61A Lecture 27

Friday, November 8

Announcements

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- Homework 8 due Tuesday 11/12 @ 11:59pm, and it's in Scheme!

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- Project 4 due Thursday 11/21 @ 11:59pm, and it's a Scheme interpreter!
  - Also, the project is very long. Get started today.

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Special form to create dynamically scoped procedures

mu

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Tail Recursion

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B*ut...* no for/while statements! Can we make basic iteration efficient? Yes!

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Tail Recursion Examples

Which Procedures are Tail Recursive?

Which of the following procedures run in constant space? $\Theta(1)$

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;; Compute the length of s.
(define (length s)
  (+ 1 (if (null? s)
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;; Return the nth Fibonacci number.
(define (fib n)
  (define (fib-iter current k)
    (if (= k n)
        current
        (fib-iter (+ current
                     (fib (- k 1)))
                  (+ k 1))          ) )
  (if (= 1 n) 0 (fib-iter 1 2)))
```

```
;; Return whether s contains v.
(define (contains s v)
  (if (null? s)
      false
      (if (= v (car s))
          true
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;; Return whether s has any repeated elements.
(define (has-repeat s)
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Map and Reduce

Example: Reduce

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(define (reduce procedure s start)
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(reduce * '(3 4 5) 2)
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```
(reduce * '(3 4 5) 2)                                                    120
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(reduce * '(3 4 5) 2)                                                    120

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Recursive call is a tail call.

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Recursive call is a tail call.

Other calls are not; constant space depends on whether procedure requires constant space.

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Example: Map with Only a Constant Number of Frames

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(define (map procedure s)
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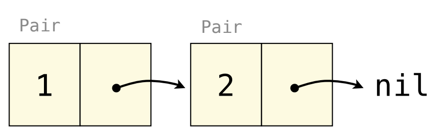

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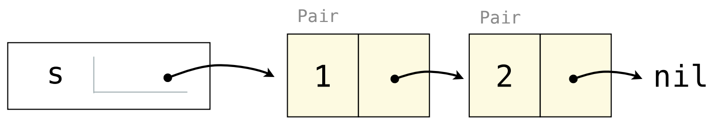

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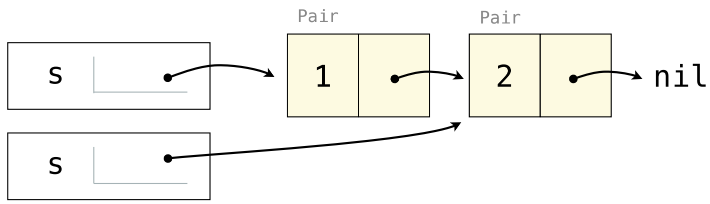

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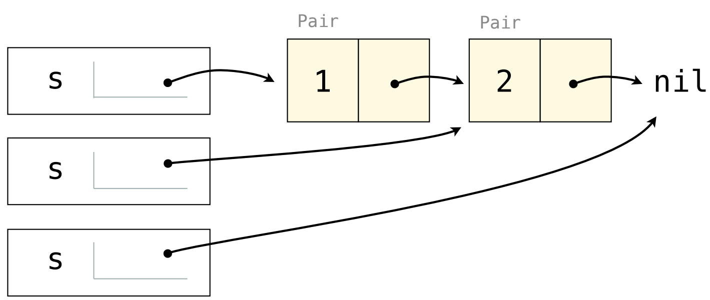

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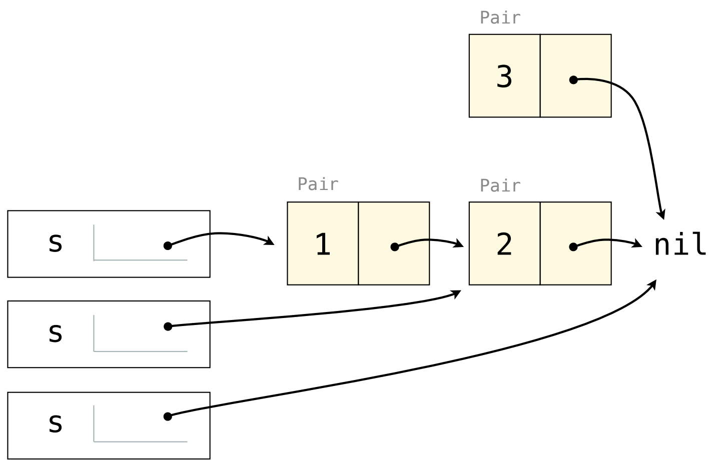

