61A Lecture 14

Friday, October 4

Announcements

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  - Topics: Data abstraction, sequences, and non-local assignment.
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- Guest lecture on Wednesday 10/9, Peter Norvig on Natural Language Processing in Python.
  - No video (except a screencast)! Come to Wheeler.

Mutable Functions

A Function with Behavior That Varies Over Time

```
Let's model a bank account that has a balance of $100
```

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```
>>> withdraw(25)
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A Function with Behavior That Varies Over Time

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Argument:
amount to withdraw

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Return value:
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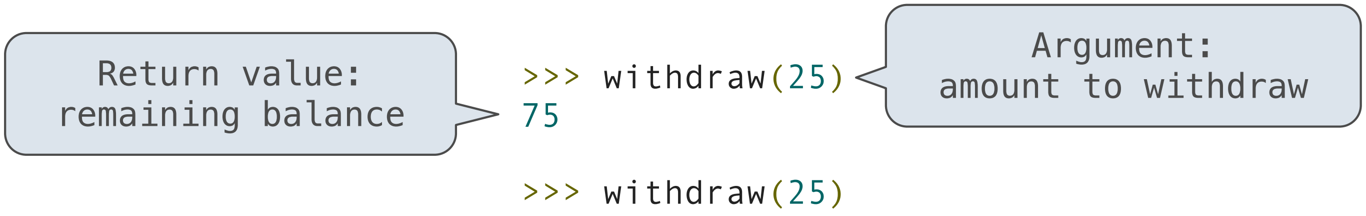

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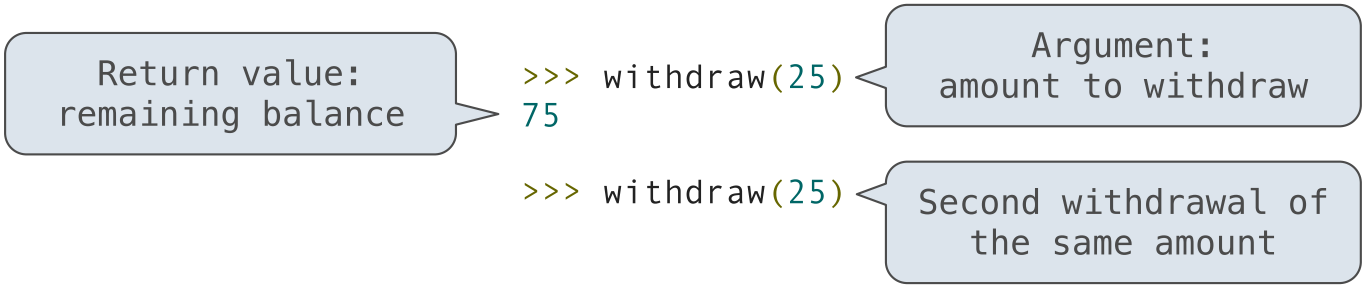

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75

>>> withdraw(25)
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Return value:
remaining balance

Argument:
amount to withdraw

Second withdrawal of
the same amount

A Function with Behavior That Varies Over Time

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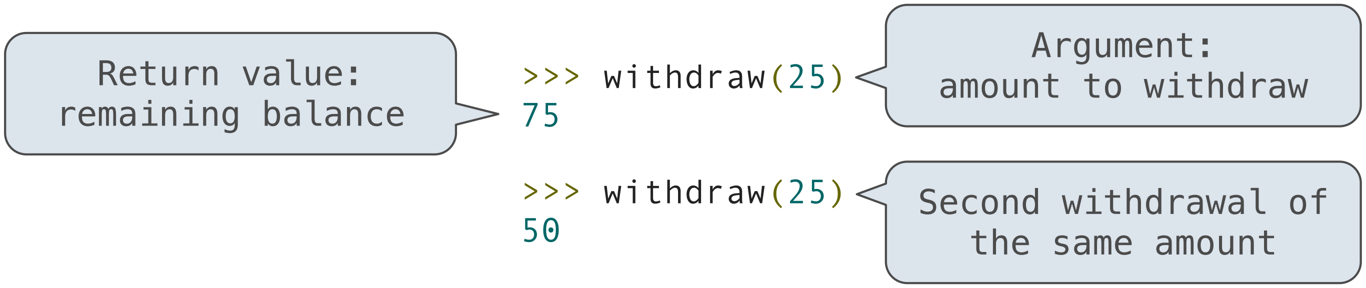

```
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75

>>> withdraw(25)
50
```

Return value:
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Argument:
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Second withdrawal of
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A Function with Behavior That Varies Over Time

Let's model a bank account that has a balance of $100

Return value:
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Argument:
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Different
return value!

