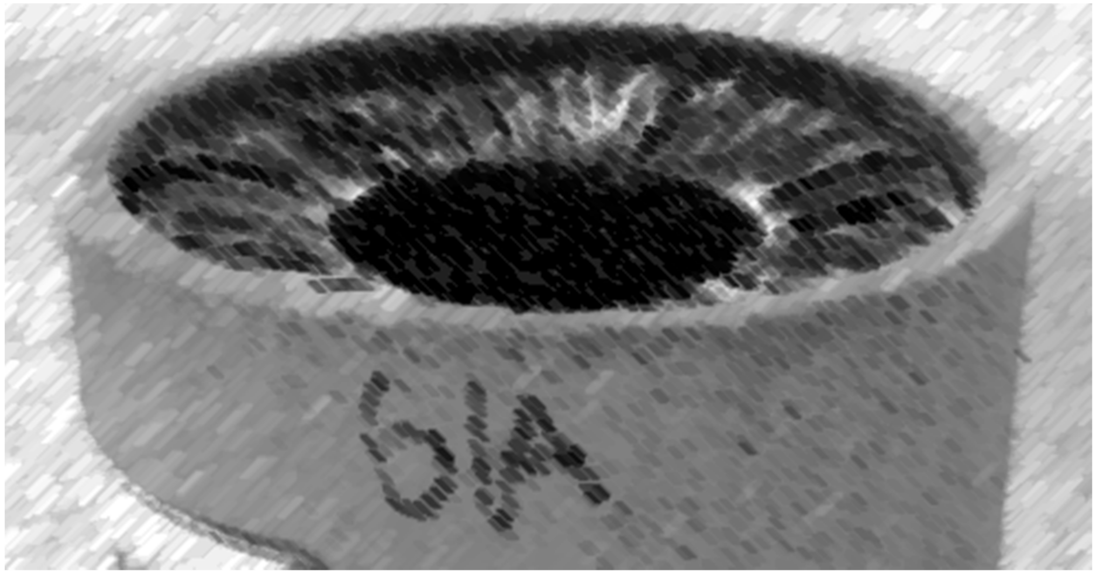

CS61A Lecture 8

Amir Kamil
UC Berkeley
February 8, 2013

Announcements

- HW3 out, due Tuesday at 7pm
- Midterm next Wednesday at 7pm
  - Keep an eye out for your assigned location
  - Old exams posted
  - Review sessions
    - Saturday 2-4pm in 2050 VLSB
    - Extended office hours Sunday 11-3pm in 310 Soda
    - HKN review session Sunday 3-6pm in 145 Dwinelle
- Environment diagram handout on website
- Code review system online
  - See Piazza post for details

Newton's Method

Begin with a function f and

an initial guess x

Visualization: http://en.wikipedia.org/wiki/File:NewtonIteration_Ani.gif

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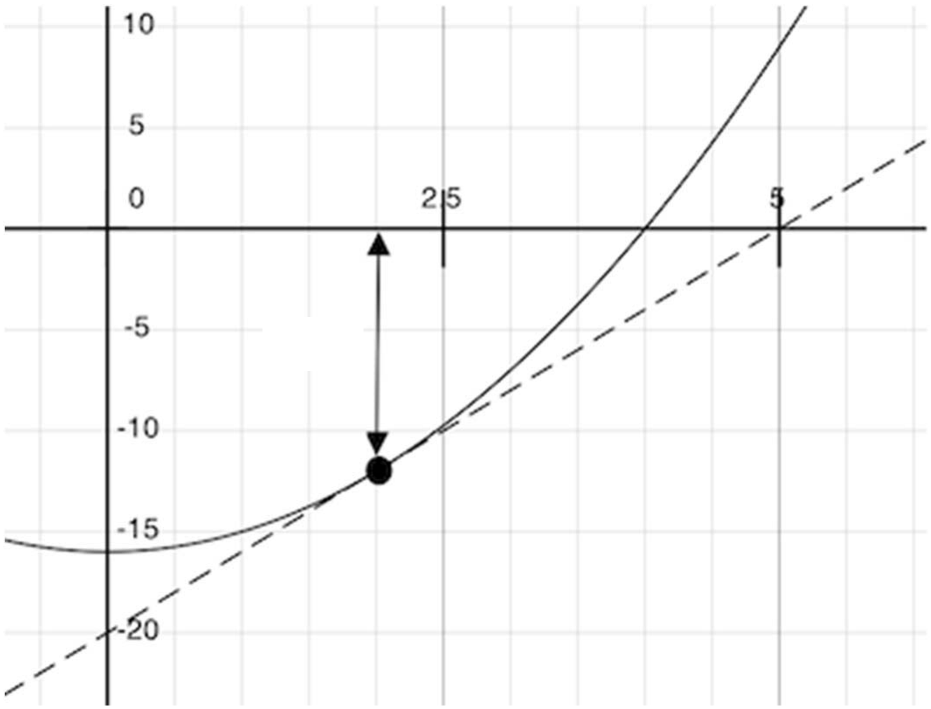

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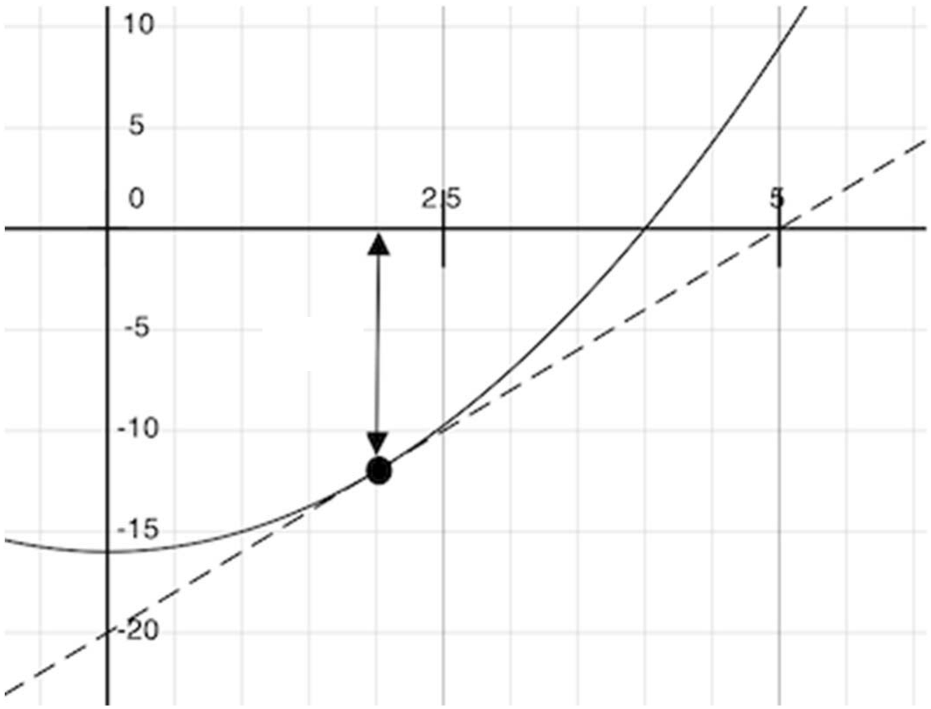