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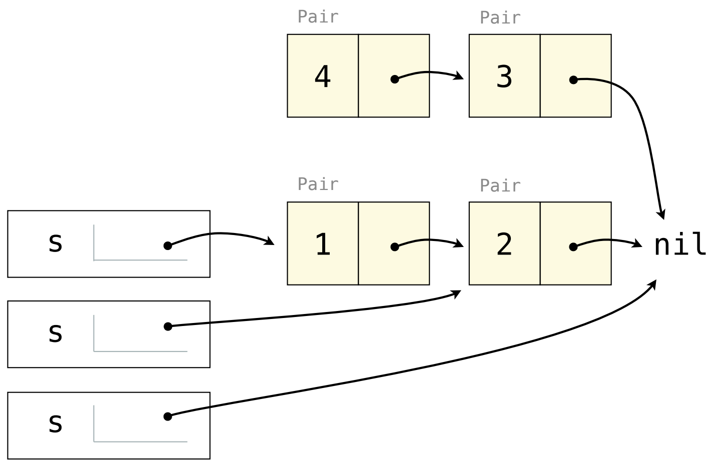

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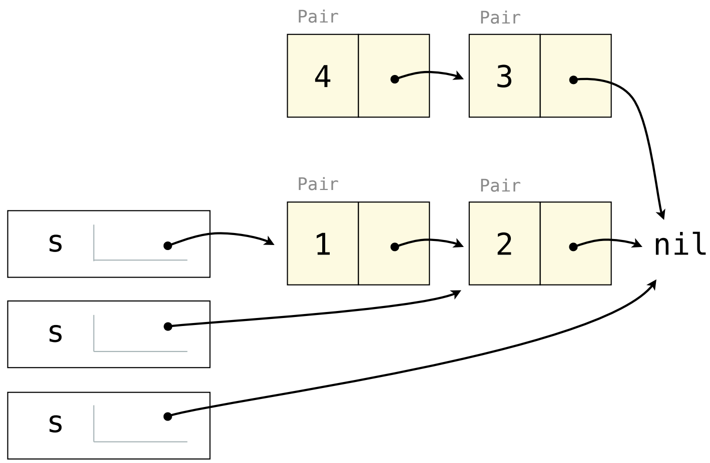


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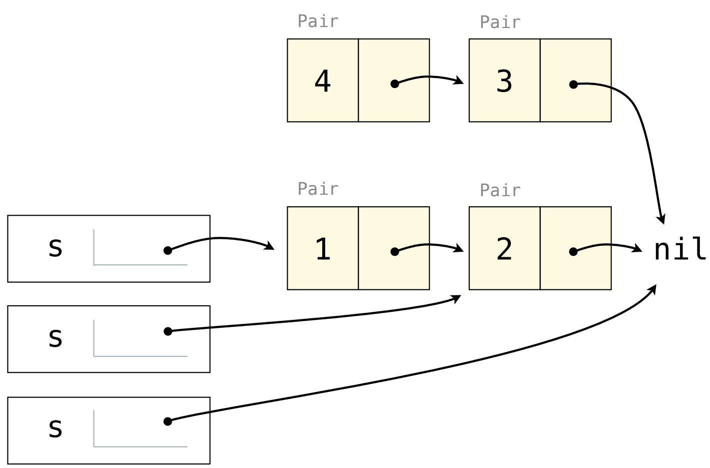