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Second withdrawal of
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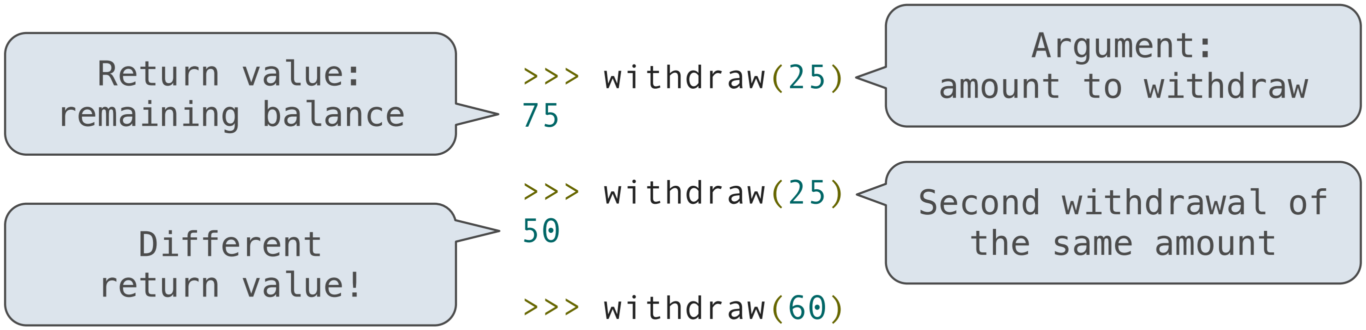
A Function with Behavior That Varies Over Time

Let's model a bank account that has a balance of $100



Return value: remaining balance

```
>>> withdraw(25)
75

>>> withdraw(25)
50

>>> withdraw(60)
```

Argument: amount to withdraw

Different return value!

Second withdrawal of the same amount

A Function with Behavior That Varies Over Time

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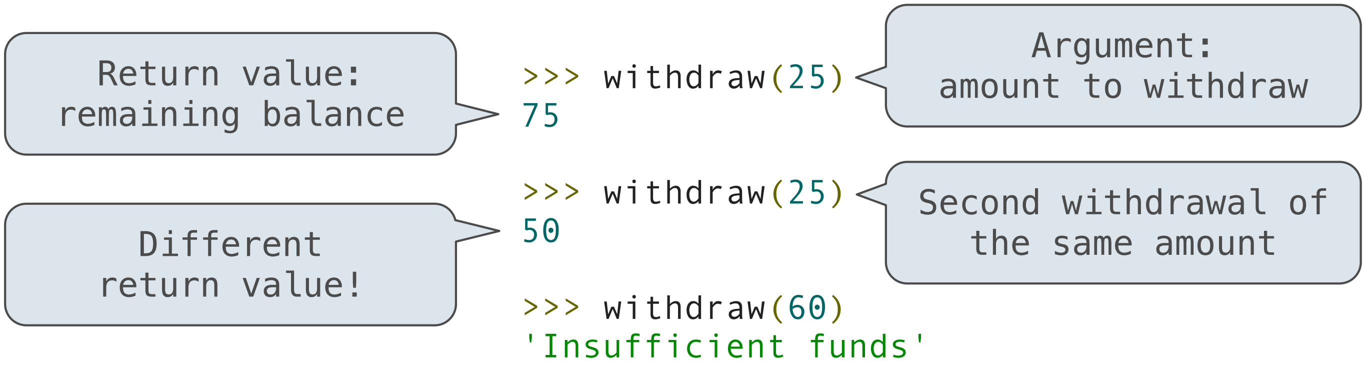

```
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'Insufficient funds'
```

Return value: remaining balance

Argument: amount to withdraw

Different return value!

