61A Lecture 19

Friday, October 18

Announcements

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  - Please let us know you are coming by filling out the Piazza poll

Comparing Orders of Growth

Comparing orders of growth (n is the problem size)

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$$\Theta(b^n)$$

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$\Theta(b^n)$ Exponential growth! Recursive fib takes

$\Theta(\phi^n)$ steps, where $\phi = \frac{1+\sqrt{5}}{2} \approx 1.61828$

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One more built-in Python container type

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{1, 2, 3, 4}
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>>> s.union({1, 5})
{1, 2, 3, 4, 5}
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{1, 2, 3, 4, 5}
>>> s.intersection({6, 5, 4, 3})
{3, 4}
```

Implementing Sets

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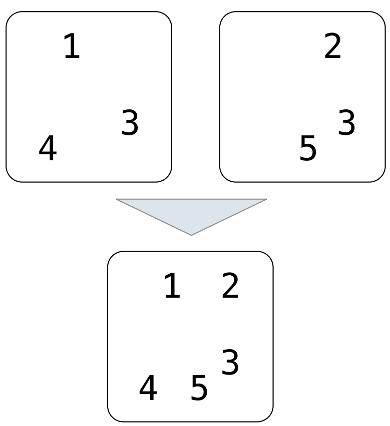

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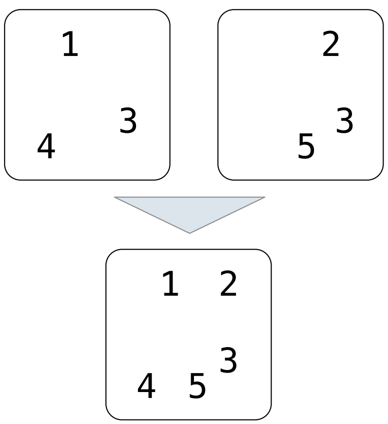

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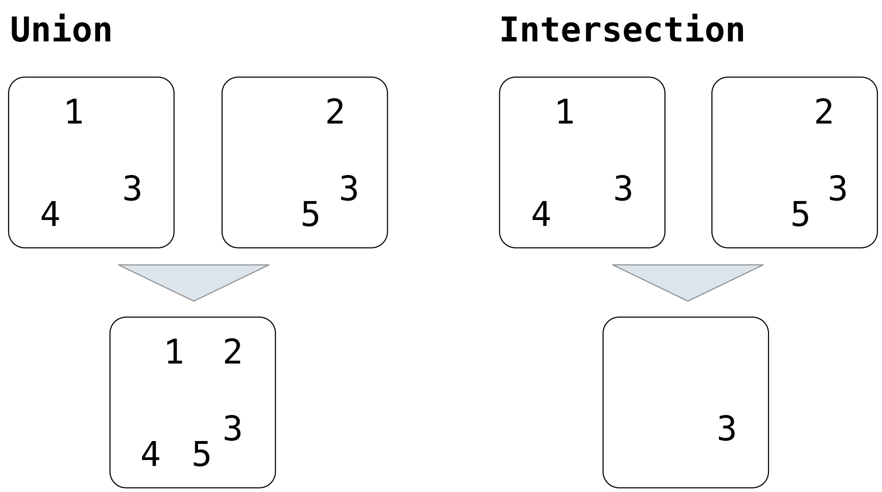

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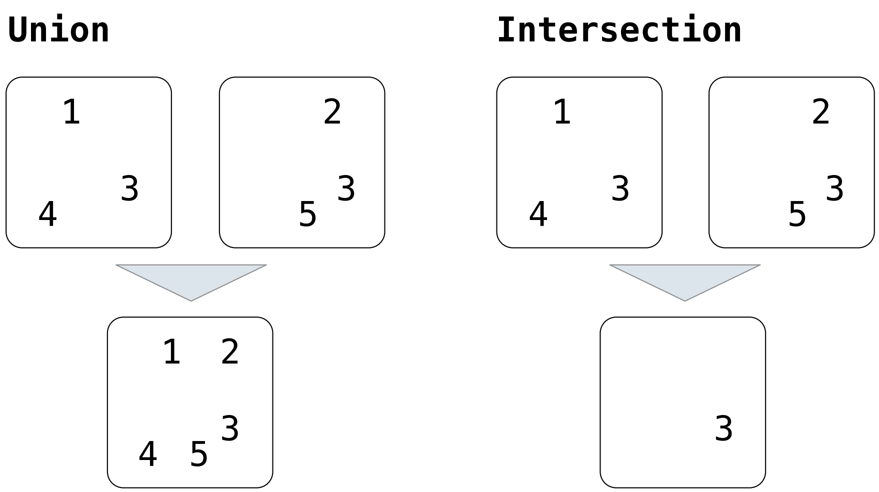

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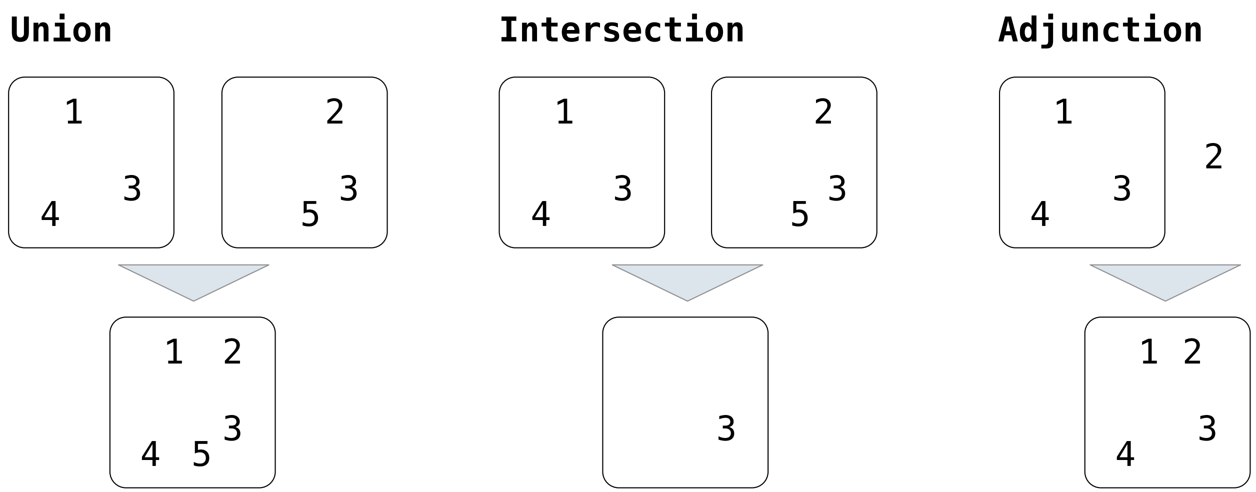

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which means that there are positive constants k_1 and k_2 such that

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for sufficiently large values of ***n*****.**

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def intersect_set(set1, set2):
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def intersect_set(set1, set2):
    in_set2 = lambda v: set_contains(set2, v)
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def set_contains(s, v):
```