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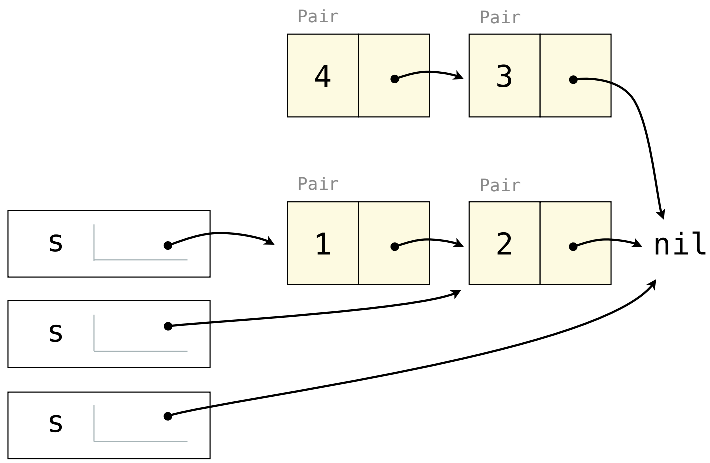

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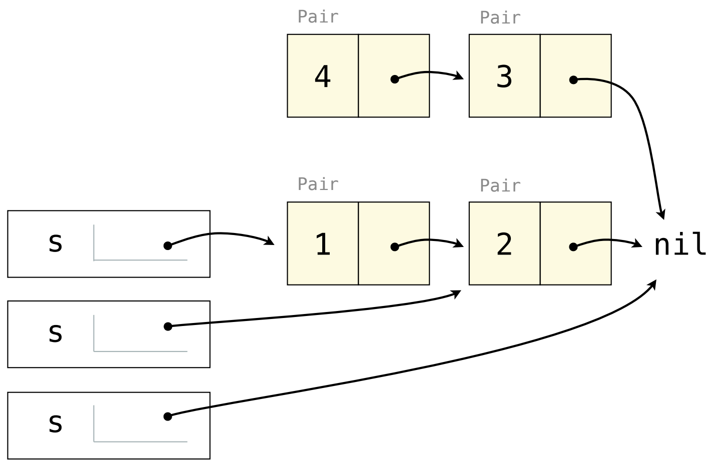


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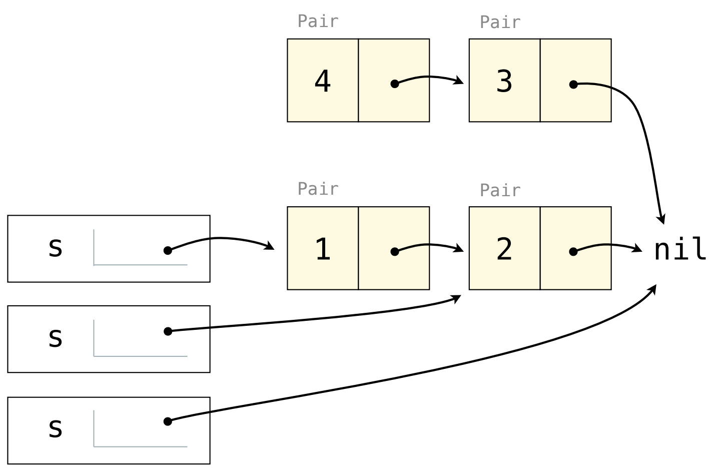

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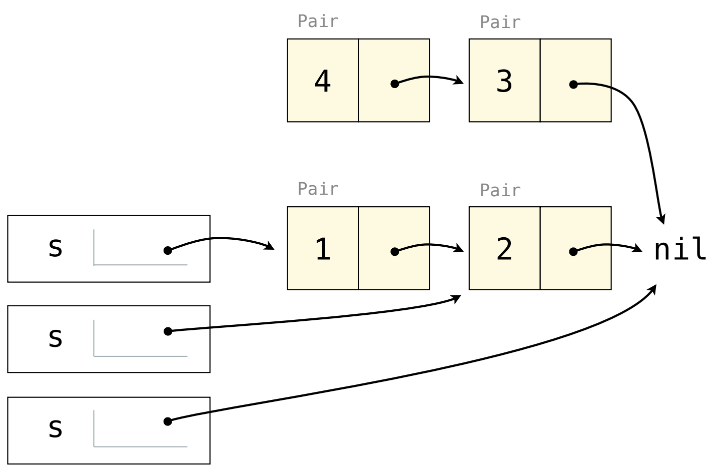