Second withdrawal of the same amount

A Function with Behavior That Varies Over Time

Let's model a bank account that has a balance of $100

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>>> withdraw(60)
'Insufficient funds'

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```

Return value: remaining balance

Argument: amount to withdraw

Different return value!

Second withdrawal of the same amount

A Function with Behavior That Varies Over Time

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Where's this balance
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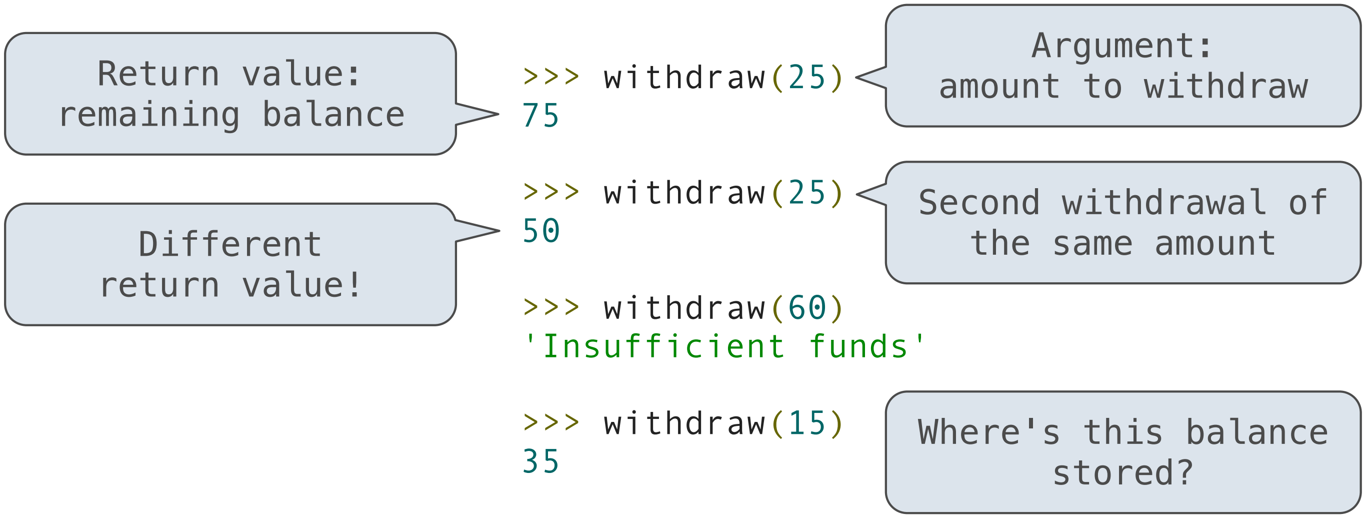

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Return value: remaining balance

Argument: amount to withdraw

Different return value!

Second withdrawal of the same amount

Where's this balance stored?

```
>>> withdraw = make_withdraw(100)
```

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Within the parent frame
of the function!

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Return value: remaining balance

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Different return value!