Sets as Ordered Sequences

Proposal 2: A set is represented by a recursive list with unique elements ordered from least to greatest

```
def set_contains(s, v):
    if empty(s) or s.first > v:
        return False
```

Sets as Ordered Sequences

Proposal 2: A set is represented by a recursive list with unique elements ordered from least to greatest

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def set_contains(s, v):
    if empty(s) or s.first > v:
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    elif s.first == v:
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```

Sets as Ordered Sequences

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```

Sets as Ordered Sequences

Proposal 2: A set is represented by a recursive list with unique elements ordered from least to greatest

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def set_contains(s, v):
    if empty(s) or s.first > v:
        return False
    elif s.first == v:
        return True
    else:
        return set_contains(s.rest, v)
```

Sets as Ordered Sequences

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Order of growth?

Sets as Ordered Sequences

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Order of growth? $\Theta(n)$

Set Intersection Using Ordered Sequences

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This algorithm *assumes* that elements are in order.

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def intersect_set(set1, set2):
    if empty(set1) or empty(set2):
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This algorithm *assumes* that elements are in order.

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    if empty(set1) or empty(set2):
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    else:
        e1, e2 = set1.first, set2.first
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    else:
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        if e1 == e2:
            return Rlist(e1, intersect_set(set1.rest, set2.rest))
```

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        elif e1 < e2:
            return intersect_set(set1.rest, set2)
        elif e2 < e1:
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```

(Demo)

Set Intersection Using Ordered Sequences

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(Demo)

Order of growth?

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(Demo)

Order of growth? $\Theta(n)$

Sets as Binary Search Trees

Tree Sets

Tree Sets

Proposal 3: A set is represented as a Tree. Each entry is:

Tree Sets

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- Larger than all entries in its left branch and

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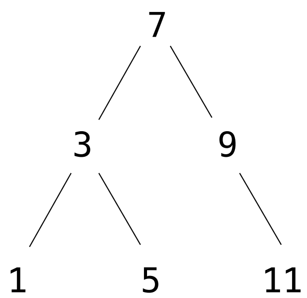
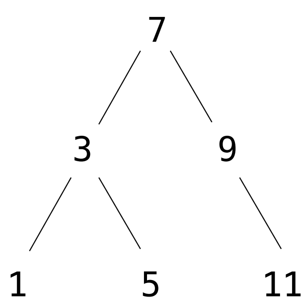
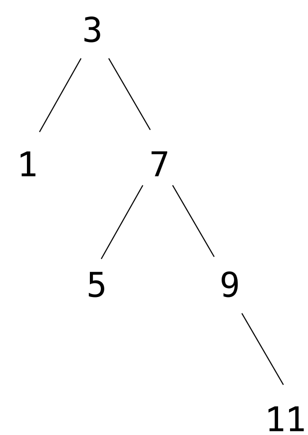


Tree Sets

Proposal 3: A set is represented as a Tree. Each entry is:

- Larger than all entries in its left branch and
- Smaller than all entries in its right branch





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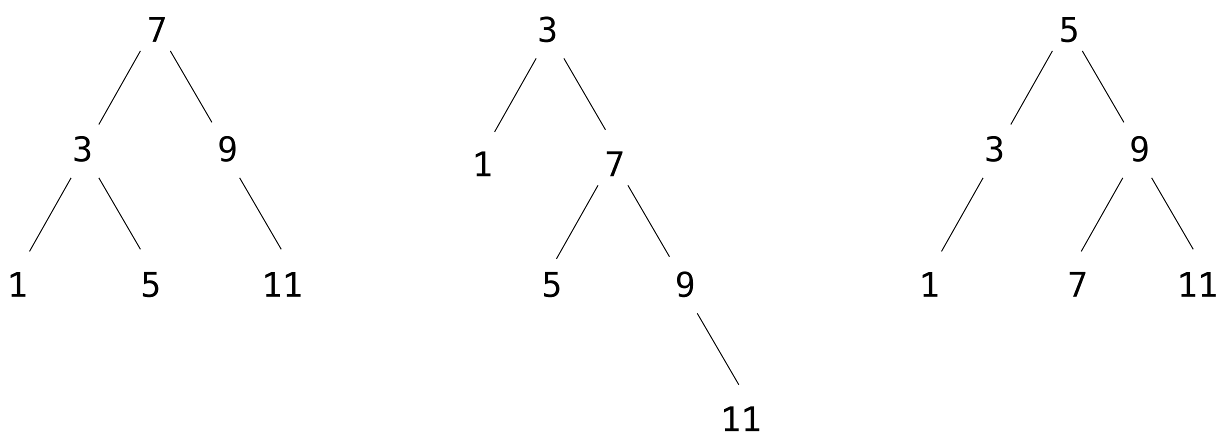

Membership in Tree Sets

Membership in Tree Sets

```
Set membership traverses the tree
```

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- The element is either in the left or right sub-branch

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```
def set_contains(s, v):
    if s is None:
```

Membership in Tree Sets

Set membership traverses the tree

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```
def set_contains(s, v):
    if s is None:
        return False
```

Membership in Tree Sets

Set membership traverses the tree

- The element is either in the left or right sub-branch
- By focusing on one branch, we reduce the set by about half

```
def set_contains(s, v):
    if s is None:
        return False
    elif s.entry == v:
```

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```
def set_contains(s, v):
    if s is None:
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```

