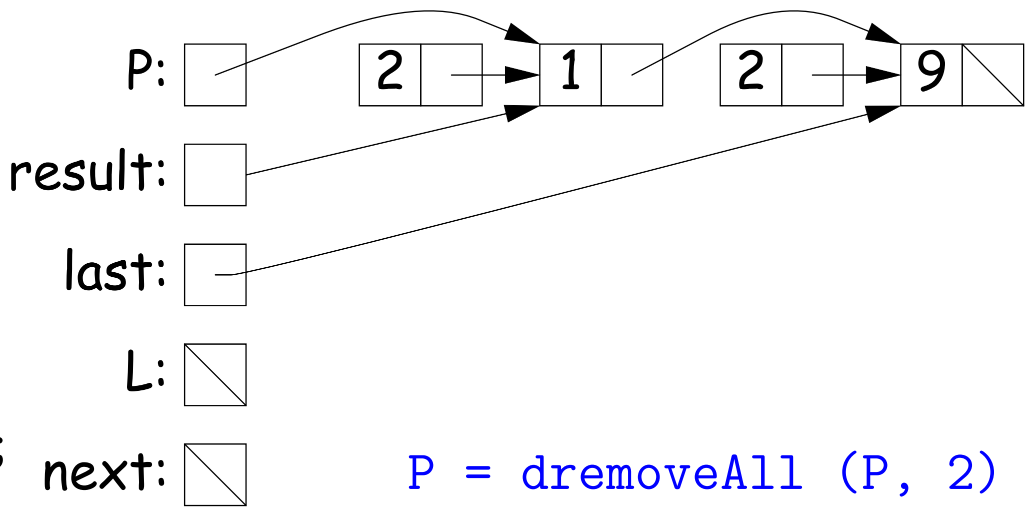

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