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```

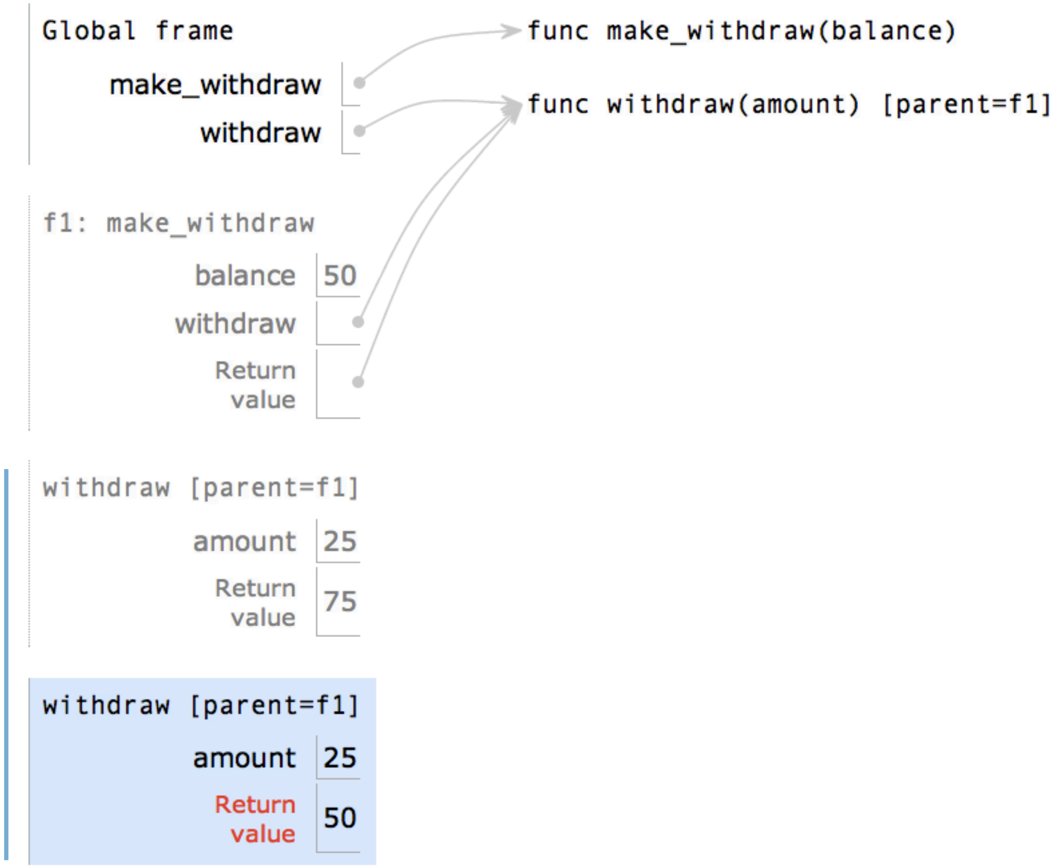
Where's this balance stored?

```
>>> withdraw = make_withdraw(100)
