

## 61A Lecture 36

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Friday, December 6

## Announcements

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- Homework 12 due Tuesday 12/10 @ 11:59pm.
  - All you have to do is vote on your favorite recursive art.
- 29 review sessions next week! Come learn about the topics that interest you the most.
  - See <http://inst.eecs.berkeley.edu/~cs61a/fa13/exams/final.html> for the schedule.
- The final exam is on Friday 12/20 @ 11:30am in the RSF gym, emphasizing:
  - Higher-order functions
  - Sequences (tuples, lists, recursive lists, Scheme lists)
  - Non-local assignment and mutation
  - Object-oriented programming
  - Recursion and recursive data
  - Iterators, generators, and streams

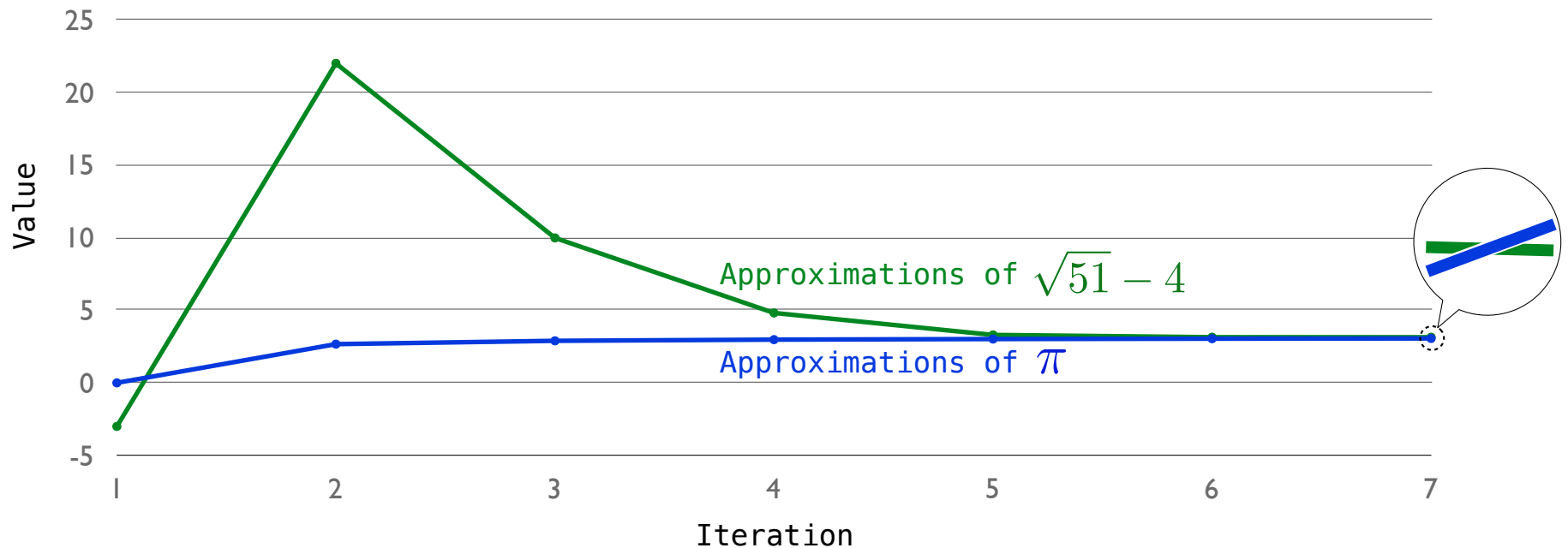
## Implicit Sequences Example

## Example: Numerical Approximations

Is  $\sqrt{51} - 4 < \pi$  ?

***No calculators/interpreters allowed!***

Let's say we have a computer that can  $+$ ,  $-$ ,  $*$ ,  $/$ . How do we answer this question?



## Approximating Square Roots

Is  $\sqrt{51} - 4 < \pi$  ?

