61A Lecture 18

Wednesday, October 16

Announcements

- Homework 6 is due Tuesday 10/22 @ 11:59pm
- Project 3 is due Thursday 10/24 @ 11:59pm
- Midterm 2 is on Monday 10/28 7pm–9pm
- Hog strategy contest winners will be announced on Wednesday 10/16 in lecture

Memoization

Memoization

Idea: Remember the results that have been computed before

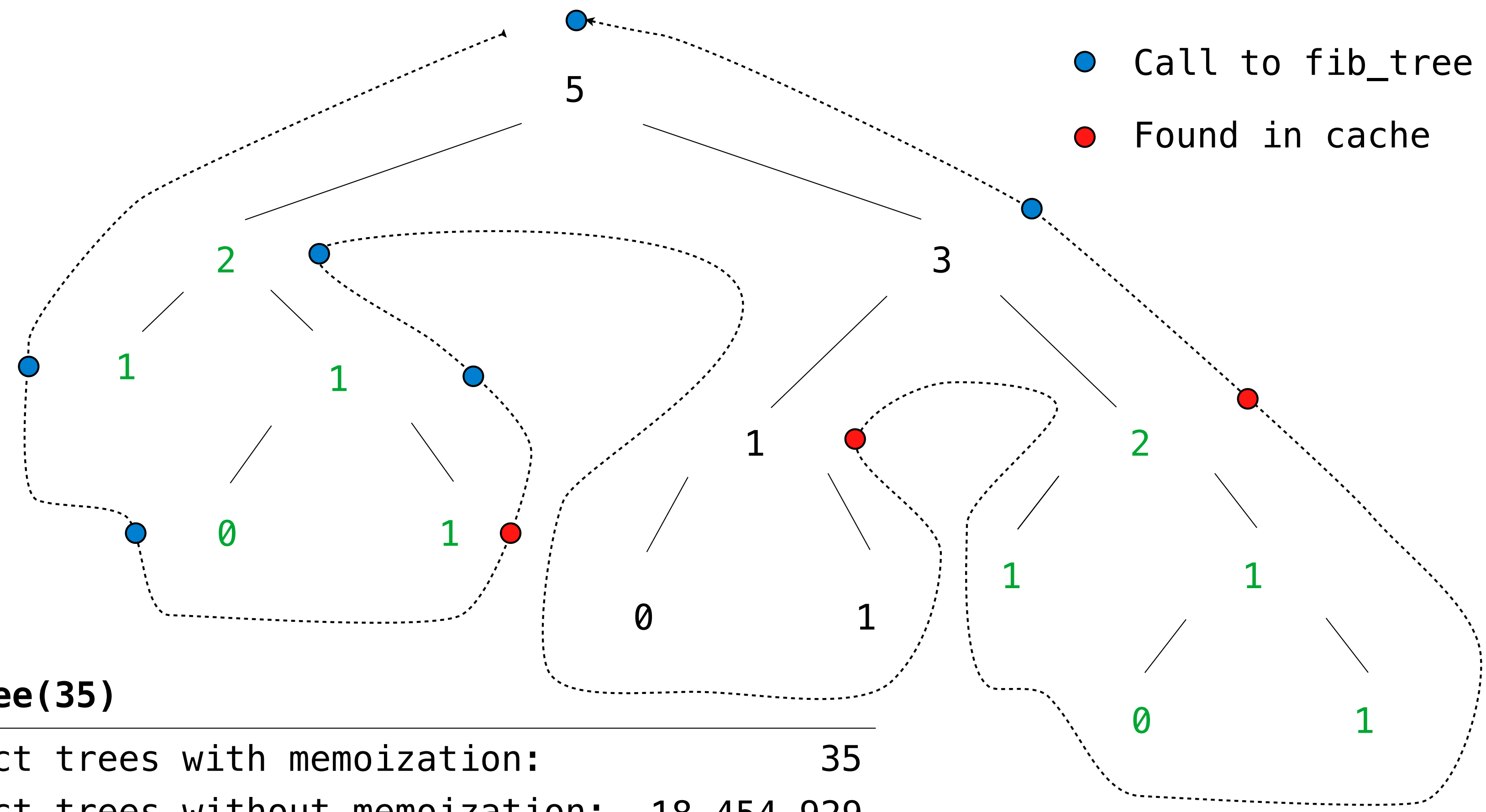
```
def memo(f):
    cache = {}
    def memoized(n):
        if n not in cache:
            cache[n] = f(n)
        return cache[n]
    return memoized
```

