

- **Today:** Minimum spanning trees, recursive graph algorithms, union-find.

### Minimum Spanning Trees

- **Problem:** Given a set of places and distances between them (assume always positive), find a set of connecting roads of minimum total length that allows travel between any two.
- The routes you get will not necessarily be shortest paths.
- Easy to see that such a set of connecting roads and places must form a tree, because removing one road in a cycle still allows all to be reached.

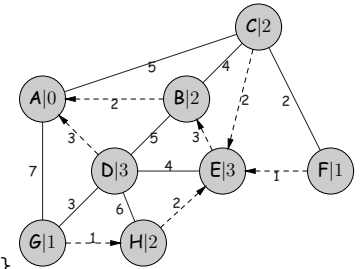
### Minimum Spanning Trees by Prim's Algorithm

- Idea is to grow a tree starting from an arbitrary node.
- At each step, add the shortest edge connecting some node already in the tree to one that isn't yet.
- Why must this work?

```

PriorityQueue fringe;
For each node v { v.dist() = ∞; v.parent() = null; }
Choose an arbitrary starting node, s;
s.dist() = 0;
fringe = priority queue ordered by smallest .dist();
add all vertices to fringe;
while (! fringe.isEmpty()) {
    Vertex v = fringe.removeFirst ();

    For each edge (v,w) {
        if (w ∈ fringe && weight(v,w) < w.dist())
            { w.dist() = weight (v, w); w.parent() = v; }
    }
}
    
```



### Minimum Spanning Trees by Kruskal's Algorithm

- Observation: the shortest edge in a graph can always be part of a minimum spanning tree.
- In fact, if we have a bunch of subtrees of a MST, then the shortest edge that connects two of them can be part of a MST, combining the two subtrees into a bigger one.
- So,...

```

Create one (trivial) subtree for each node in the graph;
MST = {};

for each edge (v,w), in increasing order of weight {
    if ( (v,w) connects two different subtrees ) {
        Add (v,w) to MST;
        Combine the two subtrees into one;
    }
}
    
```

### Recursive Depth-First Traversal

- Previously, we saw an iterative way to do depth-first traversal of a graph from a particular node.
- We are often interested in traversing all nodes of a graph, so we can repeat the procedure as long as there are unmarked nodes.
- Recursive solution is also simple:

```

void traverse (Graph G) {
    for (v ∈ nodes of G) {
        traverse (G, v);
    }
}

void traverse (Graph G, Node v) {
    if (v is unmarked) {
        mark (v);
        visit v;
        for (Edge (v, w) ∈ G)
            traverse (G, w);
    }
}
    
```

## Another Take on Topological Sort

- Observation: if we do a depth-first traversal on a DAG whose edges are reversed, and execute the recursive traverse procedure, we finish executing  $\text{traverse}(G, v)$  in proper topologically sorted order.

```
void topologicalSort (Graph G) {
  for (v ∈ nodes of G) {
    traverse (G, v);
  }

  void traverse (Graph G, Node v) {
    if (v is unmarked) {
      mark (v);
      for (Edge (w, v) ∈ G)
        traverse (G, w);
      add v to the result list;
    }
  }
}
```

Last modified: Tue Dec 4 19:14:47 2007

CS61B: Lecture #39 5

## Union Find

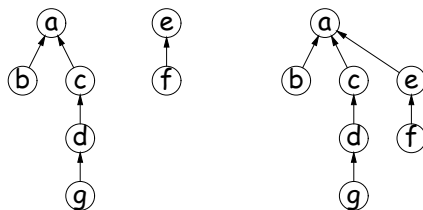
- Kruskal's algorithm required that we have a set of sets of nodes with two operations:
  - Find which of the sets a given node belongs to.
  - Replace two sets with their union, reassigning all the nodes in the two original sets to this union.
- Obvious thing to do is to store a set number in each node, making finds fast.
- Union requires changing the set number in one of the two sets being merged; the smaller is better choice.
- This means an individual union can take  $\Theta(N)$  time.
- Can union be fast?

Last modified: Tue Dec 4 19:14:47 2007

CS61B: Lecture #39 6

## A Clever Trick

- Let's choose to represent a set of nodes by *one* arbitrary representative node in that set.
- Let every node contain a pointer to another node in the same set.
- Arrange for each pointer to represent the *parent* of a node in a tree that has the representative node as its root.
- To find what set a node is in, follow parent pointers.
- To union two such trees, make one root point to the other (choose the root of the higher tree as the union representative).

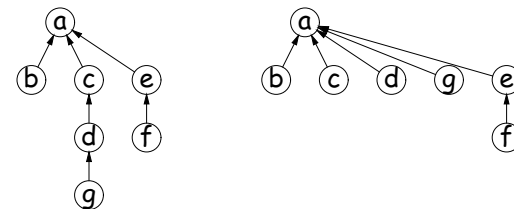


Last modified: Tue Dec 4 19:14:47 2007

CS61B: Lecture #39 7

## Path Compression

- This makes unioning really fast, but the find operation potentially slow ( $\Omega(\lg N)$ ).
- So use the following trick: whenever we do a *find* operation, *compress* the path to the root, so that subsequent finds will be faster.
- That is, make each of the nodes in the path point directly to the root.
- Now union is very fast, and sequence of unions and finds each have very, very nearly constant amortized time.
- Example: find 'g' in last tree (result of compression on right):



Last modified: Tue Dec 4 19:14:47 2007

CS61B: Lecture #39 8