61A Lecture 18

Wednesday, October 16

Announcements

- Homework 6 is due Tuesday 10/22 @ 11:59pm
- Project 3 is due Thursday 10/24 @ 11:59pm
- Midterm 2 is on Monday 10/28 7pm–9pm
- Hog strategy contest winners will be announced on Wednesday 10/16 in lecture

Memoization

Memoization

Idea: Remember the results that have been computed before

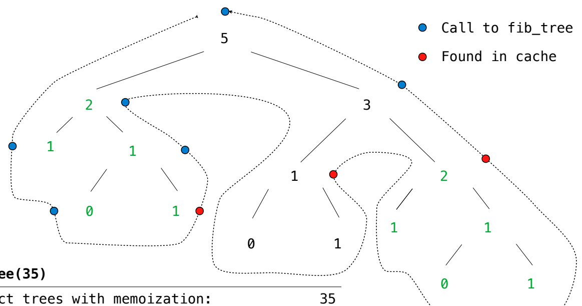
```
def memo(f):
    cache = {}
    def memoized(n):
        if n not in cache:
            cache[n] = f(n)
        return cache[n]
    return memoized
```

Keys are arguments that map to return values

Same behavior as f, if f is a pure function

(Demo)

Memoized Tree Recursion



fib_tree(35)

Distinct trees with memoization: 35

Distinct trees without memoization: 18,454,929

Time

The Consumption of Time

Implementations of the same functional abstraction can require different amounts of time to compute their result.

Problem: How many factors does a positive integer n have?

A factor k of n is a positive integer such that n/k is also a positive integer.

def count_factors(n):	**Time (number of divisions)**
Slow: Test each k from 1 through n.	n
Fast: Test each k from 1 to square root n. For every k, n/k is also a factor!	$\lfloor\sqrt{n}\rfloor$

(Demo)

Space

The Consumption of Space

Which environment frames do we need to keep during evaluation?

Each step of evaluation has a set of **active** environments.

Values and frames in active environments consume memory.

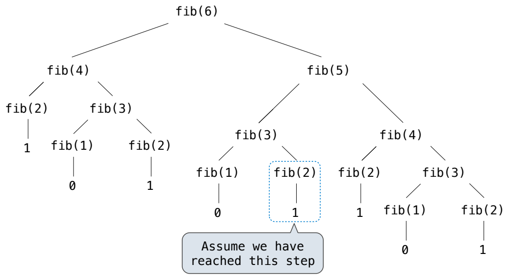
Memory used for other values and frames can be recycled.

Active environments:

- Environments for any function calls currently being evaluated
- Parent environments of functions named in active environments

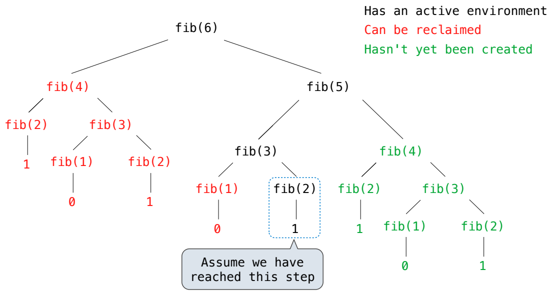
(Demo)

Fibonacci Memory Consumption



Assume we have reached this step

Fibonacci Memory Consumption



Has an active environment
Can be reclaimed
Hasn't yet been created

Assume we have reached this step

Order of Growth

Order of Growth

A method for bounding the resources used by a function by the "size" of a problem

***n*:** size of the problem

***R*(*n*):** Measurement of some resource used (time or space)

$$R(n) = \Theta(f(n))$$

means that there are positive constants k_1 and k_2 such that

$$k_1 \cdot f(n) \leq R(n) \leq k_2 \cdot f(n)$$

for sufficiently large values of ***n*.**

Iteration vs Memoized Tree Recursion

Iterative and memoized implementations are not the same.

```
def fib_iter(n):
    prev, curr = 1, 0
    for _ in range(n-1):
         prev, curr = curr, prev + curr
    return curr
```

```
@memo
def fib(n):
    if n == 1:
        return 0
    if n == 2:
        return 1
    return fib(n-2) + fib(n-1)
```

Time	Space
$\Theta(n)$	$\Theta(1)$
$\Theta(n)$	$\Theta(n)$

Counting Factors

Order of growth can still be used, even if we can quantify amounts exactly.

Problem: How many factors does a positive integer n have?

A factor k of n is a positive integer such that n/k is also a positive integer.

```
def count_factors(n)"
```

	Time	Space
Slow: Test each k from 1 to n.	$\Theta(n)$	$\Theta(1)$
Fast: Test each k from 1 to square root n. For every k, n/k is also a factor!	$\Theta(\sqrt{n})$	$\Theta(1)$

Exponentiation

Exponentiation

Goal: one more multiplication lets us double the problem size.

```
def exp(b, n):
    if n == 0:
        return 1
  else:
        return b * exp(b, n-1)

def square(x):
    return x*x

def fast_exp(b, n):
    if n == 0:
        return 1
    elif n % 2 == 0:
        return square(fast_exp(b, n//2))
    else:
        return b * fast_exp(b, n-1)
```

$$b^n = \begin{cases} 1 & \text{if } n = 0 \\ b \cdot b^{n-1} & \text{otherwise} \end{cases}$$

$$b^n = \begin{cases} 1 & \text{if } n = 0 \\ (b^{\frac{1}{2}n})^2 & \text{if } n \text{ is even} \\ b \cdot b^{n-1} & \text{if } n \text{ is odd} \end{cases}$$

(Demo)

Exponentiation

Goal: one more multiplication lets us double the problem size.

```
def exp(b, n):
    if n == 0:
        return 1
    else:
        return b * exp(b, n-1)

def square(x):
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def fast_exp(b, n):
    if n == 0:
        return 1
    elif n % 2 == 0:
        return square(fast_exp(b, n//2))
    else:
        return b * fast_exp(b, n-1)
```

Time	Space
$\Theta(n)$	$\Theta(n)$
$\Theta(\log n)$	$\Theta(\log n)$

Comparing Orders of Growth

Comparing orders of growth (n is the problem size)

$\Theta(b^n)$	Exponential growth! Recursive fib takes
	$\Theta(\phi^n)$ steps, where $\phi = \frac{1+\sqrt{5}}{2} \approx 1.61828$
$\Theta(n^6)$	Incrementing the problem scales R(n) by a factor.
$\Theta(n^2)$	Quadratic growth. E.g., operations on all pairs.
	Incrementing n increases R(n) by the problem size n.
$\Theta(n)$	Linear growth. Resources scale with the problem.
$\Theta(\sqrt{n})$	
$\Theta(\log n)$	Logarithmic growth. These processes scale well.
	Doubling the problem only increments R(n).
$\Theta(1)$	Constant. The problem size doesn't matter.