Keys are arguments that map to return values

Same behavior as f, if f is a pure function

(Demo)

Memoized Tree Recursion



fib_tree(35)

Distinct trees with memoization:	35
Distinct trees without memoization:	18,454,929

Time

The Consumption of Time

Implementations of the same functional abstraction can require different amounts of time to compute their result.

Problem: How many factors does a positive integer n have?

A factor k of n is a positive integer such that n/k is also a positive integer.

```
def count_factors(n):
```

	Time (number of divisions)
Slow: Test each k from 1 through n.	n
Fast: Test each k from 1 to square root n. For every k, n/k is also a factor!	$\lfloor\sqrt{n}\rfloor$

(Demo)

Space

The Consumption of Space

Which environment frames do we need to keep during evaluation?

Each step of evaluation has a set of **active** environments.

Values and frames in active environments consume memory.

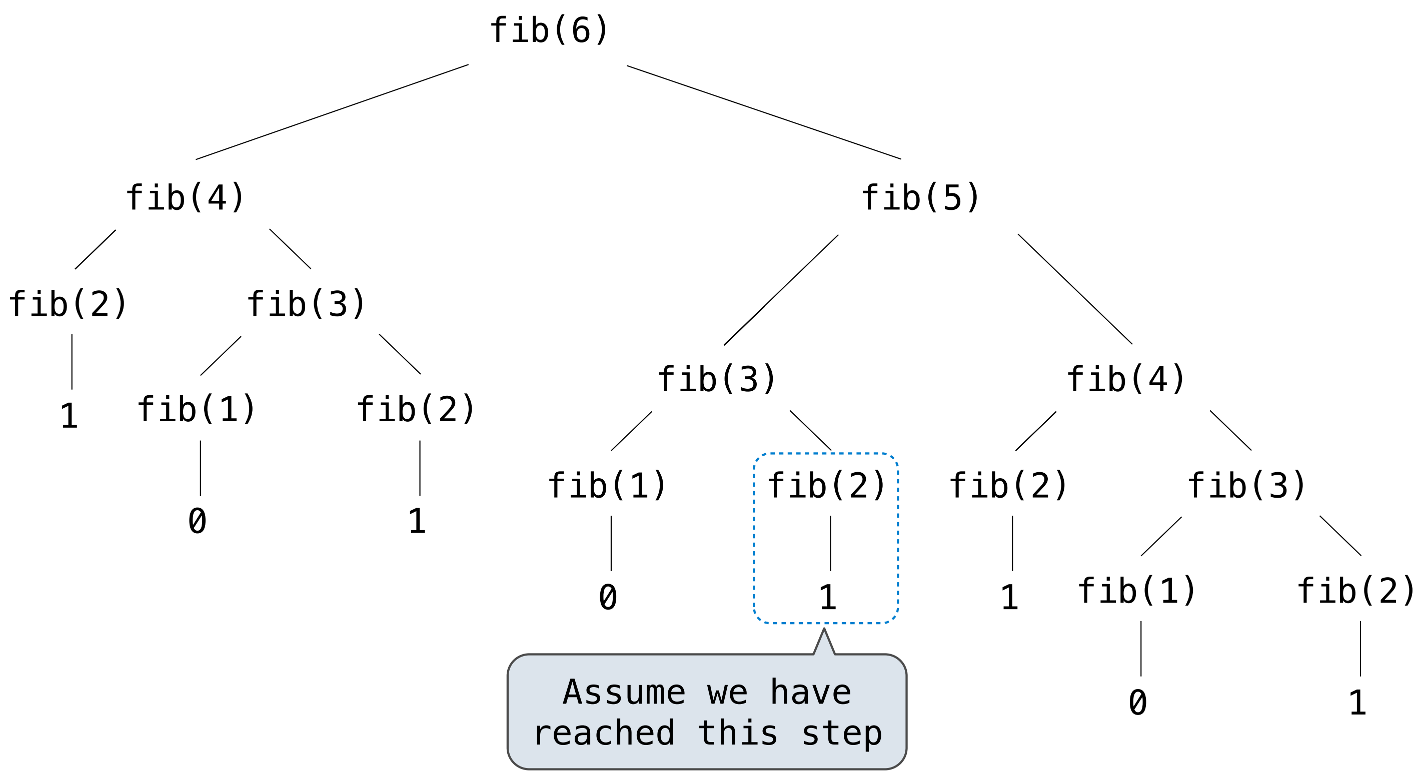
Memory used for other values and frames can be recycled.