**No calculators/interpreters allowed!**

Let's say we have a computer that can  $+$ ,  $-$ ,  $*$ ,  $/$ . How do we answer this question?

(A) A sequence of approximations (SoA) to  $y$  is an infinite sequence that converges to  $y$ .  
Implicitly define a SoA to  $\sqrt{a}$ .

```
def sqrt(a):
    x = 1
    while True:
        yield x
        x = (x + a/x)/2

>>> for x in sqrt(2):
...     print(x)
1
1.5
1.4166666666666665
1.4142156862745097
...
```

How to compute `square_root(a)`:

**Idea:** Iteratively refine a guess  $x$  about the square root of  $a$ .

$$x = \frac{x + \frac{a}{x}}{2}$$

*From lecture 6*

## Approximating Pi

Is  $\sqrt{51} - 4 < \pi$  ?

```
def sqrt(a):  
    x = 1  
    while True:  
        yield x  
        x = (x + a/x)/2
```

(B) Define a sequence of approximations to  $\pi$ .

```
def pi():  
    total, k = 0, 1  
    while True:  
        yield total  
        total += 8 / ( (4*k-3) * (4*k-1) )  
        k += 1
```

$$\sum_{k=1}^{\infty} \frac{8}{(4k-3) \cdot (4k-1)} = \pi$$

*From lecture 4*

```
>>> for x in pi():  
    ...     print(x)  
0  
2.6666666666666665  
2.895238095238095  
2.976046176046176  
3.017071817071817  
3.041839618929402  
3.0584027659273314  
...
```



## Sequences of Approximation

Is  $\sqrt{51} - \sqrt{51} - \pi < 0$ ?

```
>>> a = subtract(sqrt(51), four())
>>> less_than_0(subtract(a, pi()))
```

```
def sqrt(a):
    x = 1
    while True:
        yield x
        x = (x + a/x)/2
```

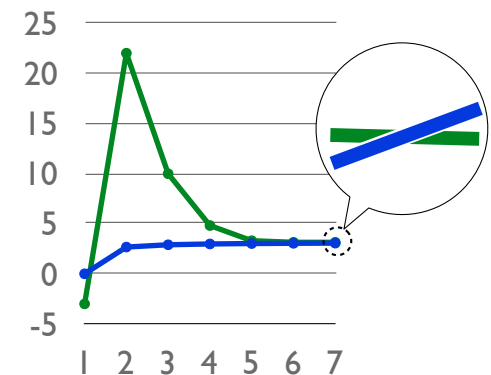
```
def pi():
    total, k = 0, 1
    while True:
        yield total
        total += 8/((4*k-3)*(4*k-1))
        k += 1
```

```
def four():
    while True:
        yield 4
def subtract(x, y):
    while True:
        yield next(x)-next(y)
```

(C) Assume that  $s$  is a SoA to  $y$  and each element of  $s$  is closer to  $y$  than the last. Define `less_than_0(s)` that returns True if it is certain that  $y < 0$ .

```
def less_than_0(s):
    current = next(s)
    while True:
        last, current = current, next(s)
        if last < 0 and current < last:
            return True
```

(Demo)



# Computer Science



## 61A was Designed to Introduce the Big Ideas in Computer Science

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What are functions, data, sequences, trees, programs, languages, and interpreters.

How to write legible programs, use recursion, measure complexity, and solve problems.

Different programming paradigms: functional, object-oriented, and declarative.

What's left to learn in CS?

- Designing and testing software
- Algorithms for solving known problems
- Low-level representations of data and programs
- Discrete mathematics and analysis of programs
- *Programming languages*
- *User interface design*
- *Networking*
- *Systems*
- *Artificial intelligence*
- *Lots of other subfields: graphics, theory, scientific computing, security, etc.*

Life

## Important Ideas Take a Long Time to Learn

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- It's a good idea to study subjects other than computer science.
- Who you spend your time with is important.
- Ideas come from people, and people think from experience.
- Don't compare.
- Contribute to the world.

Thanks for being amazing!

Please stay for the HKN survey.