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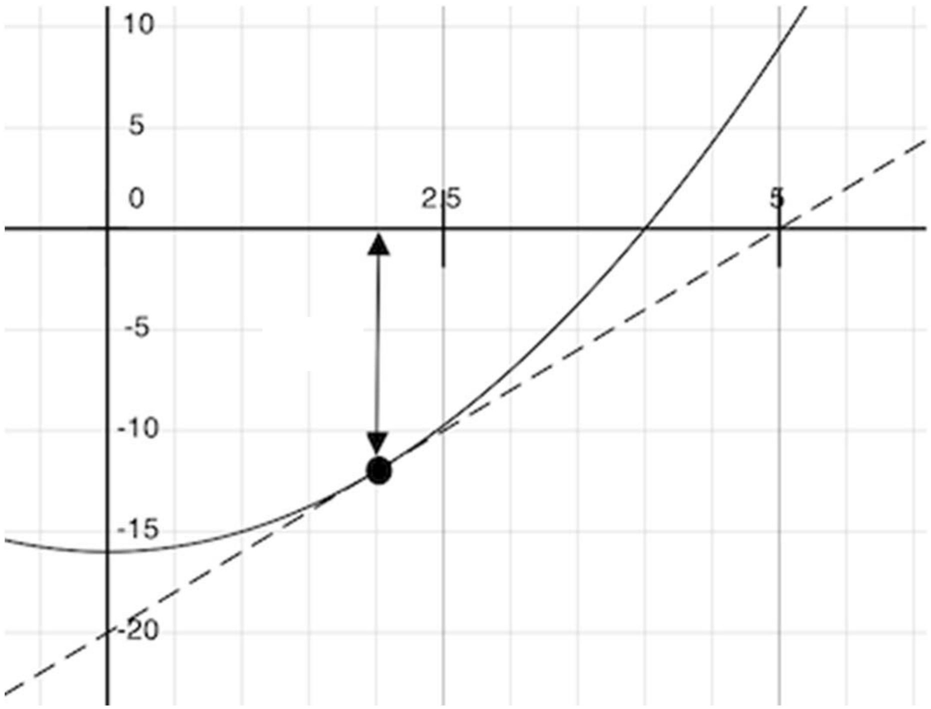

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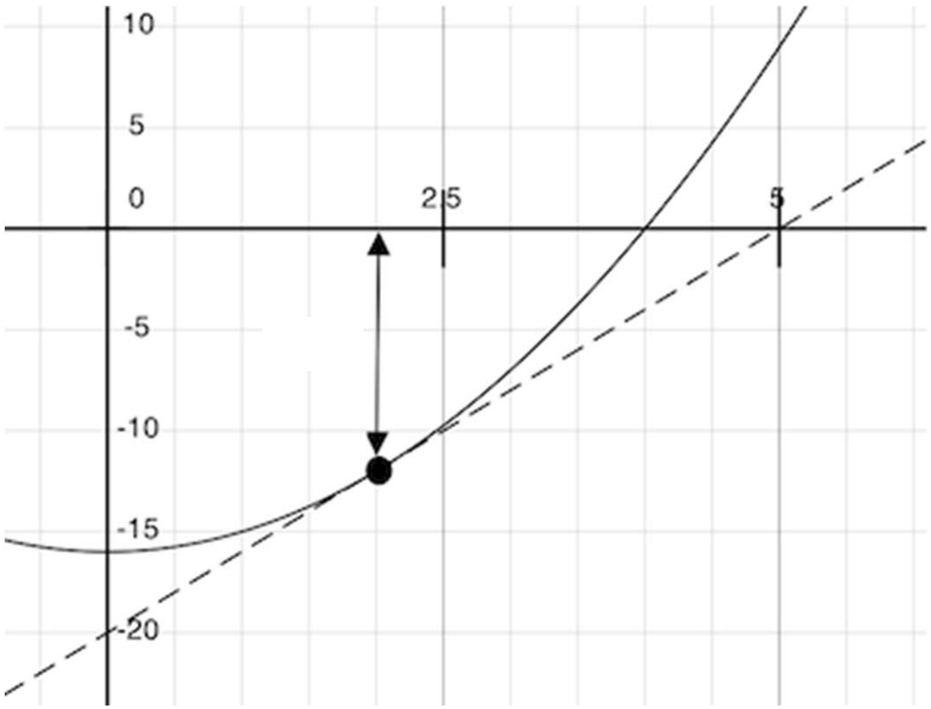

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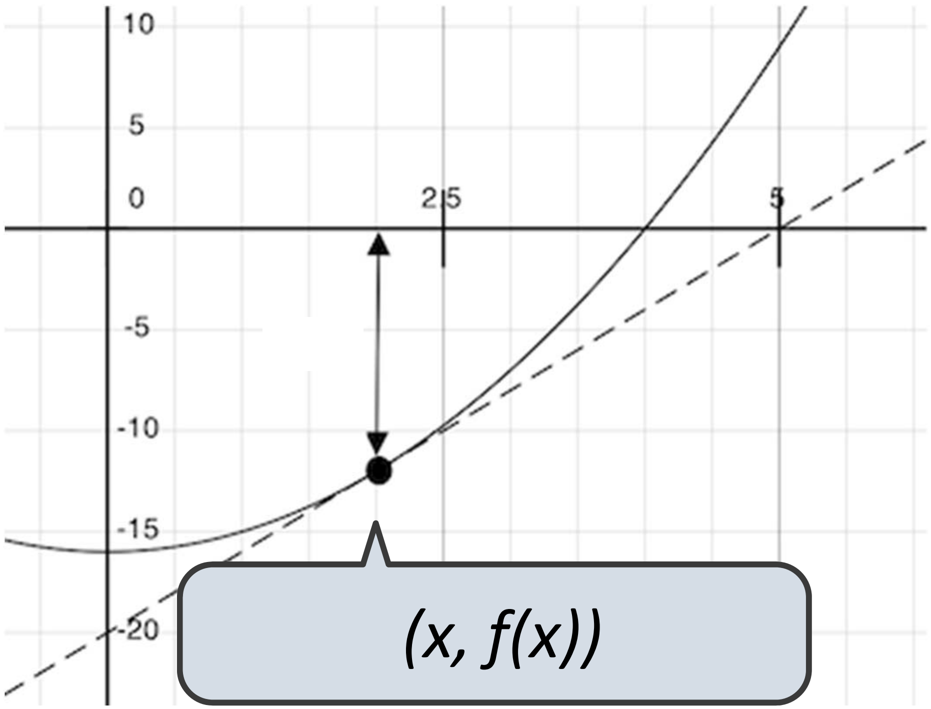