Active environments:

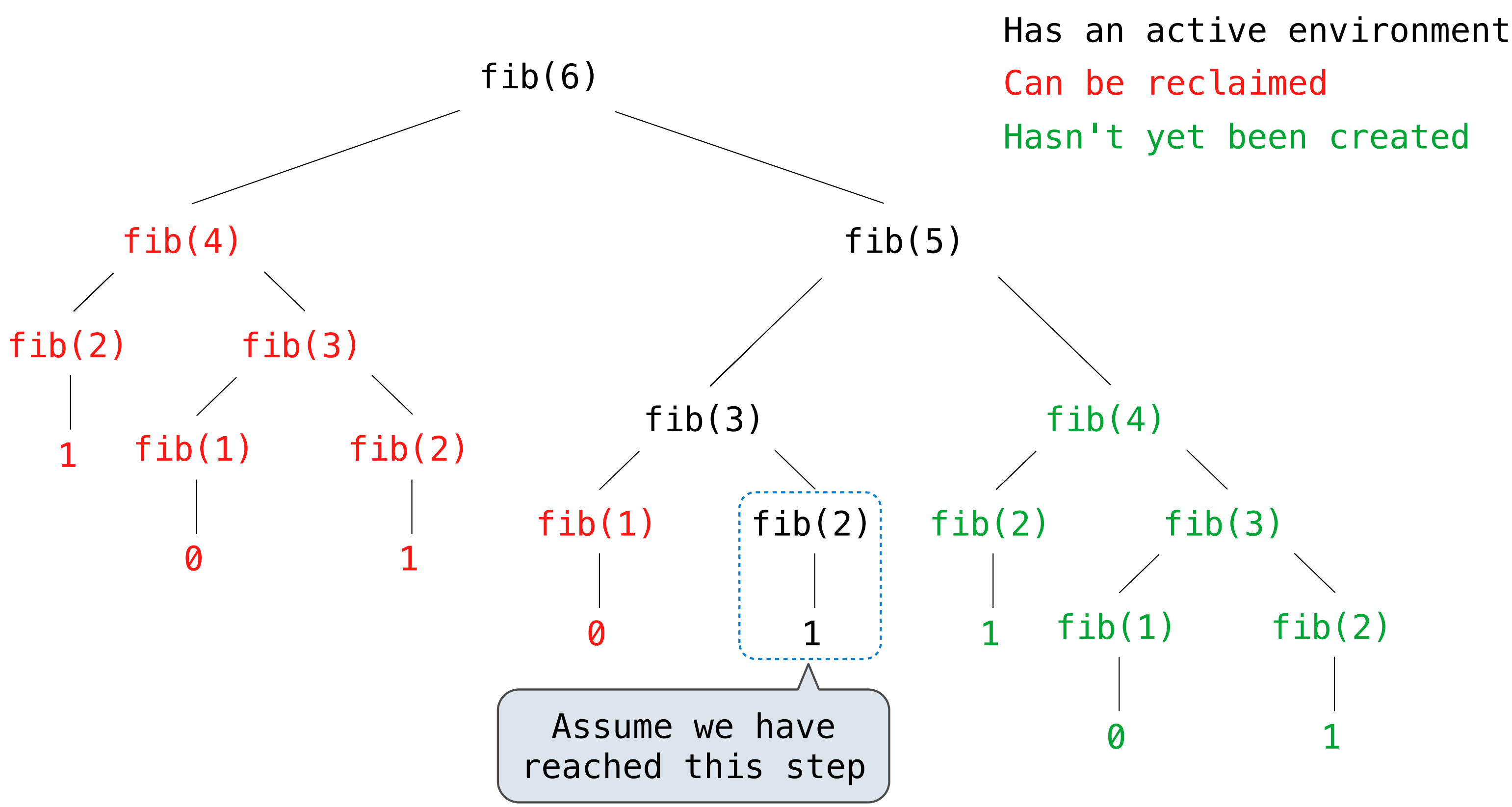
- Environments for any function calls currently being evaluated
- Parent environments of functions named in active environments

