

**Administrative:**

- Midterm graded:
  - Intended average: 20
  - Actual average: 14
  - Actual median: 13
- No; the midterm is not curved.
- Yes; I am going to test this material again.

**Today:**

- Priority queues (*Data Structures* §6.4, §6.5)
- Range queries (§6.2)
- Java utilities: SortedSet, Map, etc.

**Next topic:** Hashing (*Data Structures* Chapter 7).

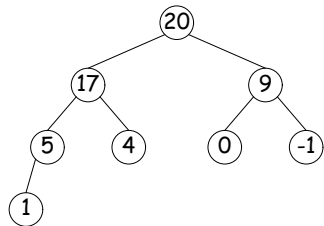
- Priority queue: defined by operations "add," "find largest," "remove largest."
- Examples: scheduling long streams of actions to occur at various future times.
- Also useful for sorting (keep removing largest).
- Heap is common implementation.
- Enforces *heap property*: all labels in *both* children of node are less (or greater) than node's label.
- So node at top has largest (or smallest) label.
- Are free to add smaller value to less bushy subtree, thus maintaining bushiness (keeping tree balanced).
- Insertion and deletion always proportional to  $\lg N$  in worst case.

**Example: Inserting into a simple heap**

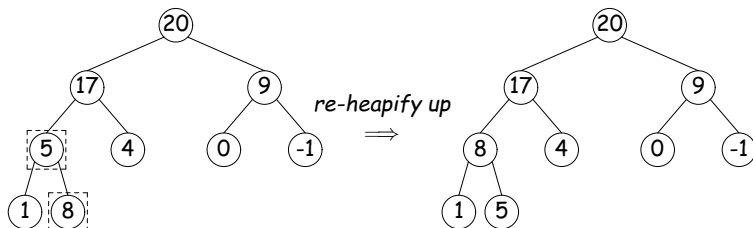
**Data:**

1 17 4 5 9 0 -1 20

**Initial Heap:**

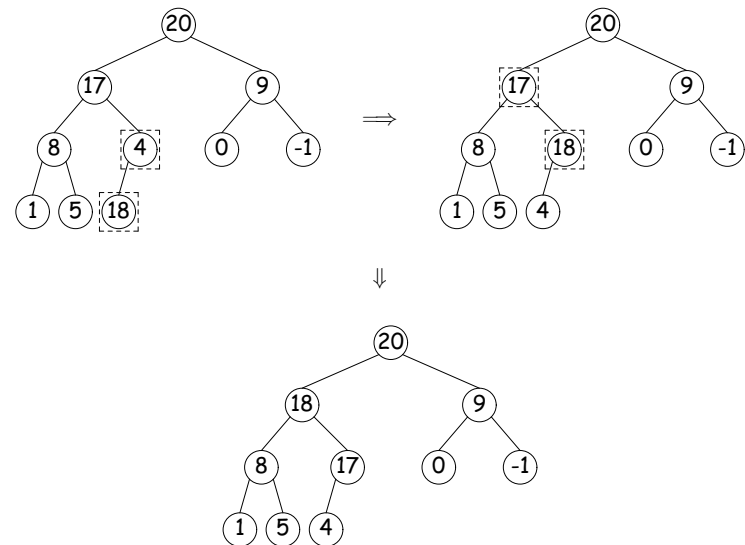


**Add 8:** Dashed boxes show where heap property violated



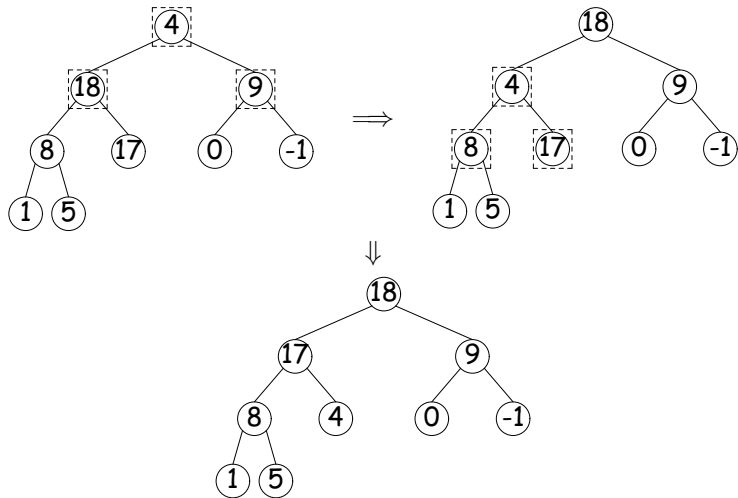
**Heap insertion continued**

**Now insert 18:**



## Removing Largest from Heap

To remove largest: Move bottommost, rightmost node to top, then re-heapify down as needed (swap offending node with larger child) to re-establish heap property.

