Public Service Announcement

"CodeBears is a rapidly growing organization on campus aimed at providing members with the opportunity to compete with each other and against teams in other universities in algorithmic software competitions that require strong problem solving abilities. Winners of our intracollege programming competitions will be able to meet with industry leaders, network with founders of various organizations, and represent UCB at inter-college competitions. We hope to create a new hacking culture at Cal! Learn more about us at

`https://callink.berkeley.edu/organization/codebears`

Info session tonight at 6:00PM Dwinelle 229. RSVP at

`https://www.facebook.com/events/536143506535287/`

We already have almost 400 students attending."

CS61B Lecture #25

Today: Hashing (*Data Structures* Chapter 7).

Next topic: Sorting (*Data Structures* Chapter 8).

Back to Simple Search: Hashing

- Linear search is OK for small data sets, bad for large.
- So linear search would be OK *if* we could rapidly narrow the search to a few items.
- Suppose that in constant time could put any item in our data set into a numbered *bucket*, where # buckets stays within a constant factor of # keys.
- Suppose also that buckets contain roughly equal numbers of keys.
- Then search would be constant time.

Hash functions

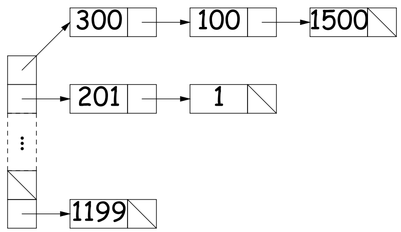
- To do this, must have way to convert key to bucket number: a *hash function*.

  "**hash** /hæʃ/ **2 a** a mixture; a jumble. **b** a mess."
  Concise Oxford Dictionary, eighth edition

- Example:
  - $N = 200$ data items.
  - keys are `longs`, evenly spread over the range $0..2^{63} - 1$.
  - Want to keep maximum search to $L = 2$ items.
  - Use hash function $h(K) = K\%M$, where $M = N/L = 100$ is the number of buckets: $0 \leq h(K) < M$.
  - So 100232, 433, and 10002332482 go into different buckets, but 10, 400210, and 210 all go into the same bucket.

External chaining

- Array of M buckets.
- Each bucket is a list of data items.



- Not all buckets have same length, but average is $N/M = L$, the *load factor*.
- To work well, hash function must avoid *collisions:* keys that "hash" to equal values.

Ditching the Chains: Open Addressing

- Idea: Put one data item in each bucket.
- When there is a collision, and bucket is full, just use another.
- Various ways to do this:
  - Linear probes: If there is a collision at $h(K)$, try $h(K)+m$, $h(K)+2m$, etc. (wrap around at end).
  - Quadratic probes: $h(K) + m$, $h(K) + m^2$, ...
  - Double hashing: $h(K) + h'(K)$, $h(K) + 2h'(K)$, etc.
- Example: $h(K) = K\%M$, with $M = 10$, linear probes with $m = 1$.
  - Add 1, 2, 11, 3, 102, 9, 18, 108, 309 to empty table.

108	1	2	11	3	102	309		18	9

- Things can get slow, even when table is far from full.
- Lots of literature on this technique, but
- Personally, I just settle for external chaining.

Filling the Table

- To get (likely to be) constant-time lookup, need to keep #buckets within constant factor of #items.
- So resize table when load factor gets higher than some limit.
- In general, must *re-hash* all table items.
- Still, this operation constant time per item,
- So by doubling table size each time, get constant *amortized* time for insertion and lookup
- (Assuming, that is, that our hash function is good).

Hash Functions: Strings

- For String, "$s_0 s_1 \cdots s_{n-1}$" want function that takes all characters and their positions into account.
- What's wrong with $s_0 + s_1 + \ldots + s_{n-1}$?
- For strings, Java uses

$$h(s) = s_0 \cdot 31^{n-1} + s_1 \cdot 31^{n-2} + \ldots + s_{n-1}$$

  computed modulo 2^{32} as in Java `int` arithmetic.
- To convert to a table index in $0..N-1$, compute `h(s)%N` (but *don't* use table size that is multiple of 31!)
- Not as hard to compute as you might think; don't even need multiplication!

```
int r; r = 0;
for (int i = 0; i < s.length (); i += 1)
   r = (r << 5) - r + s.charAt (i);
```