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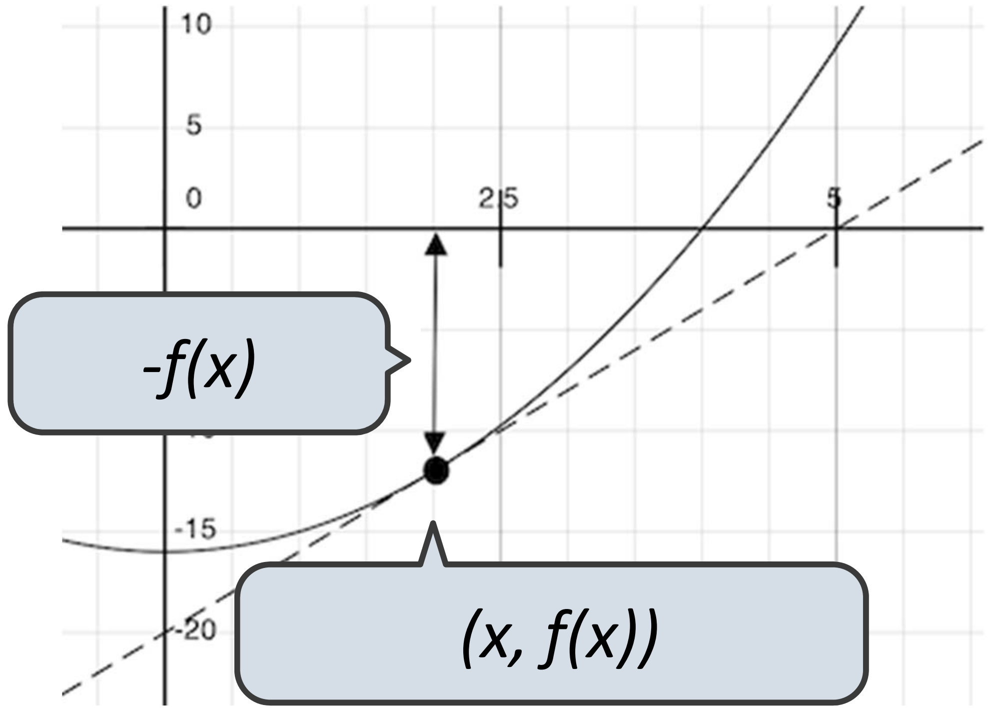

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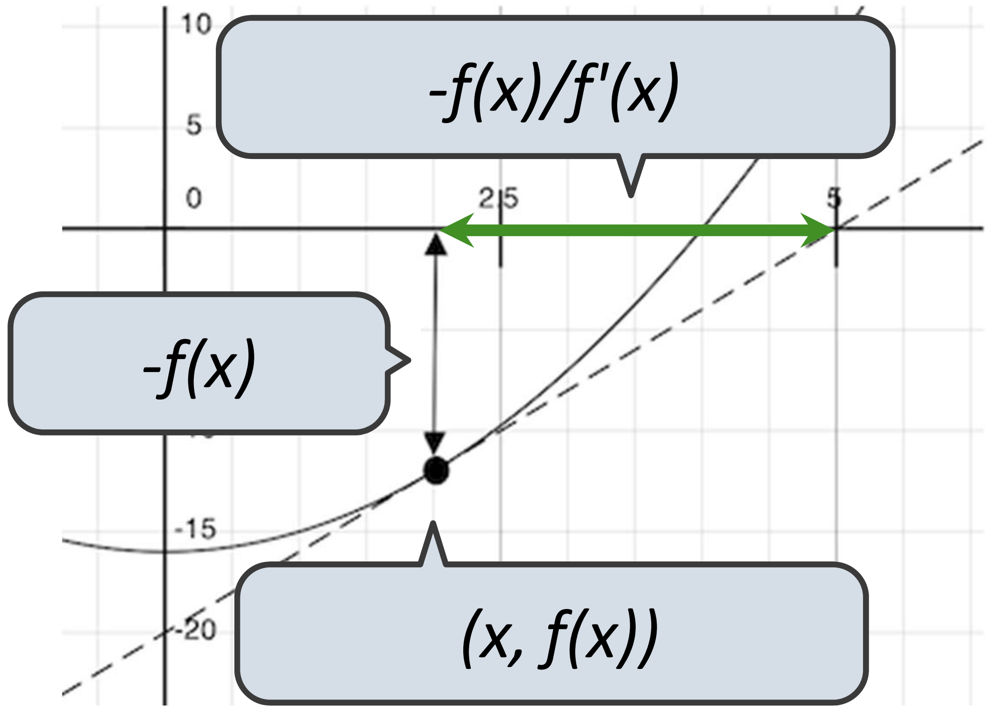

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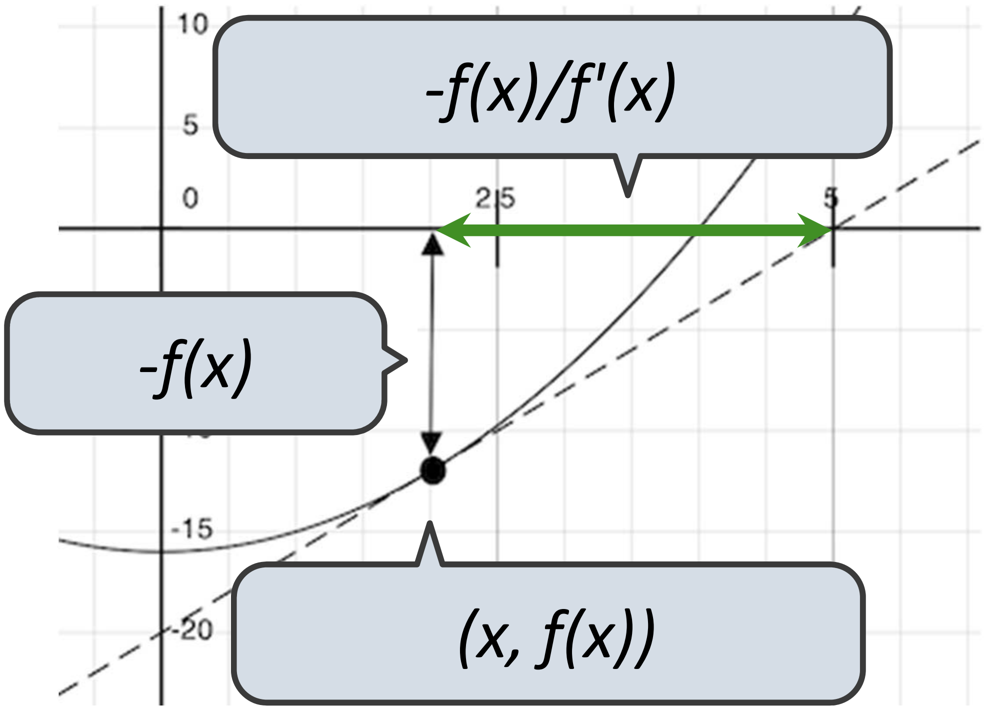

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```
def iter_improve(update, done, guess=1, max_updates=1000):
    """Iteratively improve guess with update until done
    returns a true value.