```

Within the parent frame of the function!

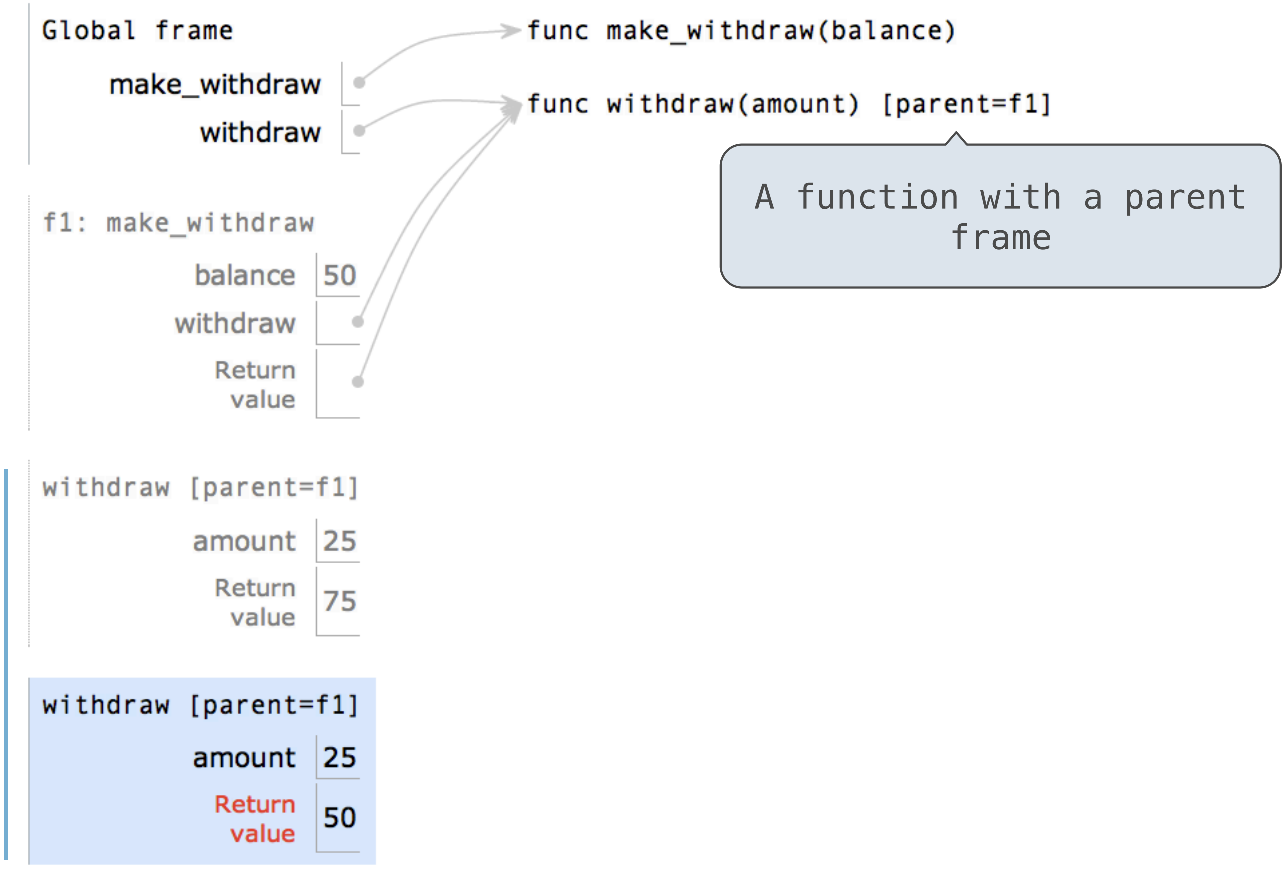
A function has a body and a parent environment