Membership in Tree Sets

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```
def set_contains(s, v):
    if s is None:
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        return True
    elif s.entry < v:
        return set_contains(s.right, v)
```

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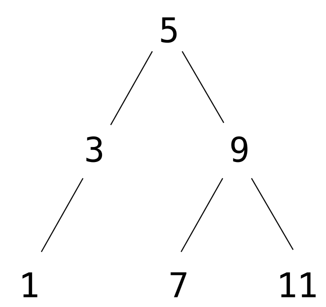

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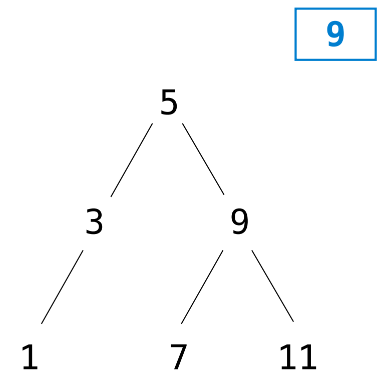

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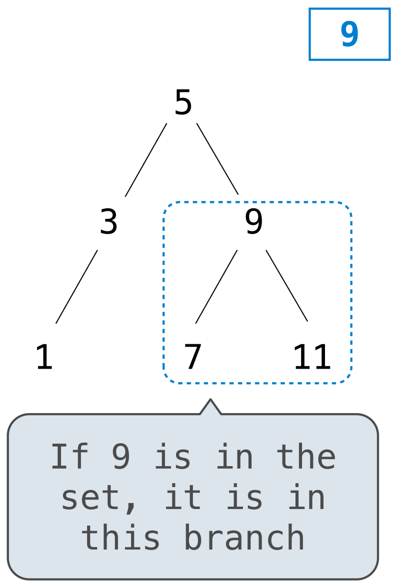