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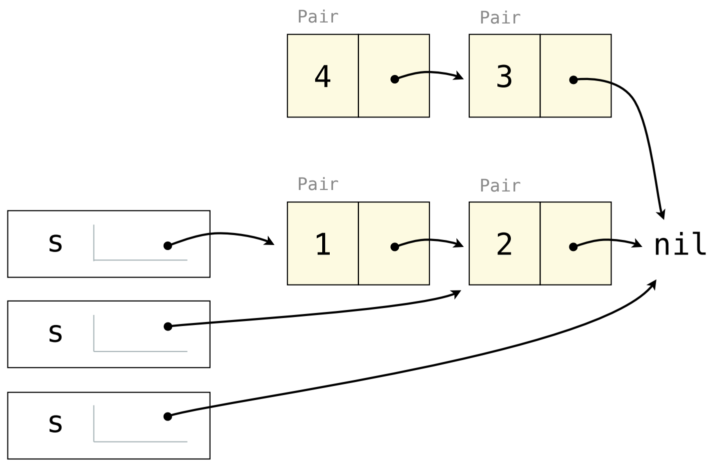


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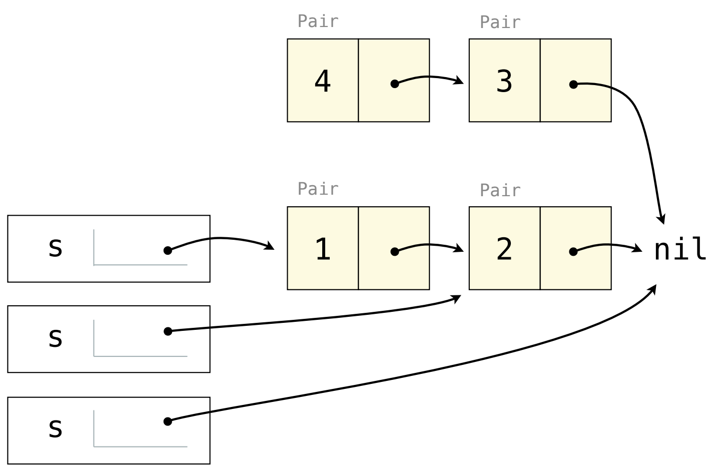


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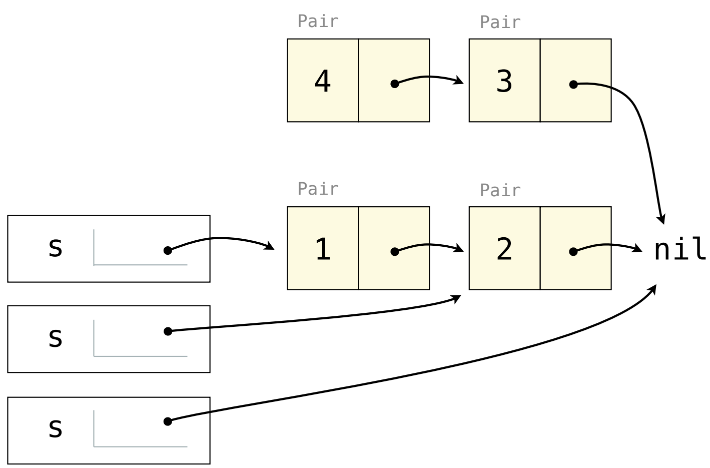