    >>> iter_improve(golden_update, golden_test)
    1.618033988749895
    """
    k = 0
    while not done(guess) and k < max_updates:
        guess = update(guess)
        k = k + 1
    return guess
```

Newton's Method for nth Roots

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```
def nth_root_func_and_derivative(n, a):
    def root_func(x):
        return pow(x, n) - a
    def derivative(x):
        return n * pow(x, n-1)
    return root_func, derivative

def nth_root_newton(a, n):
    """Return the nth root of a.

    >>> nth_root_newton(8, 3)
    2.0
    """
    root_func, deriv = nth_root_func_and_derivative(n, a)
    def update(x):
        return x - root_func(x) / deriv(x)
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Definition of a function zero

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Can we write code to compute factorial using the same pattern?

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def factorial(n):
    if n == 0 or n == 1:
        return 1
    total = 1
    while n >= 1:
        total, n = total * n, n - 1
    return total
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This is much shorter! But can a function call itself?

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Example: http://goo.gl/NjCKG

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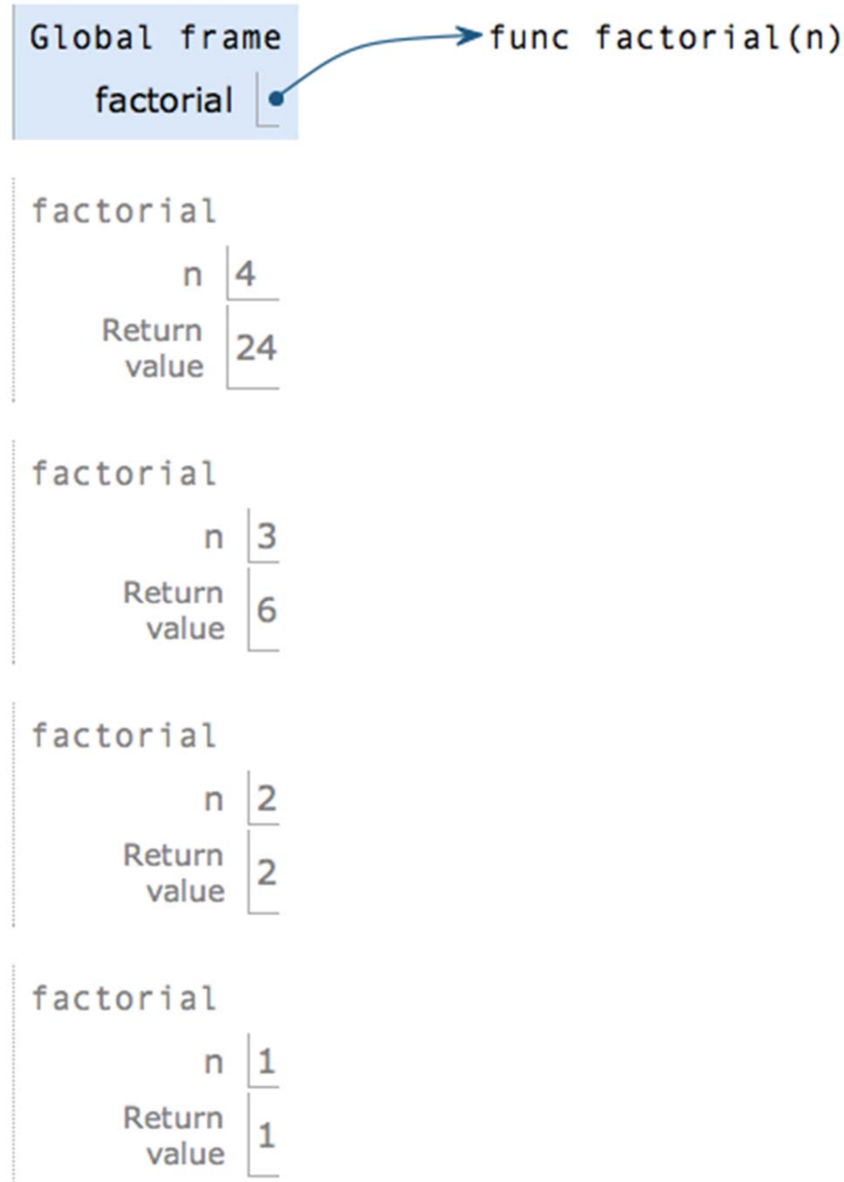

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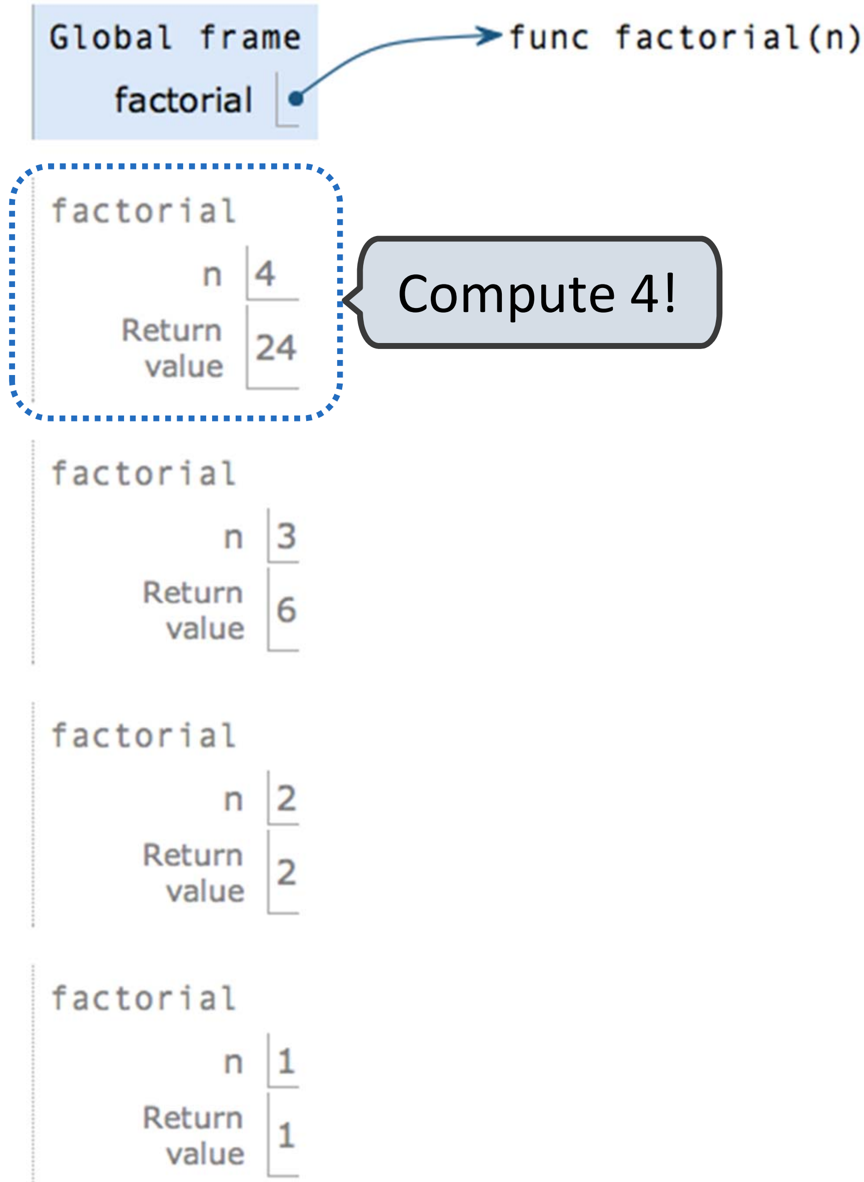

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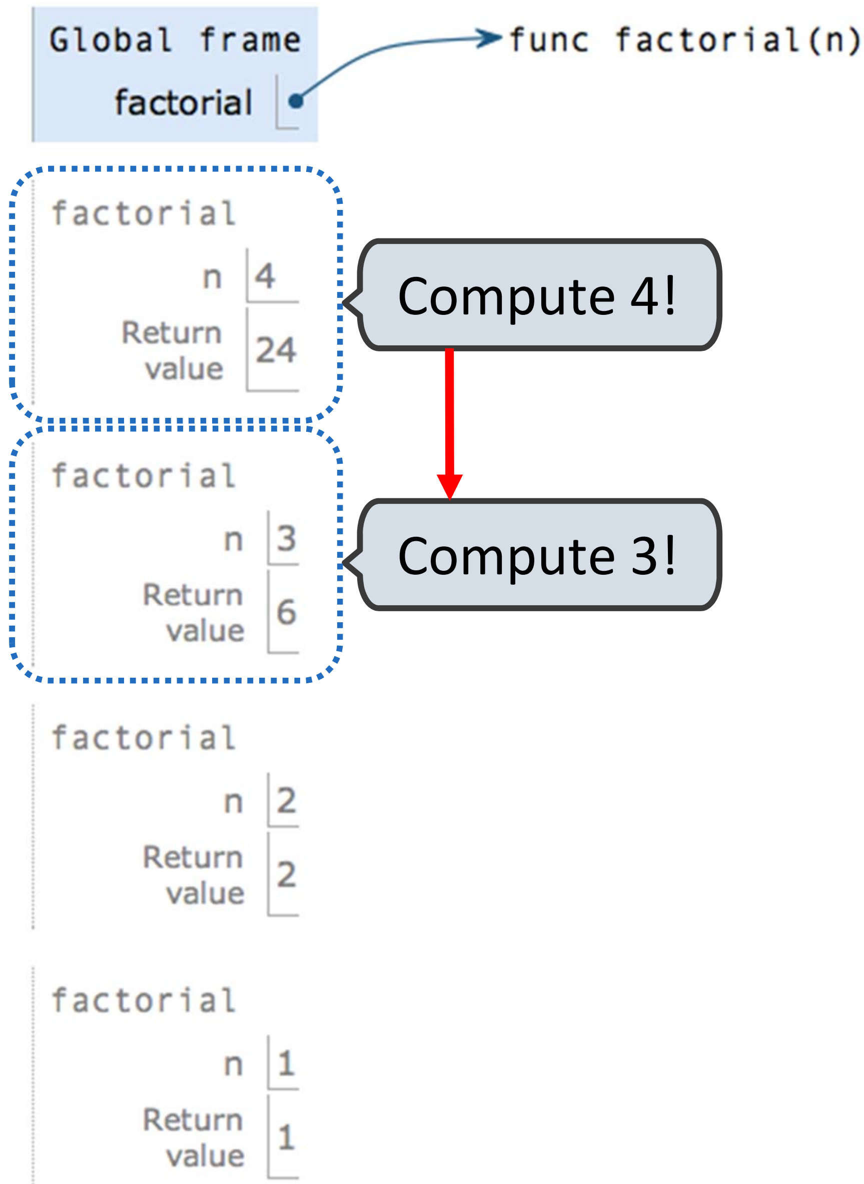

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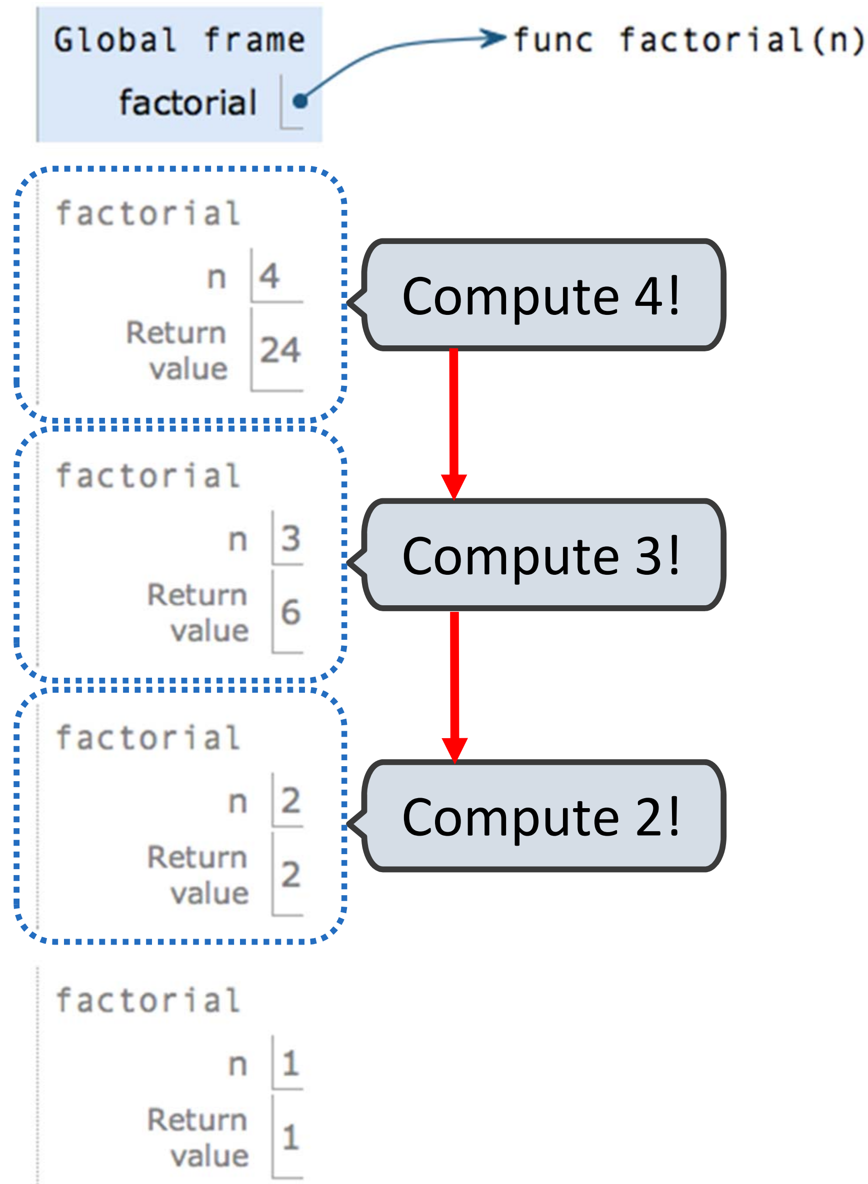

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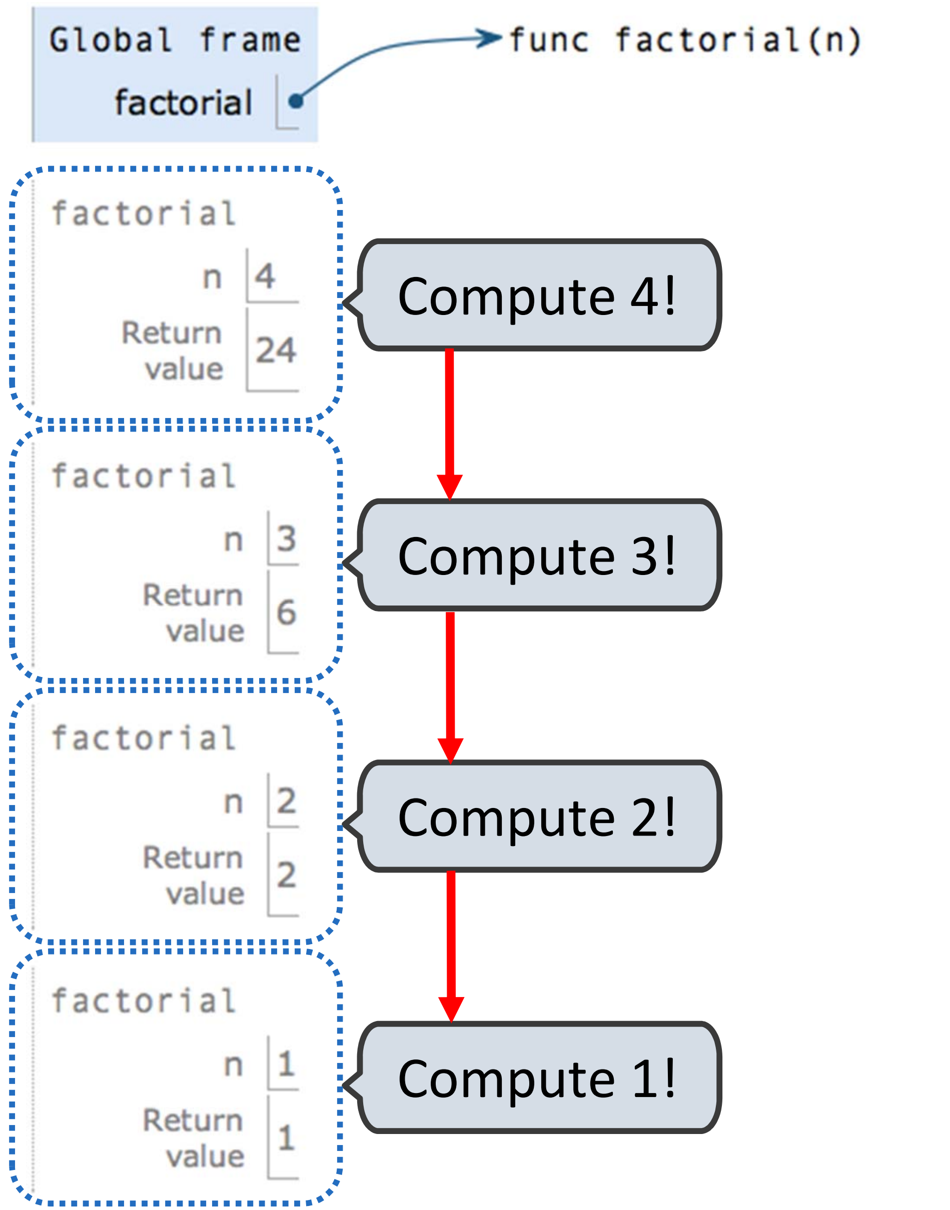

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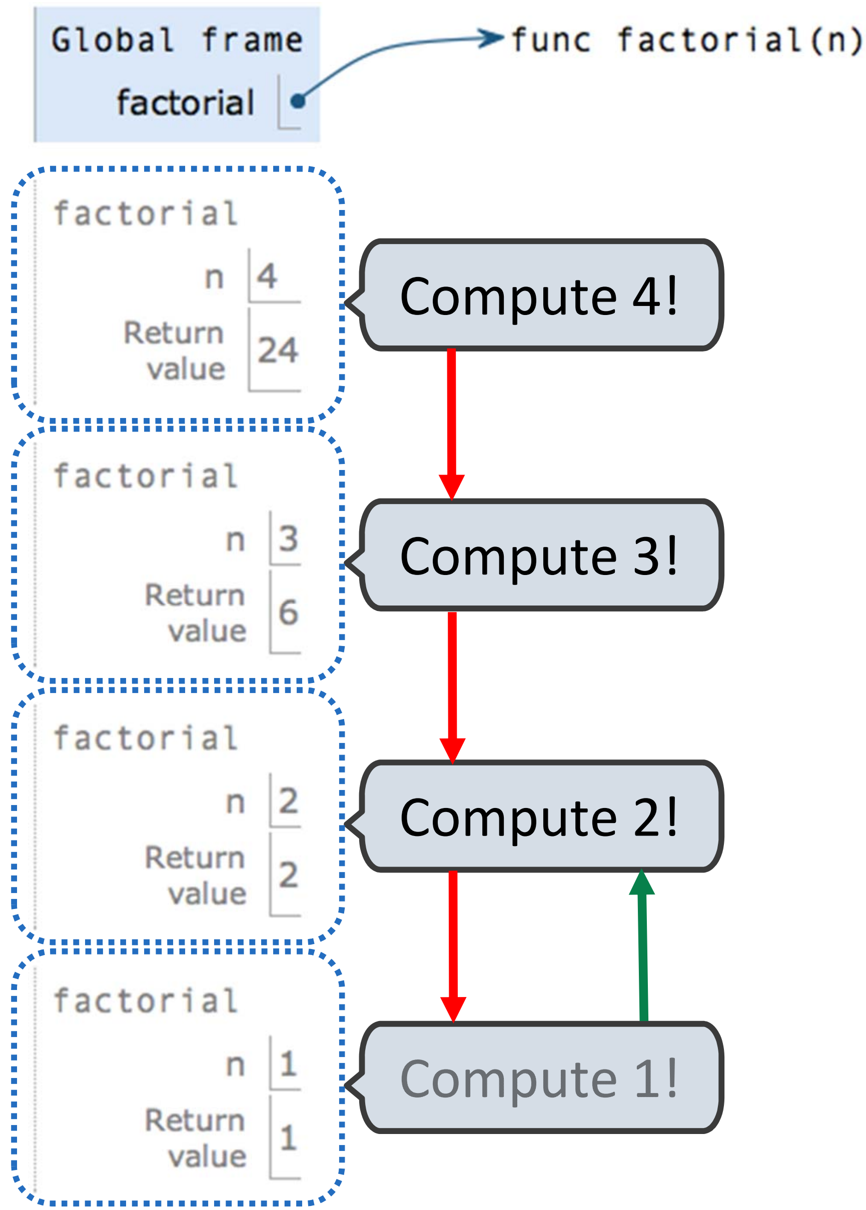

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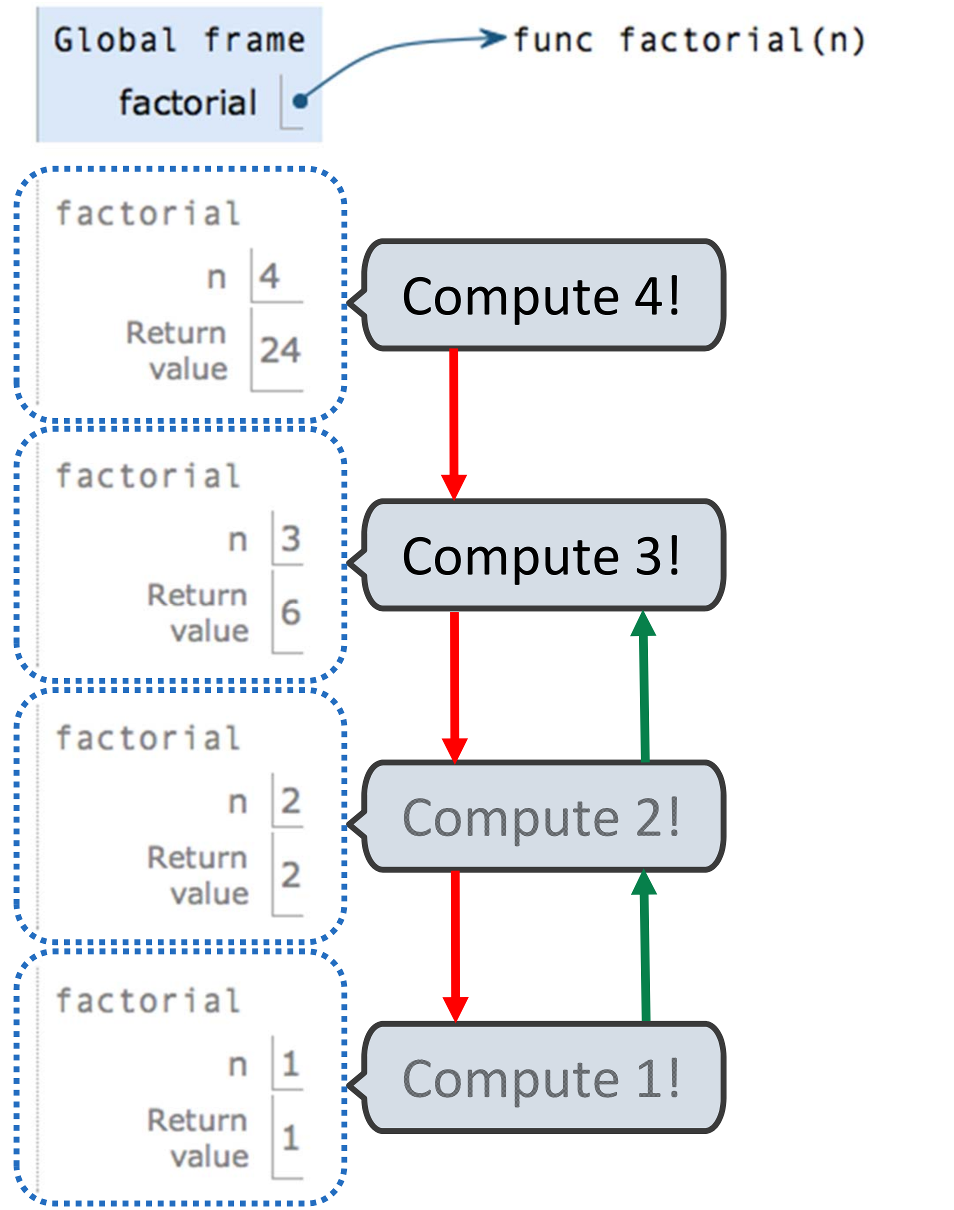