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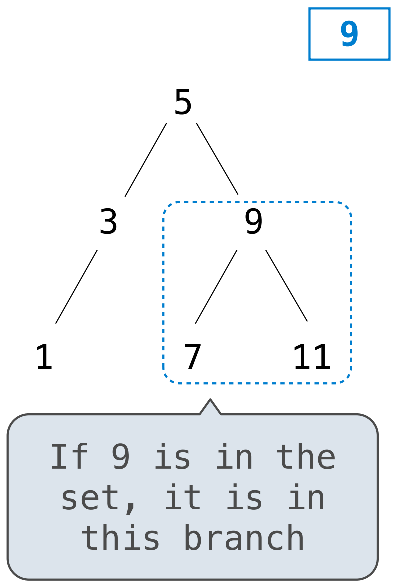
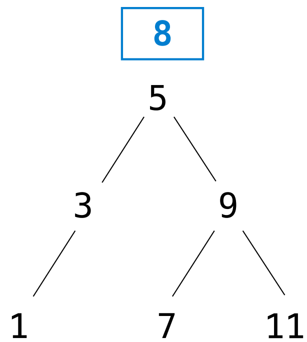


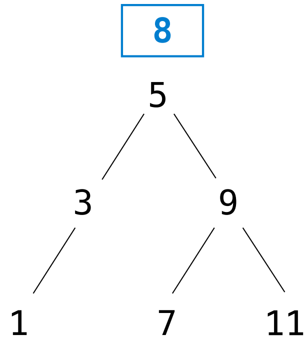
Order of growth?

Adjoining to a Tree Set

Adjoining to a Tree Set



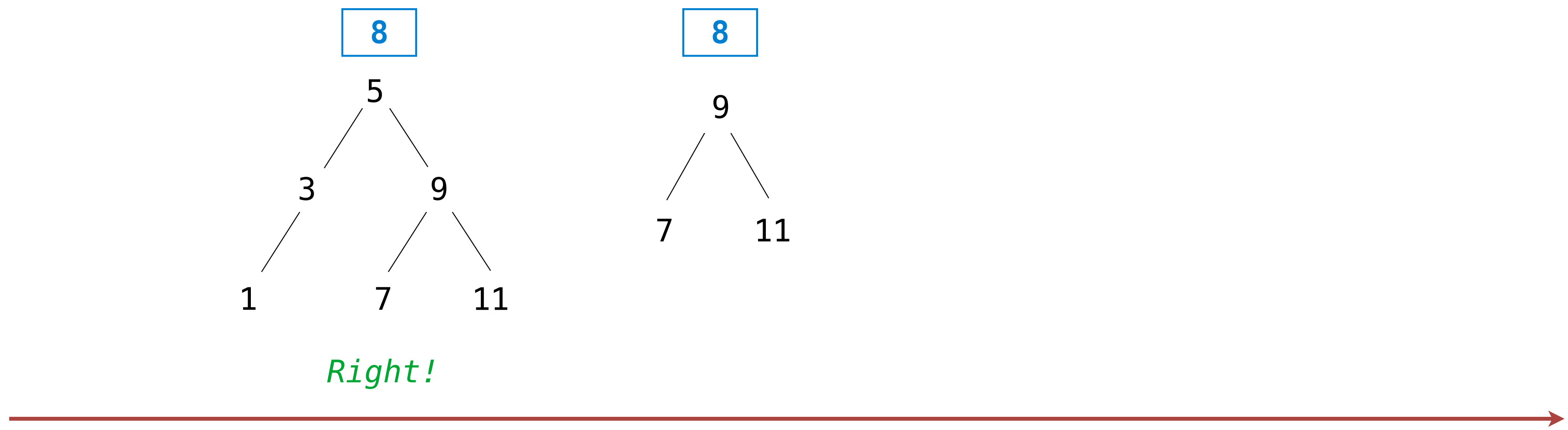
Adjoining to a Tree Set



Right!

Adjoining to a Tree Set

Adjoining to a Tree Set



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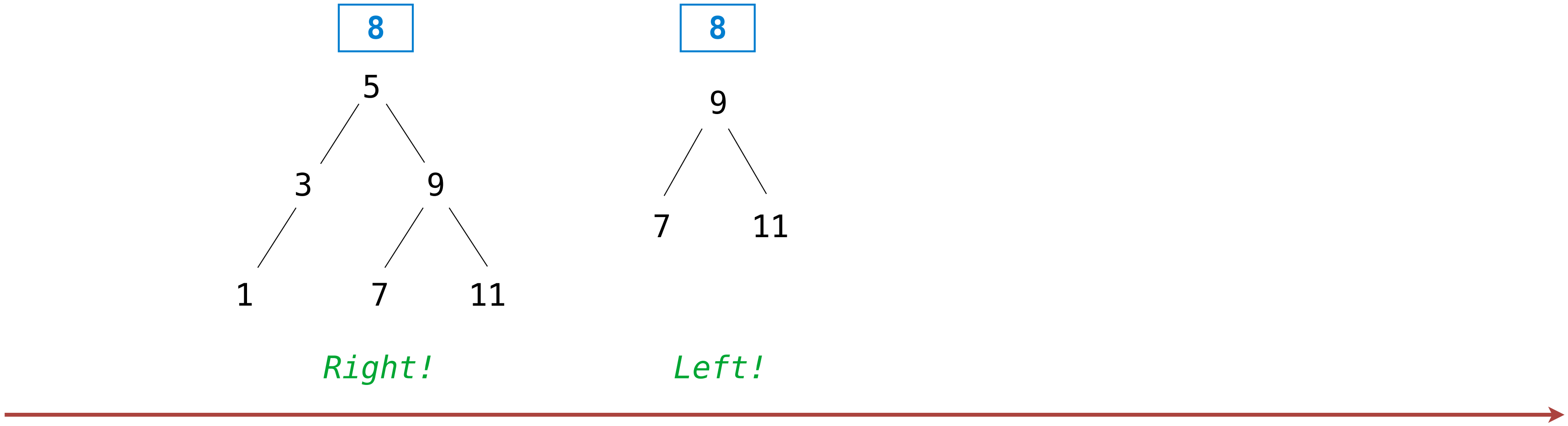

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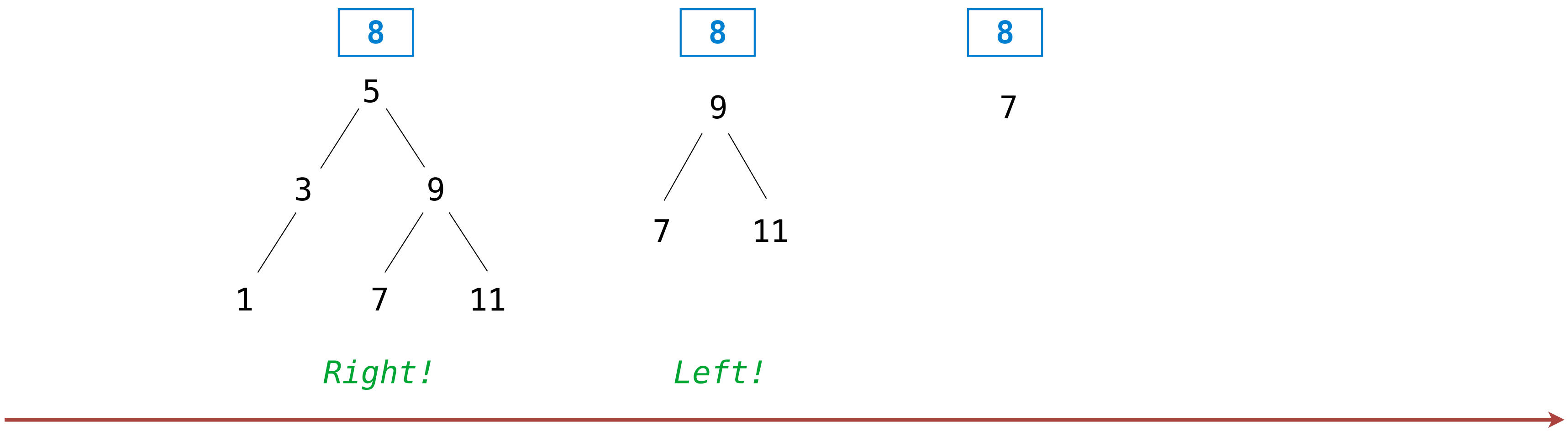