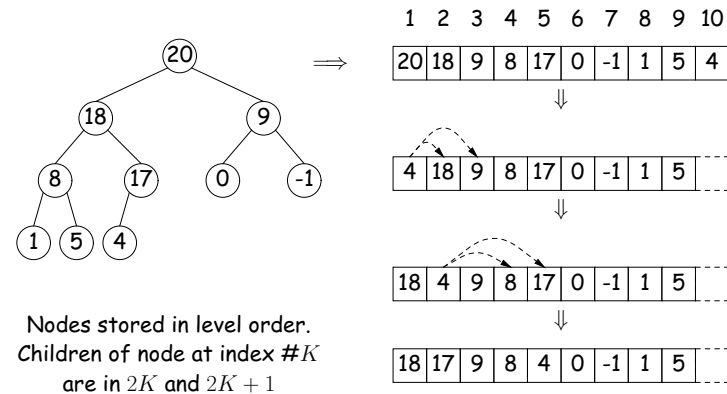


Last modified: Tue Oct 23 18:24:38 2007

CS61B: Lecture #25 5

## Heaps in Arrays

- Since heaps are complete (missing items only at bottom level), can use arrays for compact representation.
- Example of removal from last slide (dashed arrows show children):



Last modified: Tue Oct 23 18:24:38 2007

CS61B: Lecture #25 6

## Ranges

- So far, have looked for specific items
- But for BSTs, need an ordering anyway, and can also support looking for *ranges of values*.
- Example: perform some action on all values in a BST that are within some range (in natural order):

```

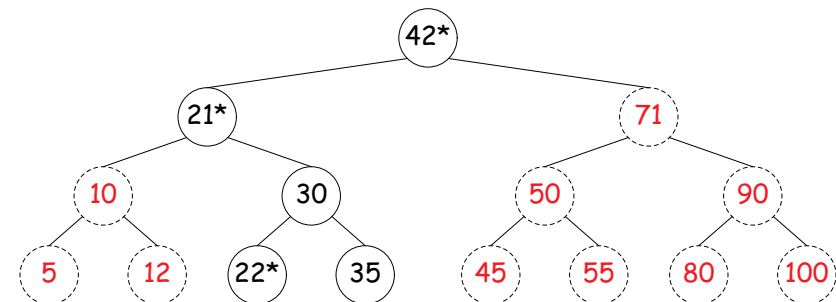
/** Apply WHATTODO to all labels in T that are
 * >= L and < U, in ascending natural order. */
static void visitRange (BST T, Comparable<Key> L, Comparable<Key> U,
                        Action whatToDo)
{
    if (T != null) {
        int compLeft = L.compareTo (T.label ()),
            compRight = U.compareTo (T.label ());
        if (compLeft < 0) /* L < label */
            visitRange (T.left (), L, U, whatToDo);
        if (compLeft <= 0 && compRight > 0) /* L <= label < U */
            whatToDo.action (T);
        if (compRight > 0) /* label < U */
            visitRange (T.right (), L, U, whatToDo);
    }
}
    
```

Last modified: Tue Oct 23 18:24:38 2007

CS61B: Lecture #25 7

## Time for Range Queries

- Time for range query  $\in O(h + M)$ , where  $h$  is height of tree, and  $M$  is number of data items that turn out to be in the range.
- Consider searching the tree below for all values,  $x$ , such that  $25 \leq x < 40$ .
- In this example, the  $h$  comes from the starred nodes; the  $M$  comes from other non-dashed nodes. **Dashed** nodes are never looked at.



Last modified: Tue Oct 23 18:24:38 2007

CS61B: Lecture #25 8

## Ordered Sets and Range Queries in Java

- Class SortedSet supports range queries with *views* of set:

- S.headSet(U): subset of S that is  $< U$ .
- S.tailSet(L): subset that is  $\geq L$ .
- S.subSet(L,U): subset that is  $\geq L, < U$ .

- Changes to views modify S.

- Attempts to, e.g., add to a headSet beyond U are disallowed.

- Can iterate through a view to process a range:

```
SortedSet<String> fauna = new TreeSet<String>
    (Arrays.asList ("axolotl", "elk", "dog", "hartebeest", "duck"));
for (String item : fauna.subSet ("bison", "gnu"))
    System.out.printf ("%s, ", item);
```

would print "dog, duck, elk,"

- Java library type TreeSet<T> requires either that T be Comparable, or that you provide a Comparator:

```
SortedSet<String> rev_fauna = new TreeSet<String> (Collections.reverseOrder());
```

## Example of Representation: BSTSet

- Use binary search tree to represent set. Can use same representation for both BSTSet and its subsets.
- Each set has pointer to BST, plus bounds (if any).
- In this representation, size is rather expensive!

```
SortedSet<String>
    fauna = new BSTSet<String> (collection of stuff);
    subset = fauna.subSet ("bison", "gnu");
Iterator<String> i = subset.iterator ();
```