Persistent Local State Using Environments



Example: http://goo.gl/cUC09s

Persistent Local State Using Environments



Global frame

make_withdraw

withdraw

func make_withdraw(balance)

func withdraw(amount) [parent=f1]

f1: make_withdraw

balance 50

withdraw

Return value

withdraw [parent=f1]

amount 25

Return value 75

withdraw [parent=f1]

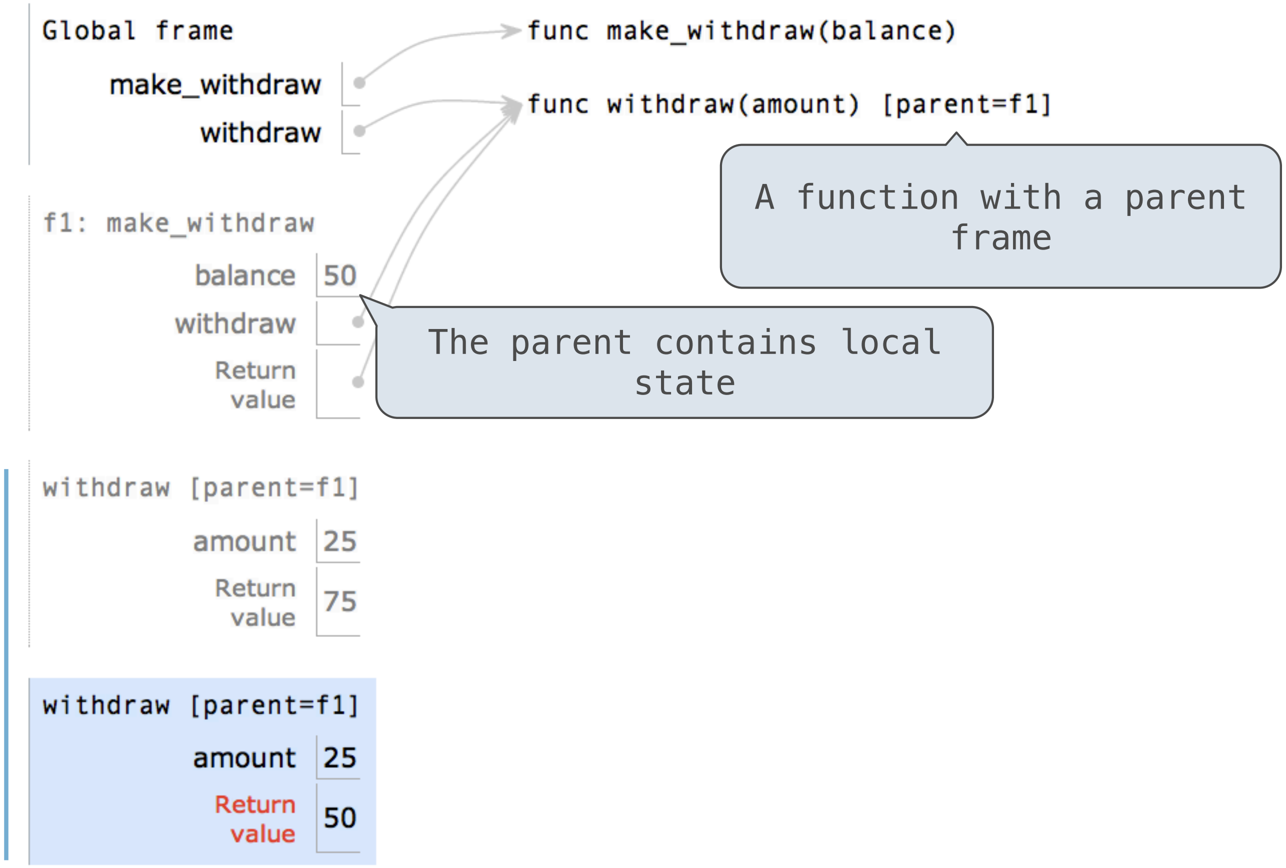
amount 25

Return value 50