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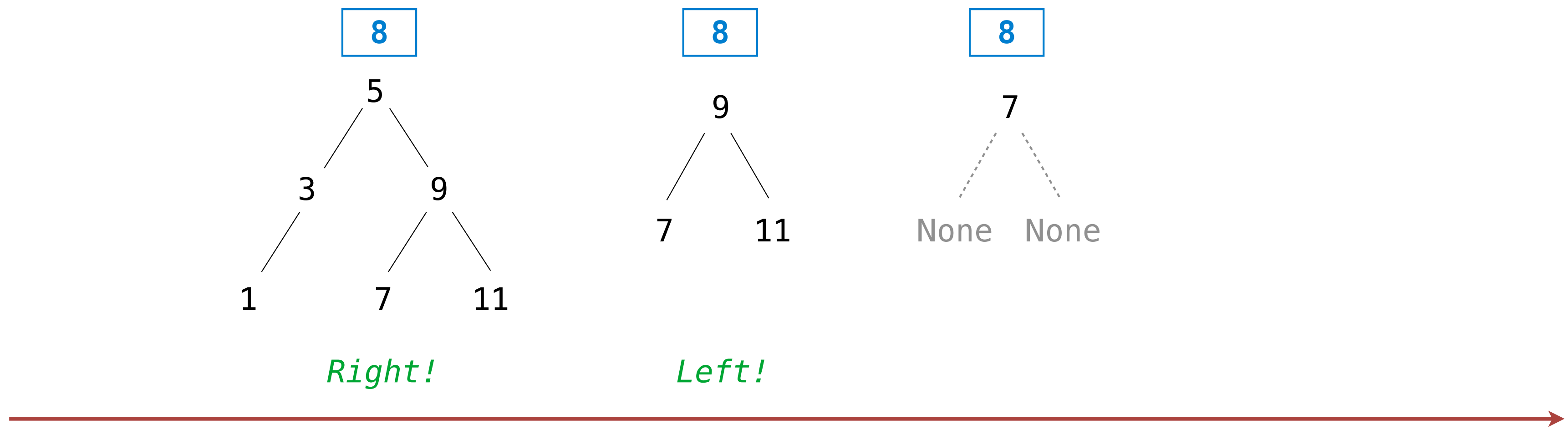

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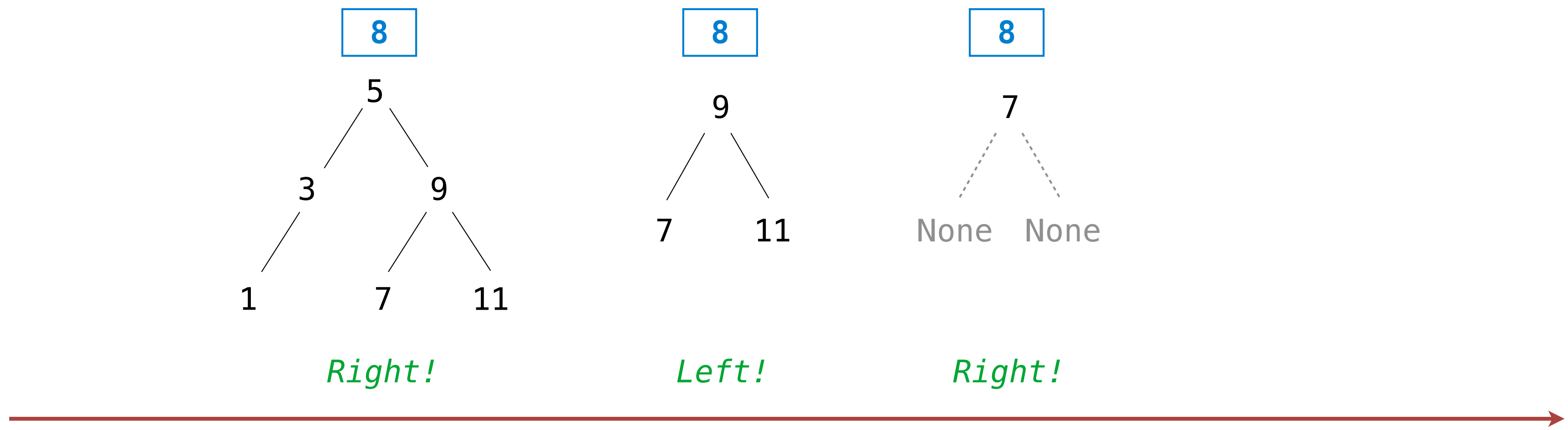

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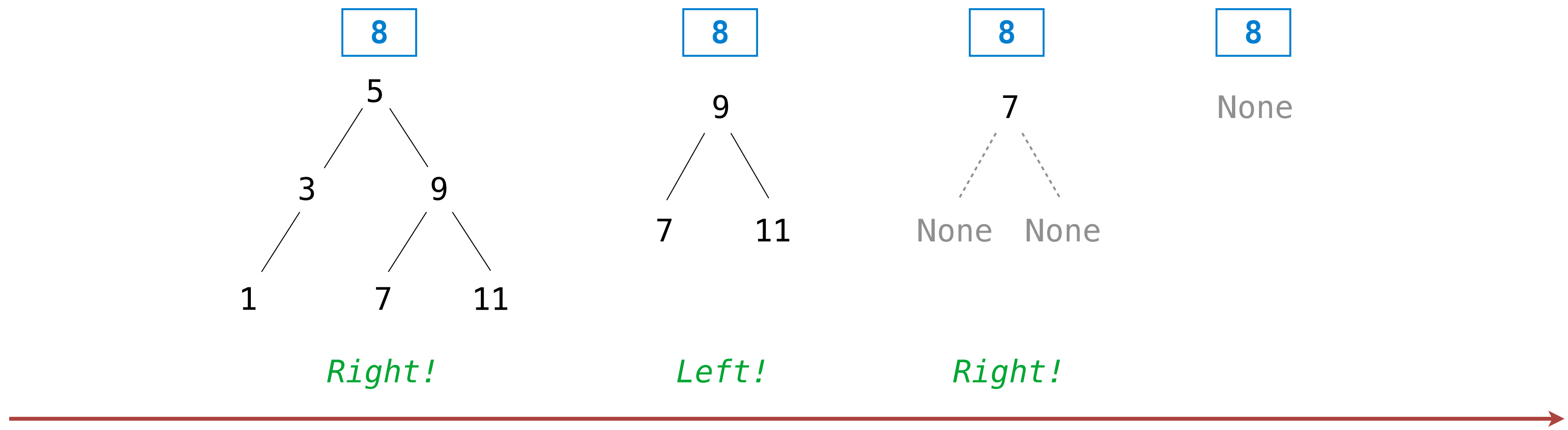

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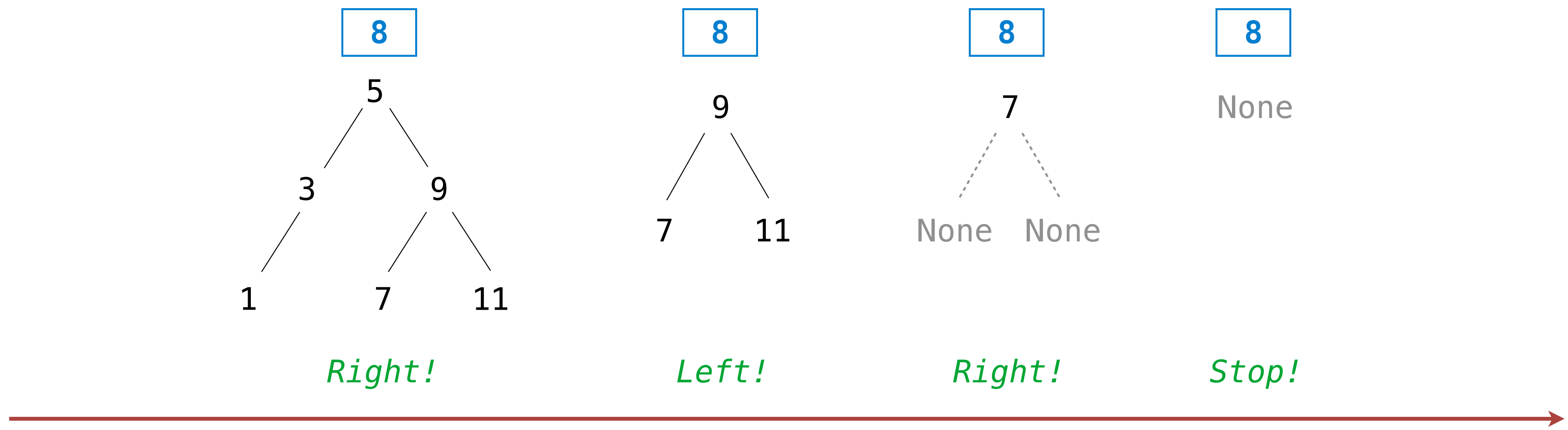

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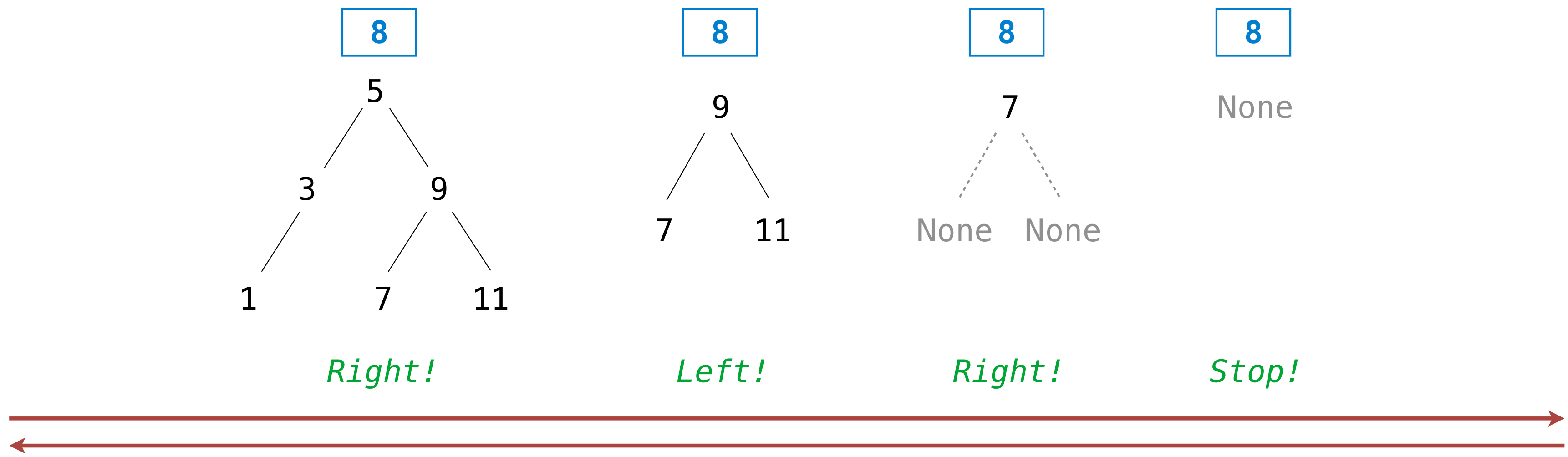