Hash Functions: Other Data Structures I

- Lists (`ArrayList`, `LinkedList`, etc.) are analagous to strings: e.g., Java uses

```
hashCode = 1; Iterator i = list.iterator();
while (i.hasNext()) {
   Object obj = i.next();
   hashCode =
     31*hashCode
     + (obj==null ? 0 : obj.hashCode());
}
```

- Can limit time spent computing hash function by not looking at entire list. For example: look only at first few items (if dealing with a `List` or `SortedSet`).
- Causes more collisions, but does *not* cause equal things to go to different buckets.

Hash Functions: Other Data Structures II

- Recursively defined data structures ⇒ recursively defined hash functions.
- For example, on a binary tree, one can use something like

```
hash(T):
    if (T == null)
       return 0;
    else return someHashFunction (T.label ())
                ^ hash(T.left ()) ^ hash(T.right ());
```


Identity Hash Functions

- Can use address of object ("hash on identity") if distinct (`!=`) objects are never considered equal.
- But careful! Won't work for Strings, because `.equal` Strings could be in different buckets:

```
String H  = "Hello",
       S1 = H + ", world!",
       S2 = "Hello, world!";
```

- Here `S1.equals(S2)`, but `S1 != S2`.

What Java Provides

- In class `Object`, is function `hashCode()`.
- By default, returns the identity hash function, or something similar. [Why is this OK as a default?]
- Can override it for your particular type.
- For reasons given on last slide, is overridden for type `String`, as well as many types in the Java library, like all kinds of `List`.
- The types `Hashtable`, `HashSet`, and `HashMap` use `hashCode` to give you fast look-up of objects.

```
HashMap<KeyType,ValueType> map =
     new HashMap<>(approximate size, load factor);
map.put (key, value);         // Map KEY -> VALUE.
... map.get (someKey)         // VALUE last mapped to by SOMEKEY.
... map.containsKey(someKey)  // Is SOMEKEY mapped?
... map.keySet()              // All keys in MAP (a Set)
```

Special Case: Monotonic Hash Functions

- Suppose our hash function is *monotonic:* either nonincreasing or nondescreasing.
- So, e.g., if key $k_1 > k_2$, then $h(k_1) \geq h(k_2)$.
- Example:
  - Items are time-stamped records; key is the time.
  - Hashing function is to have one bucket for every hour.
- In this case, you *can* use a hash table to speed up range queries [How?]
- Could this be applied to strings? When would it work well?

Perfect Hashing

- Suppose set of keys is *fixed*.
- A tailor-made hash function might then hash every key to a different value: *perfect hashing*.
- In that case, there is no search along a chain or in an open-address table: either the element at the hash value is or is not equal to the target key.
- For example, might use first, middle, and last letters of a string (read as a 3-digit base-26 numeral). Would work if those letters differ among all strings in the set.
- Or might use the Java method, but tweak the multipliers until all strings gave different results.

Characteristics

- Assuming good hash function, add, lookup, deletion take $\Theta(1)$ time, amortized.
- Good for cases where one looks up *equal* keys.
- Usually bad for *range queries:* "Give me every name between Martin and Napoli." [Why?]
- Hashing is probably not a good idea for small sets that you rapidly create and discard [why?]

Comparing Search Structures

Here, N is #items, k is #answers to query.

Function	Unordered List	Sorted Array	Bushy Search Tree	"Good" Hash Table	Heap
find	$\Theta(N)$	$\Theta(\lg N)$	$\Theta(\lg N)$	$\Theta(1)$	$\Theta(N)$
add	$\Theta(1)$	$\Theta(N)$	$\Theta(\lg N)$	$\Theta(1)$	$\Theta(\lg N)$
range query	$\Theta(N)$	$\Theta(k + \lg N)$	$\Theta(k + \lg N)$	$\Theta(N)$	$\Theta(N)$
find largest	$\Theta(N)$	$\Theta(1)$	$\Theta(\lg N)$	$\Theta(N)$	$\Theta(1)$
remove largest	$\Theta(N)$	$\Theta(1)$	$\Theta(\lg N)$	$\Theta(N)$	$\Theta(\lg N)$