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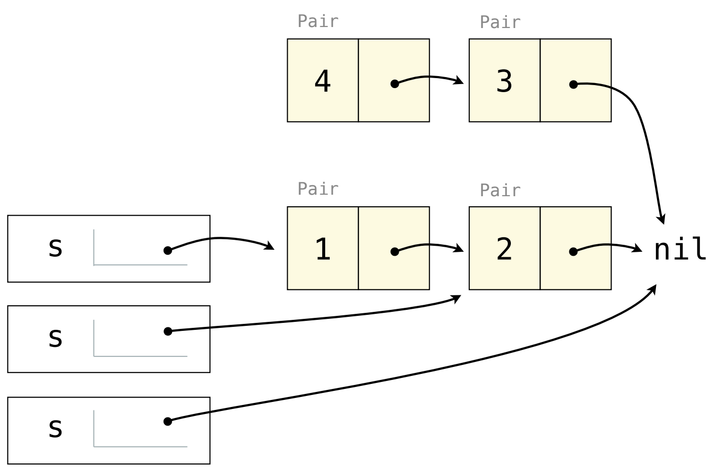


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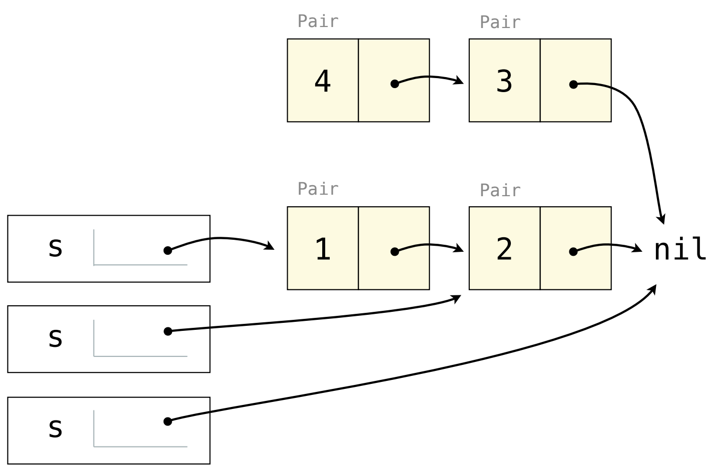


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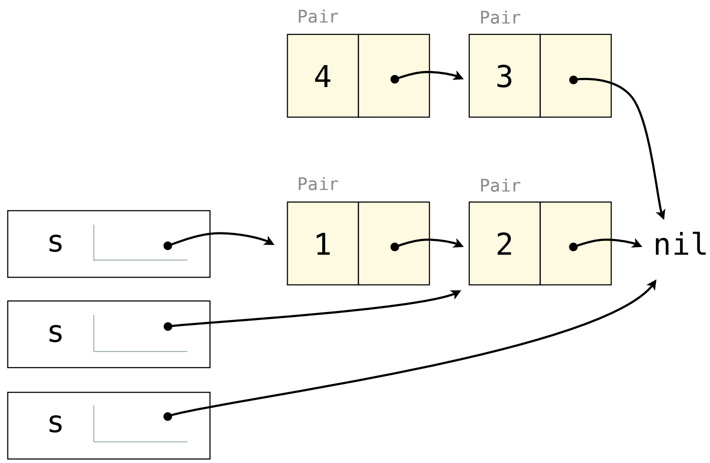


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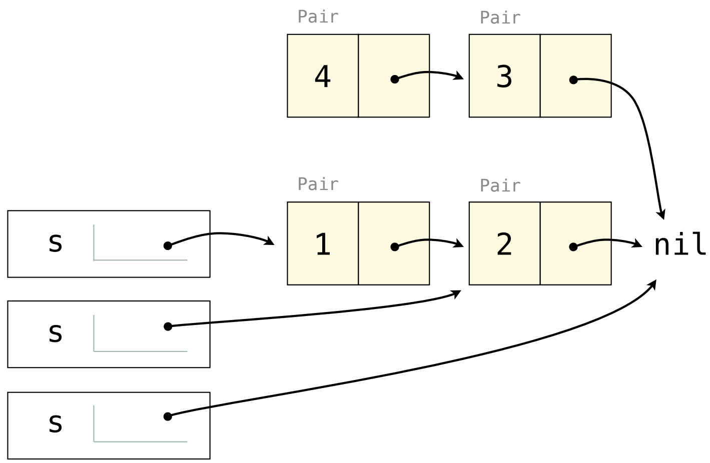