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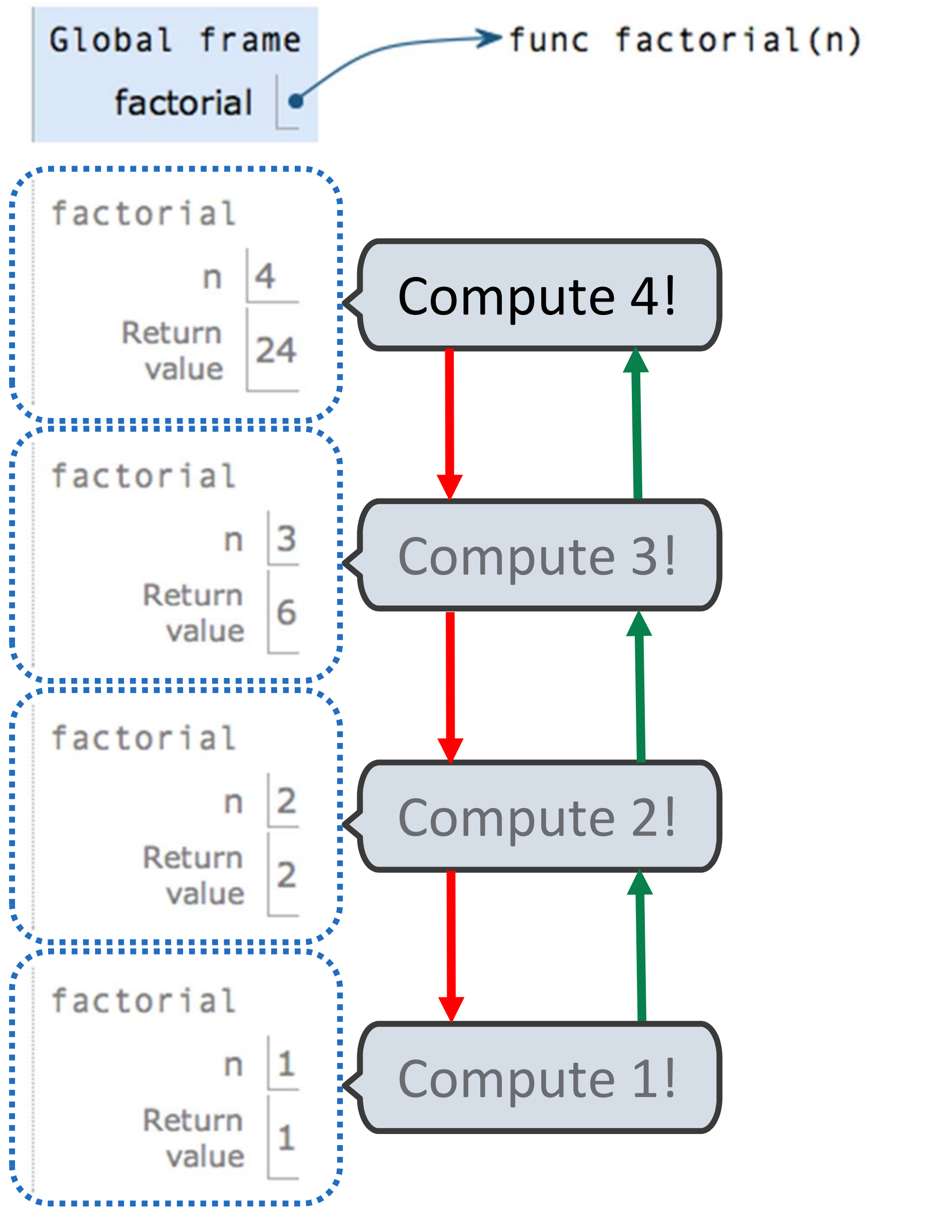

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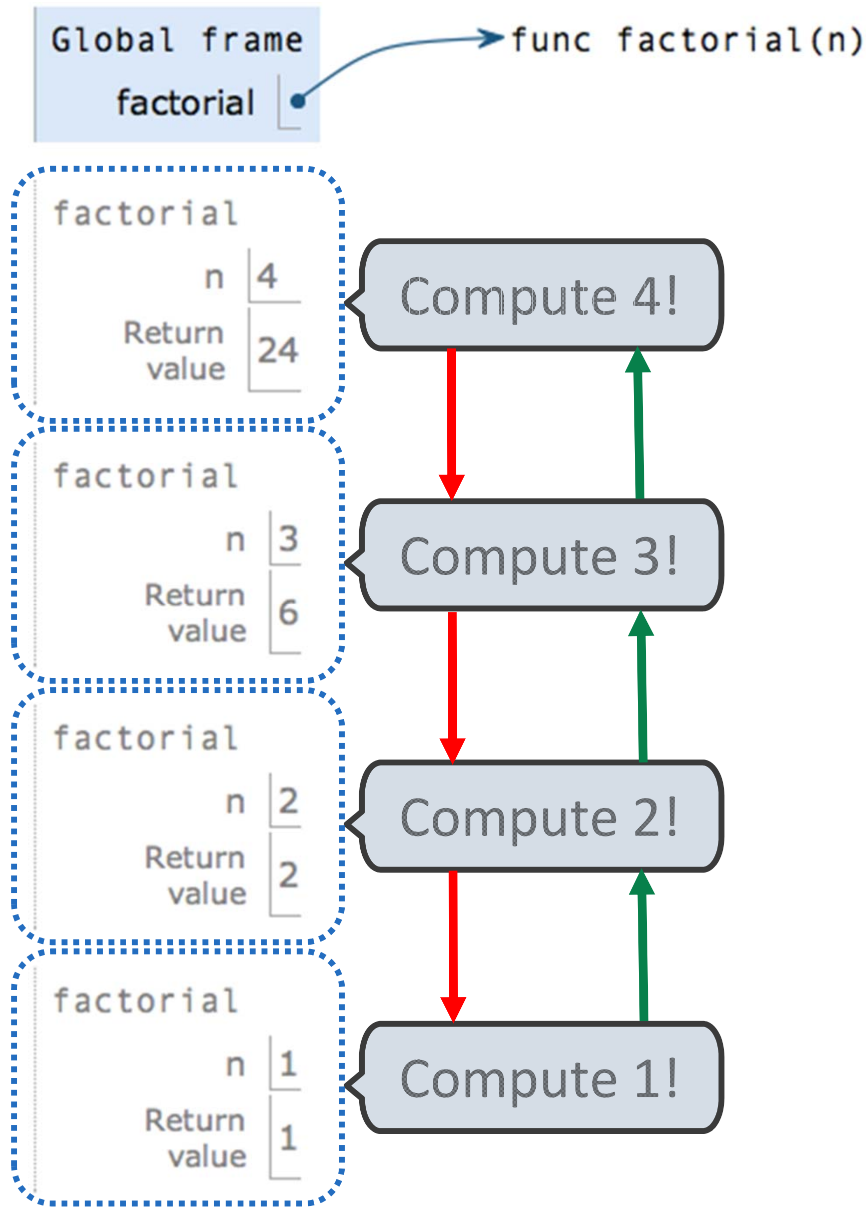

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Recursive Functions

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Recursive Functions

A function is *recursive* if the body calls the function itself, either directly or indirectly

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Use names for repeated compound expressions

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```
if sqrt(square(a) + square(b)) > 1:
    x = x + sqrt(square(a) + square(b))
```