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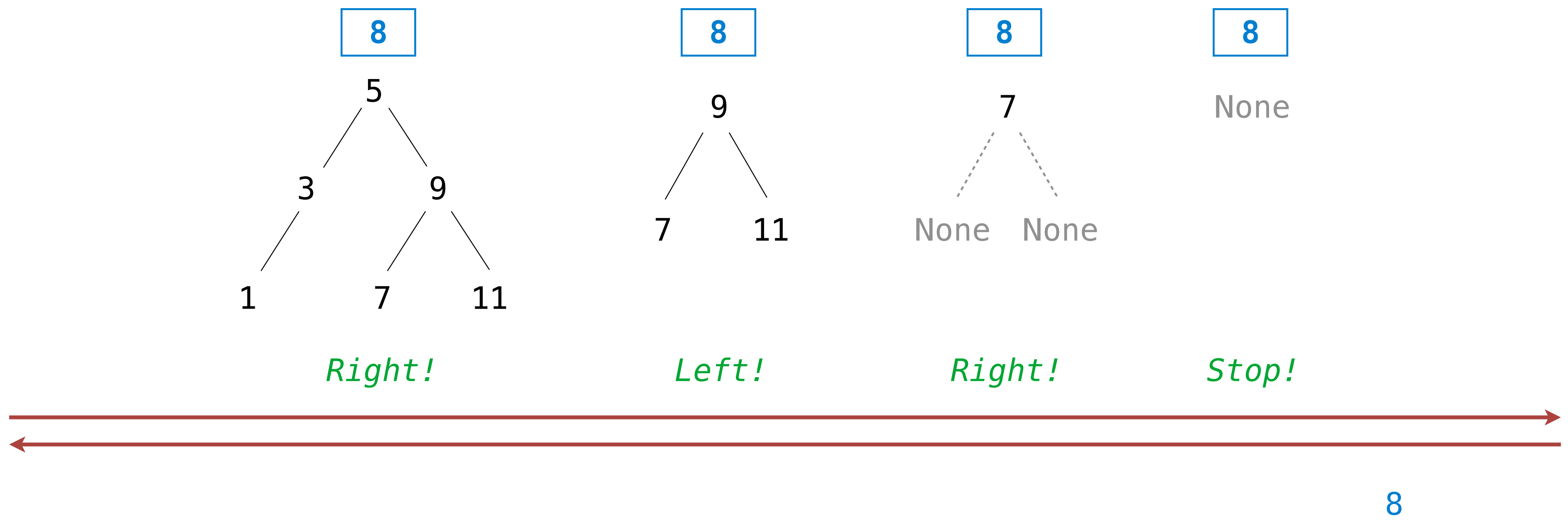

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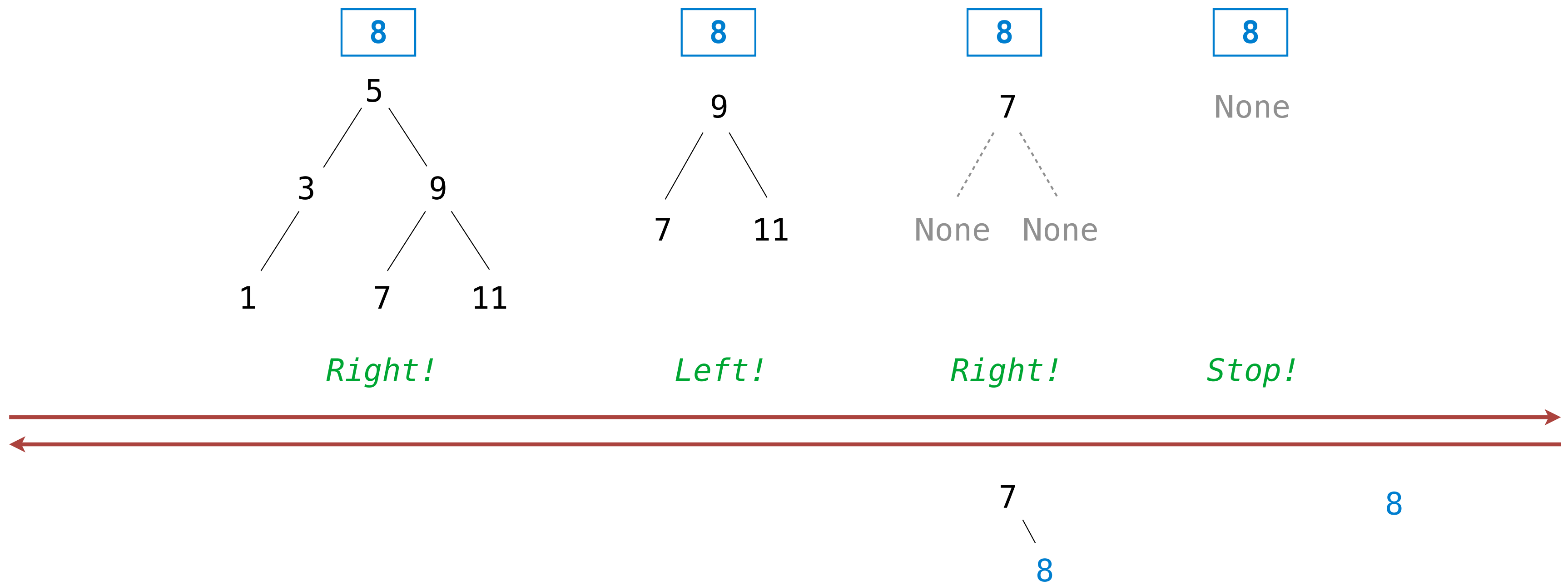

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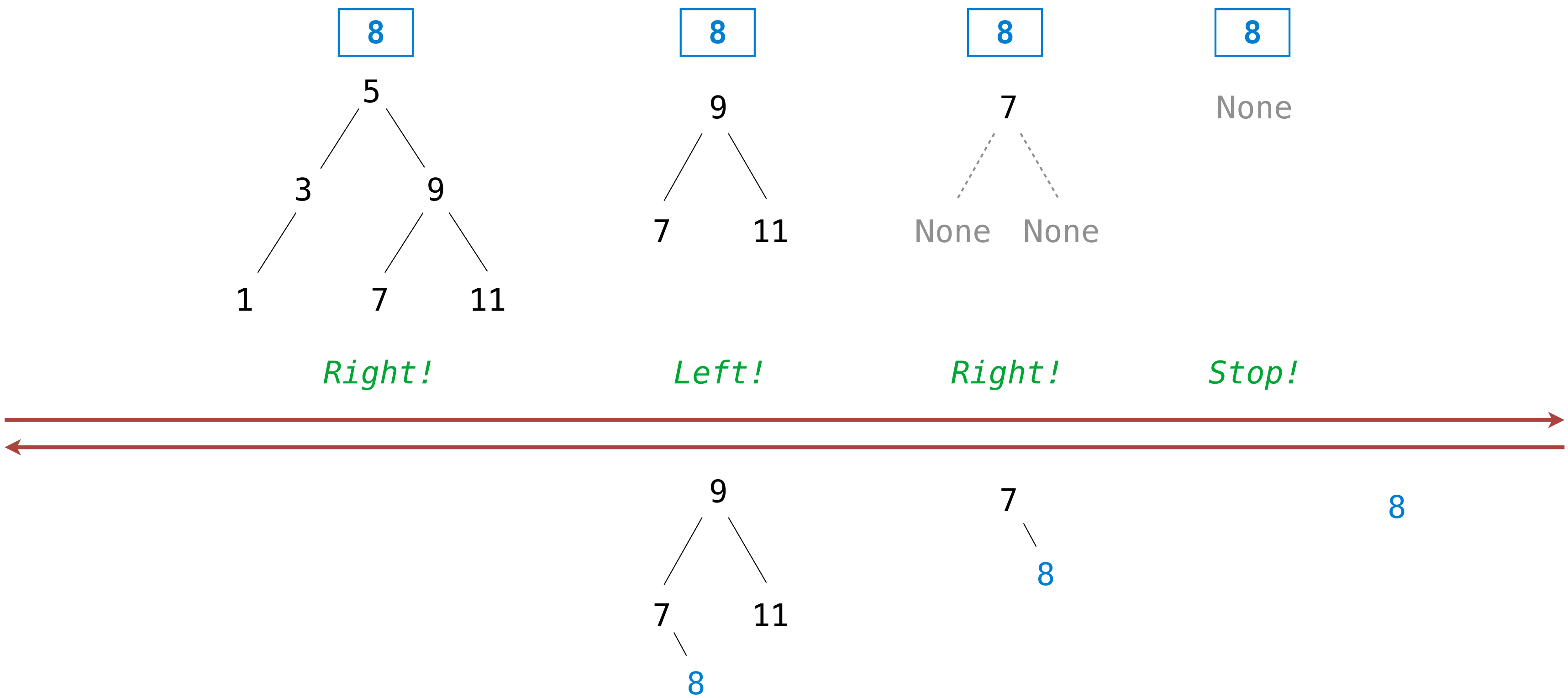

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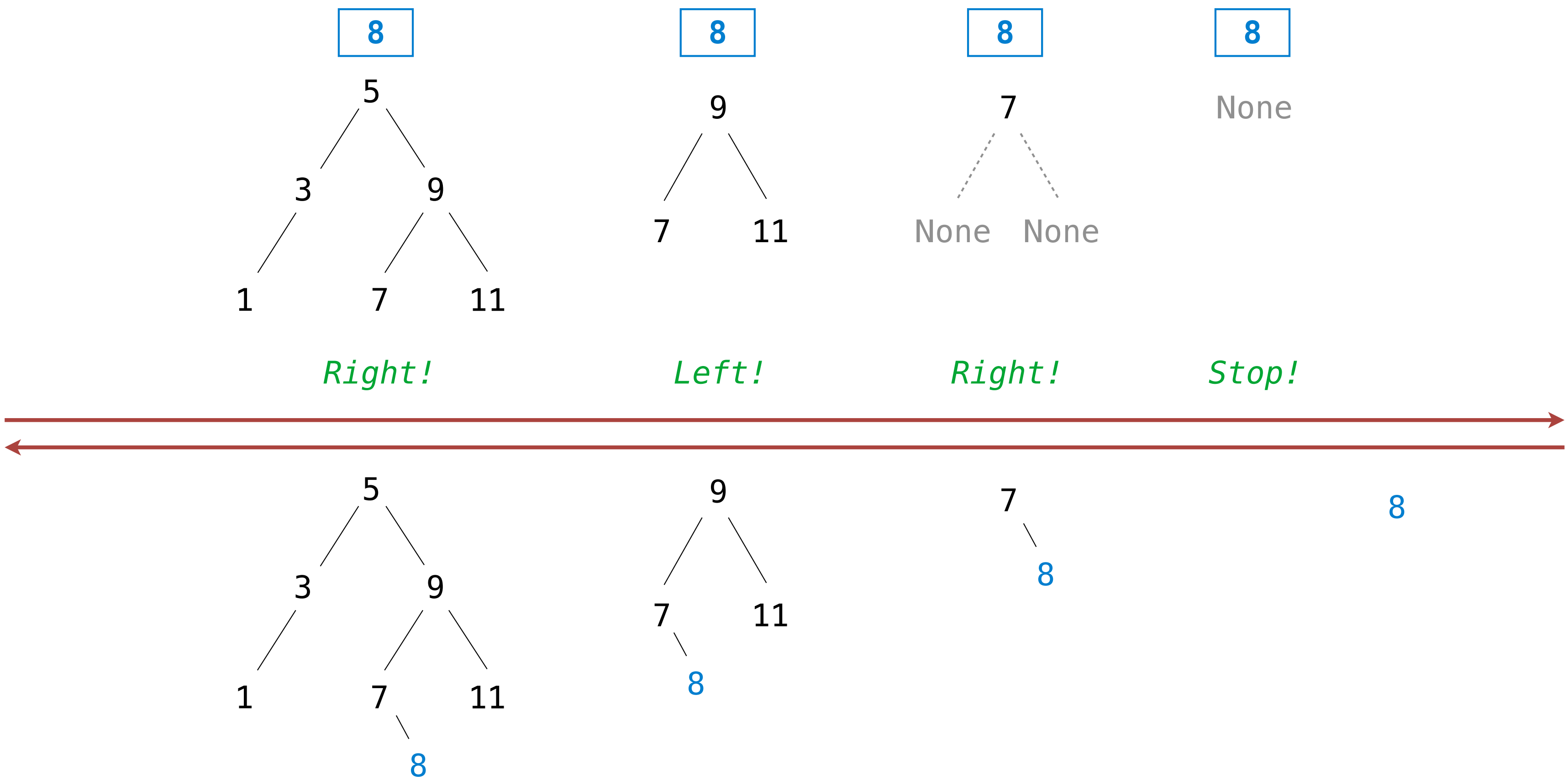

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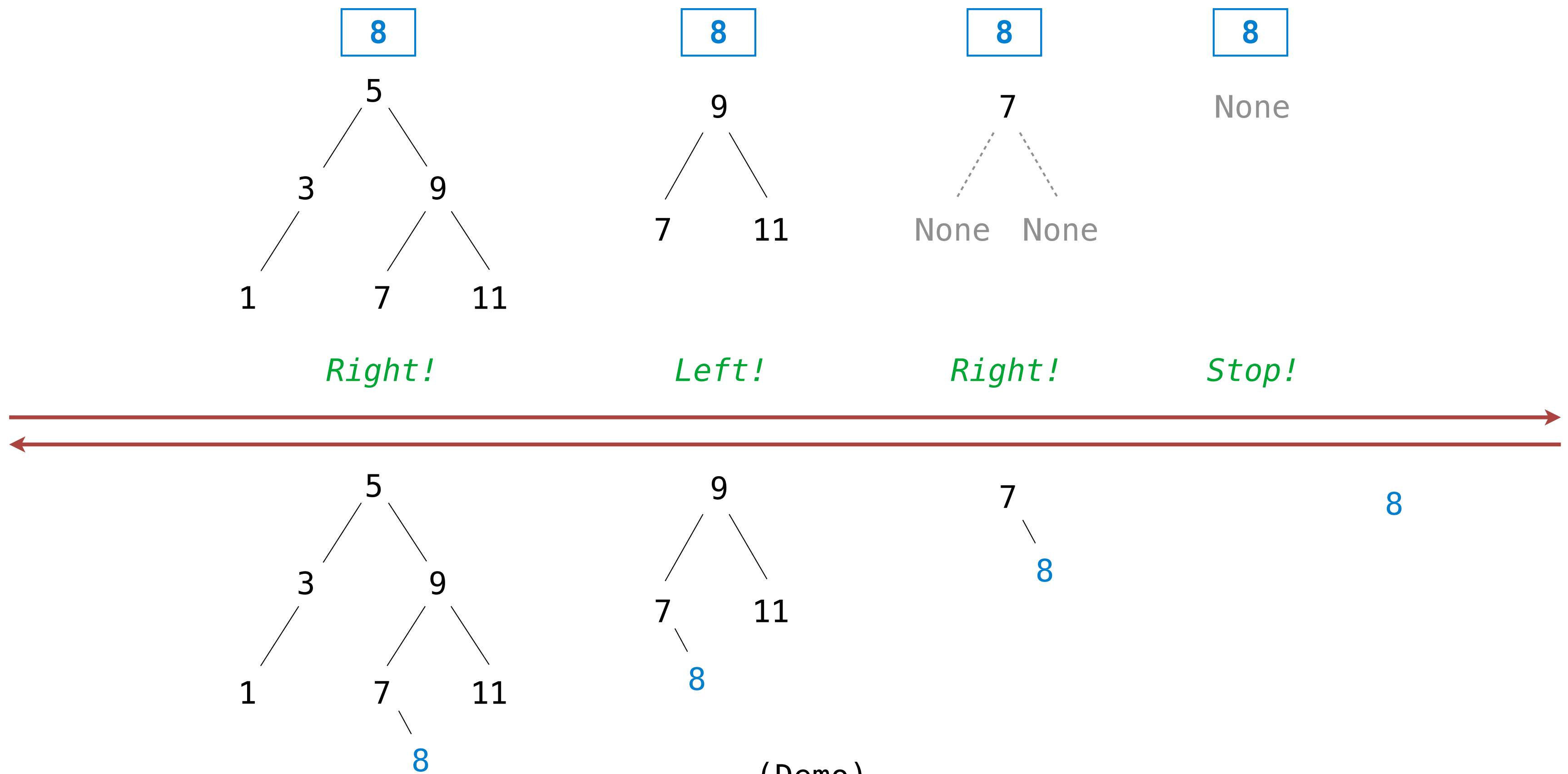

(Demo)

More Set Operations

What Did I Leave Out?

What Did I Leave Out?

```
Sets as ordered sequences:
```

What Did I Leave Out?

Sets as ordered sequences:

- Adjoining an element to a set

What Did I Leave Out?

Sets as ordered sequences:

- Adjoining an element to a set
- Union of two sets

What Did I Leave Out?

Sets as ordered sequences:

- Adjoining an element to a set
- Union of two sets

Sets as binary trees:

What Did I Leave Out?

Sets as ordered sequences:

- Adjoining an element to a set
- Union of two sets

Sets as binary trees:

- Intersection of two sets

What Did I Leave Out?

Sets as ordered sequences:

- Adjoining an element to a set
- Union of two sets

Sets as binary trees:

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- Union of two sets

What Did I Leave Out?

Sets as ordered sequences:

- Adjoining an element to a set
- Union of two sets

Sets as binary trees:

- Intersection of two sets
- Union of two sets
- Balancing a tree

What Did I Leave Out?

Sets as ordered sequences:

- Adjoining an element to a set
- Union of two sets

Sets as binary trees:

- Intersection of two sets
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That's all on homework 7!