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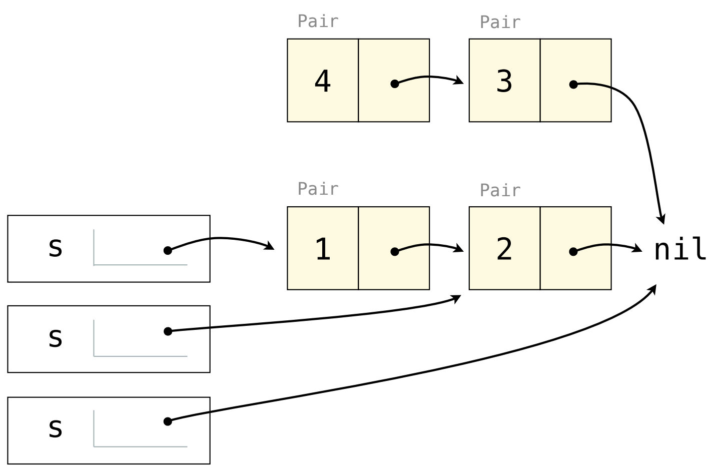


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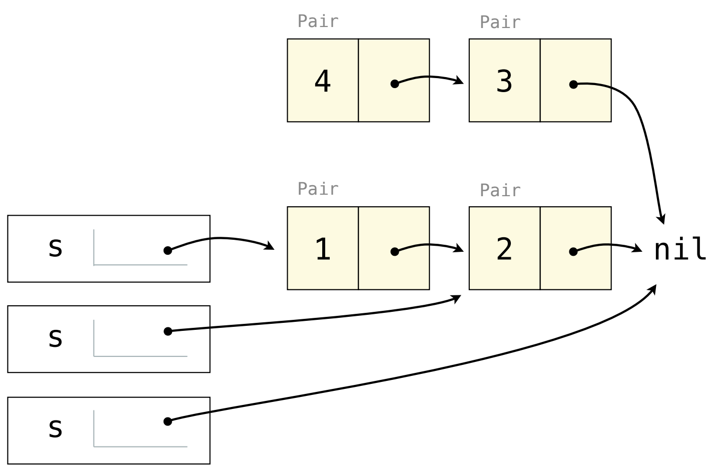


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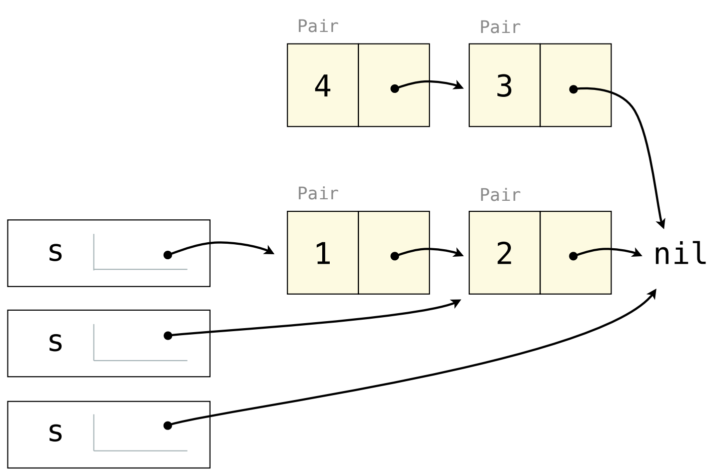