A function with a parent frame

Example: http://goo.gl/cUC09s

Persistent Local State Using Environments



Global frame

make_withdraw

withdraw

func make_withdraw(balance)

func withdraw(amount) [parent=f1]

A function with a parent frame

f1: make_withdraw

balance 50

withdraw

Return value

The parent contains local state

withdraw [parent=f1]

amount 25

Return value 75

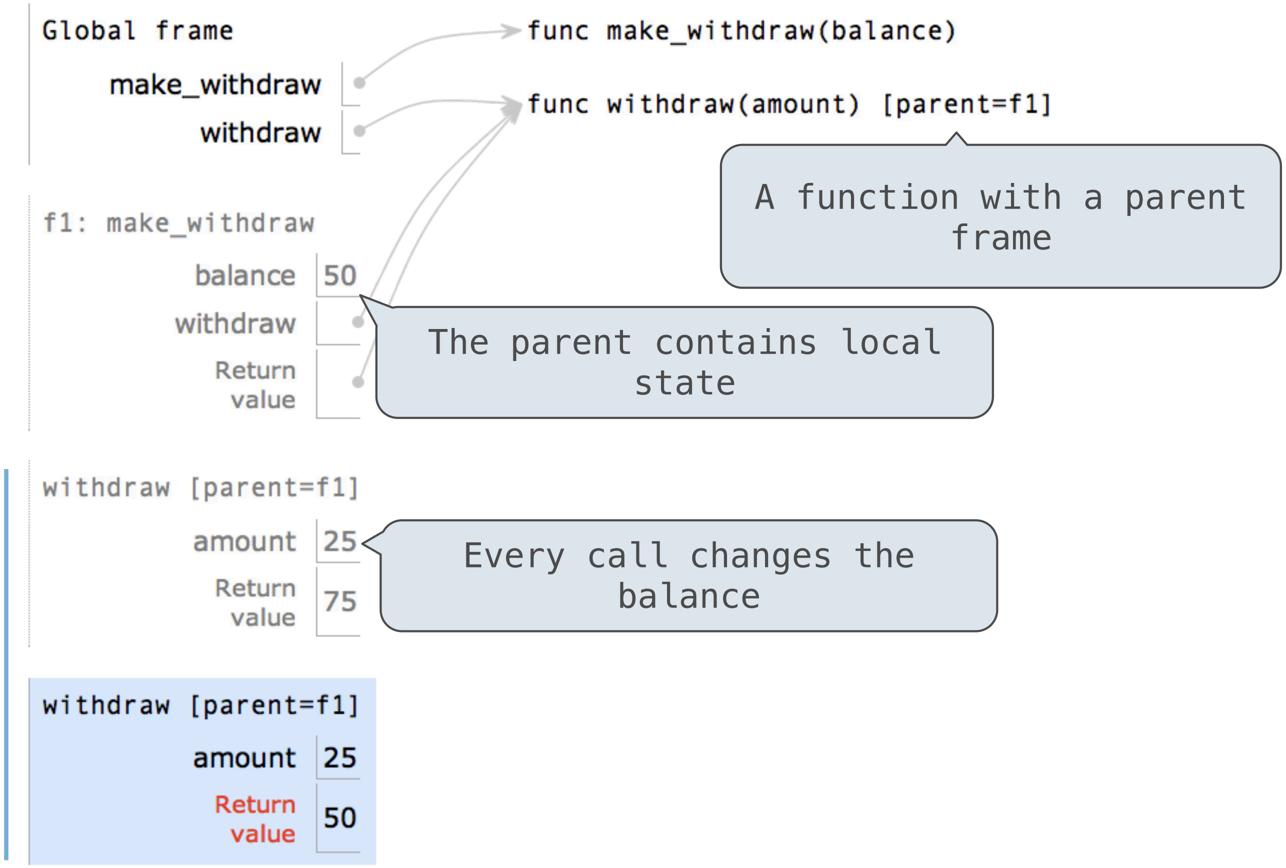
withdraw [parent=f1]