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if sqrt(square(a) + square(b)) > 1:
    x = x + sqrt(square(a) + square(b))
```

↘

```
h = sqrt(square(a) + square(b))
if h > 1:
    x = x + h
```

Practical Guidance: Choosing Names

Names typically don't matter for correctness,

but they matter tremendously for legibility

`boolean` → `turn_is_over`   `d` → `dice`   `play_helper` → `take_turn`

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Use names for meaningful parts of compound expressions

```
x = (-b + sqrt(square(b) - 4 * a * c)) / (2 * a)
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x = (-b + sqrt(square(b) - 4 * a * c)) / (2 * a)
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↓

```
disc_term = sqrt(square(b) - 4 * a * c)
x = (-b + disc_term) / (2 * a)
```

Practical Guidance: DRY

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```
def find_quadratic_root(a, b, c, plus=True):
    """Applies the quadratic formula to the polynomial
    ax^2 + bx + c."""
    if plus:
        return (-b + sqrt(square(b) - 4 * a * c)) / (2 * a)
    else:
        return (-b - sqrt(square(b) - 4 * a * c)) / (2 * a)
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```
def find_quadratic_root(a, b, c, plus=True):
    """Applies the quadratic formula to the polynomial
    ax^2 + bx + c."""
    disc_term = sqrt(square(b) - 4 * a * c)
    if not plus:
        disc_term *= -1
    return (-b + disc_term) / (2 * a)
```

Test-Driven Development

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- Run your old tests again after you make new changes