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General Computing Machines

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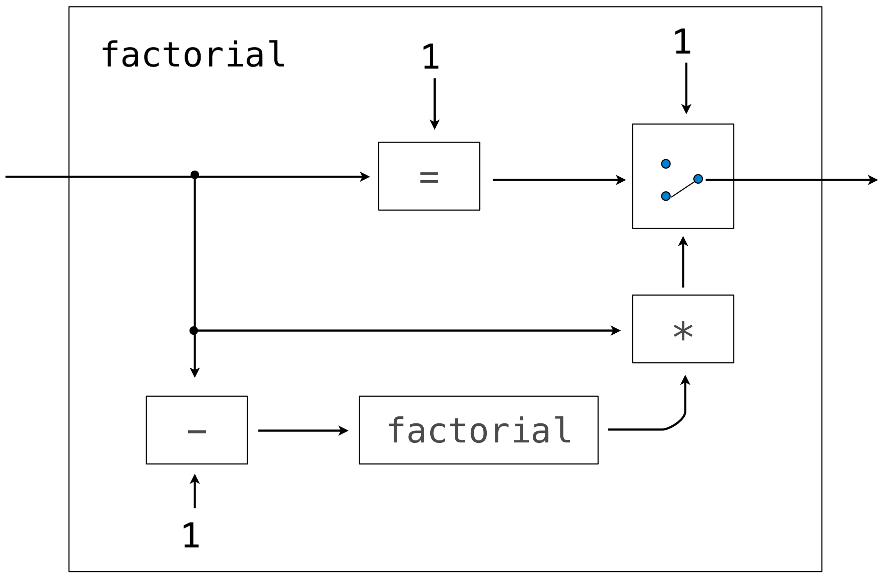

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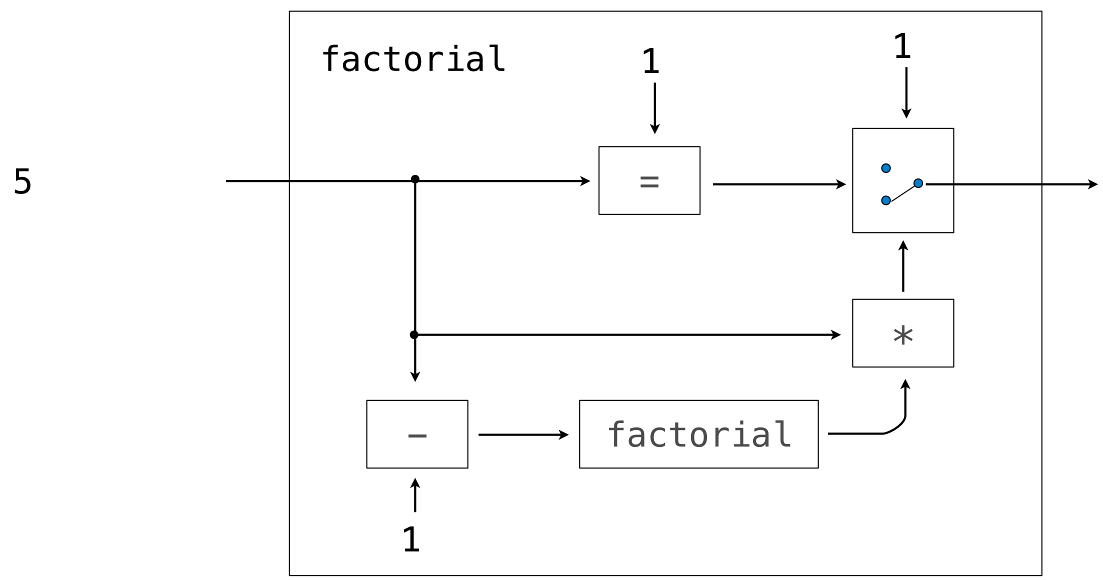