(Demo)

Fibonacci Memory Consumption



Fibonacci Memory Consumption



Order of Growth

Order of Growth

A method for bounding the resources used by a function by the "size" of a problem

n**:** size of the problem

R**(*****n*****):** Measurement of some resource used (time or space)

$$R(n) = \Theta(f(n))$$

means that there are positive constants k_1 and k_2 such that

$$k_1 \cdot f(n) \leq R(n) \leq k_2 \cdot f(n)$$

for sufficiently large values of ***n*****.**

Iteration vs Memoized Tree Recursion

Iterative and memoized implementations are not the same.

```
def fib_iter(n):
    prev, curr = 1, 0
    for _ in range(n-1):
        prev, curr = curr, prev + curr
    return curr
```

```
@memo
def fib(n):
    if n == 1:
        return 0
    if n == 2:
        return 1
    return fib(n-2) + fib(n-1)
```

	Time	**Space**
`fib_iter`	$\Theta(n)$	$\Theta(1)$
`fib` (memoized)	$\Theta(n)$	$\Theta(n)$

Counting Factors

Order of growth can still be used, even if we can quantify amounts exactly.

Problem: How many factors does a positive integer n have?

A factor k of n is a positive integer such that n/k is also a positive integer.

```
def count_factors(n)"
```

	Time	Space
Slow: Test each k from 1 to n.	$\Theta(n)$	$\Theta(1)$
Fast: Test each k from 1 to square root n. For every k, n/k is also a factor!	$\Theta(\sqrt{n})$	$\Theta(1)$

Exponentiation

Exponentiation

Goal: one more multiplication lets us double the problem size.

```
def exp(b, n):
    if n == 0:
        return 1
   else:
        return b * exp(b, n-1)
```

$$b^n = \begin{cases} 1 & \text{if } n = 0 \\ b \cdot b^{n-1} & \text{otherwise} \end{cases}$$

```
def square(x):
    return x*x

def fast_exp(b, n):
    if n == 0:
        return 1
    elif n % 2 == 0:
        return square(fast_exp(b, n//2))
    else:
        return b * fast_exp(b, n-1)
```

$$b^n = \begin{cases} 1 & \text{if } n = 0 \\ (b^{\frac{1}{2}n})^2 & \text{if } n \text{ is even} \\ b \cdot b^{n-1} & \text{if } n \text{ is odd} \end{cases}$$

(Demo)

Exponentiation

Goal: one more multiplication lets us double the problem size.

```
def exp(b, n):
    if n == 0:
        return 1
    else:
        return b * exp(b, n-1)

def square(x):
    return x*x

def fast_exp(b, n):
    if n == 0:
        return 1
    elif n % 2 == 0:
        return square(fast_exp(b, n//2))
    else:
        return b * fast_exp(b, n-1)
```

	Time	Space
exp	$\Theta(n)$	$\Theta(n)$
fast_exp	$\Theta(\log n)$	$\Theta(\log n)$

Comparing Orders of Growth

Comparing orders of growth (n is the problem size)

$\Theta(b^n)$ Exponential growth! Recursive fib takes

$\Theta(\phi^n)$ steps, where $\phi = \frac{1+\sqrt{5}}{2} \approx 1.61828$

$\Theta(n^6)$ Incrementing the problem scales R(n) by a factor.

$\Theta(n^2)$ Quadratic growth. E.g., operations on all pairs.

Incrementing n increases R(n) by the problem size n.

$\Theta(n)$ Linear growth. Resources scale with the problem.

$\Theta(\sqrt{n})$

$\Theta(\log n)$ Logarithmic growth. These processes scale well.

Doubling the problem only increments R(n).

$\Theta(1)$ Constant. The problem size doesn't matter.