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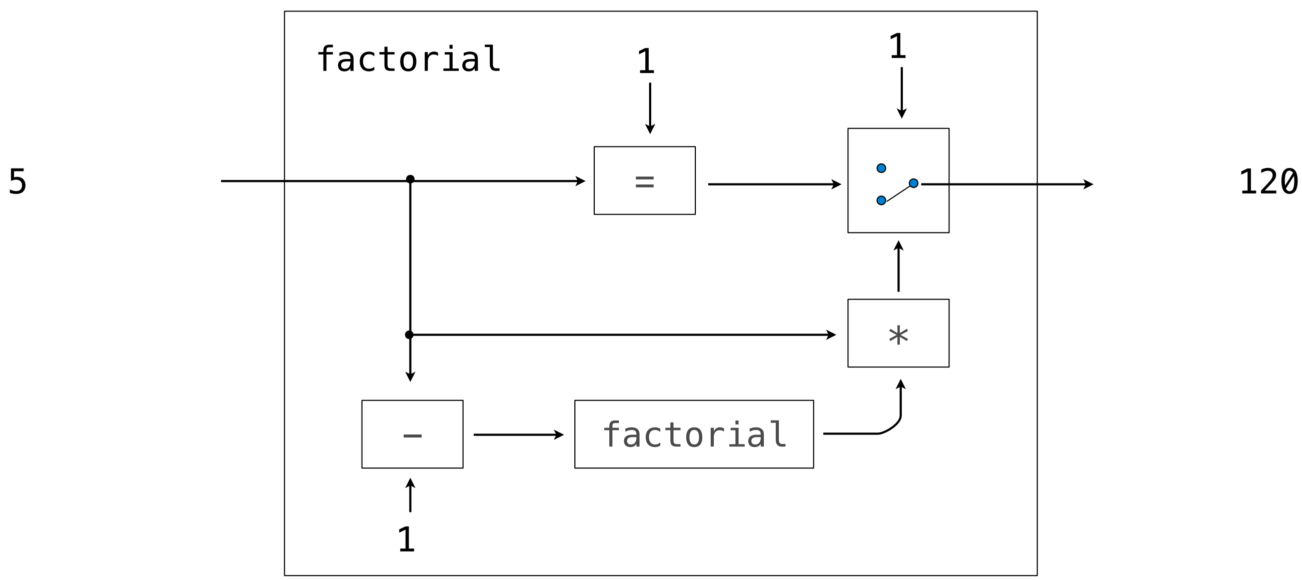

Interpreters are General Computing Machine

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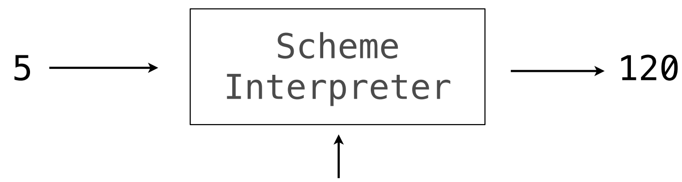


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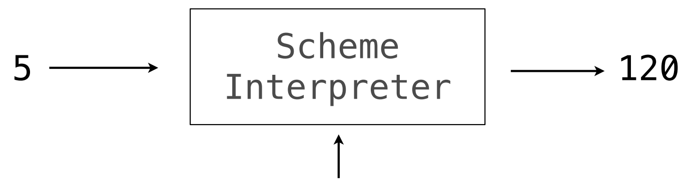


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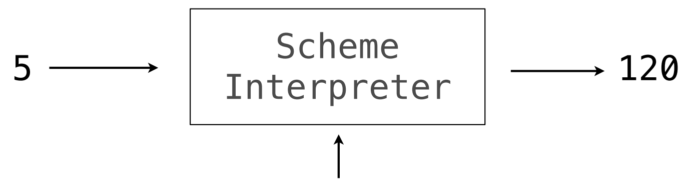


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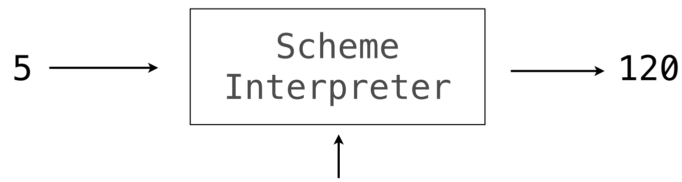


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Internally, it is just a set of evaluation rules