amount 25

Return value 50

Example: http://goo.gl/cUC09s

Persistent Local State Using Environments



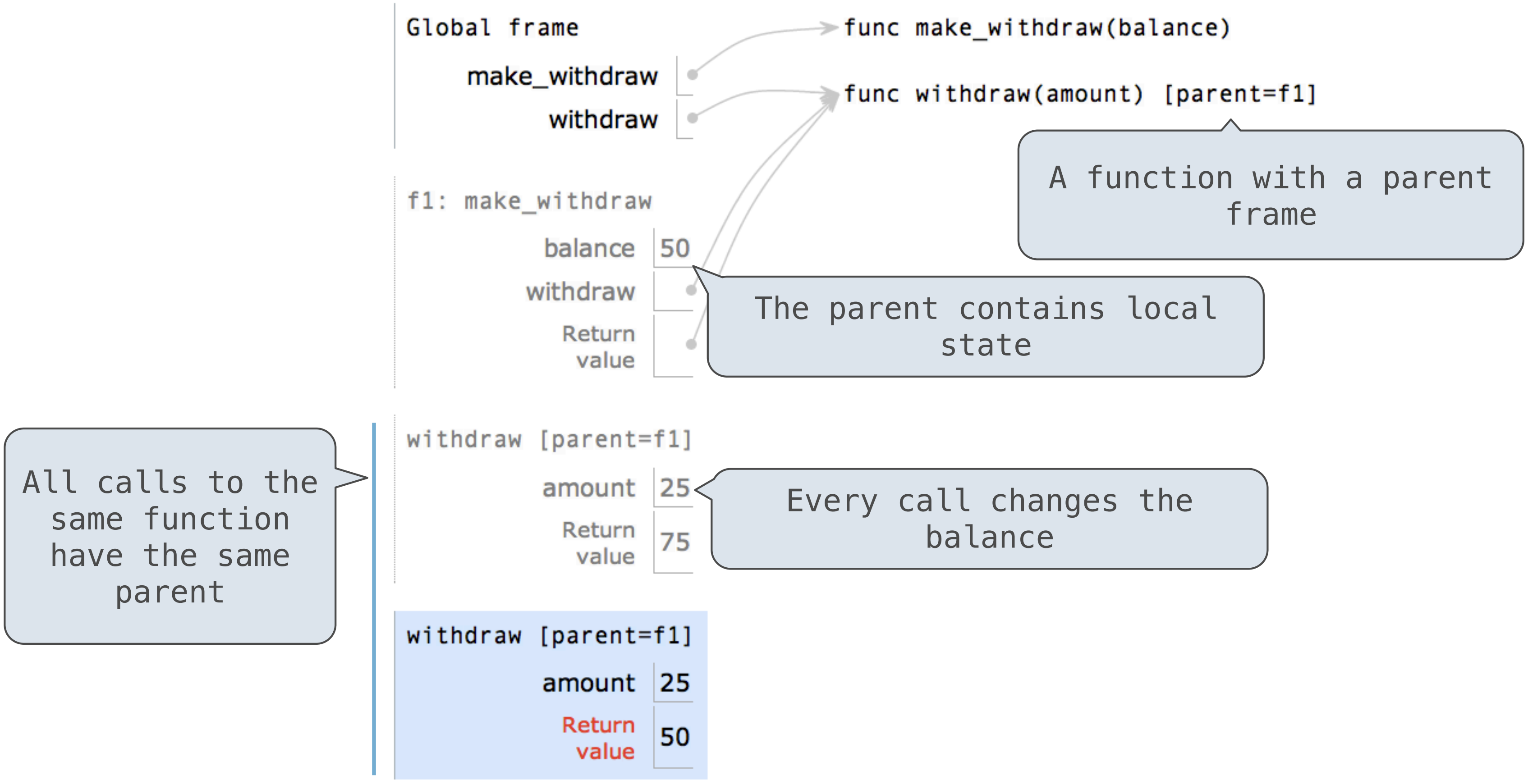
A function with a parent frame

The parent contains local state

Every call changes the balance

Example: http://goo.gl/cUC09s

Persistent Local State Using Environments



A function with a parent frame

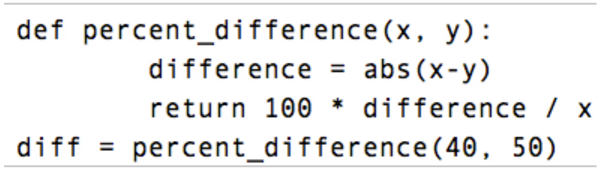
The parent contains local state

Every call changes the balance

All calls to the same function have the same parent

Example: http://goo.gl/cUC09s

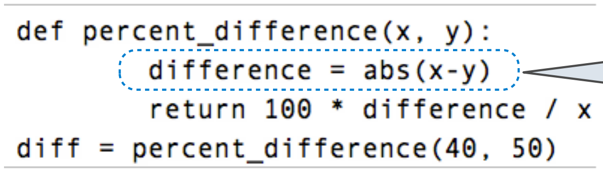
Reminder: Local Assignment



```
def percent_difference(x, y):
        difference = abs(x-y)
        return 100 * difference / x
diff = percent_difference(40, 50)
```

Example: http://goo.gl/Wxpg5Z

Reminder: Local Assignment



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Assignment binds name(s) to value(s) in the first frame of the current environment

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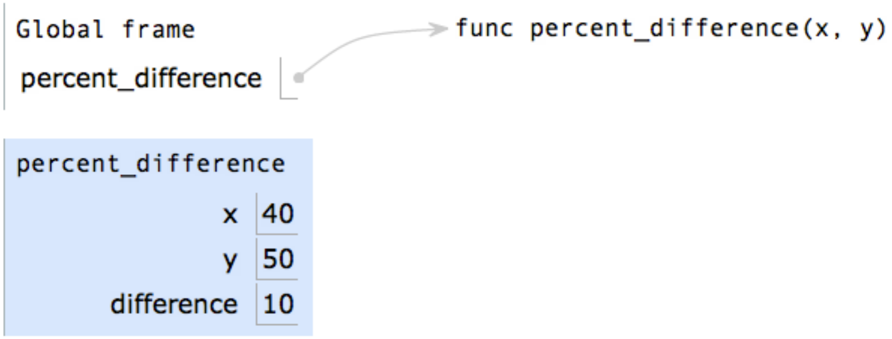

Global frame

percent_difference → func percent_difference(x, y)

percent_difference

x	40
y	50
difference	10

Example: http://goo.gl/Wxpg5Z

Reminder: Local Assignment

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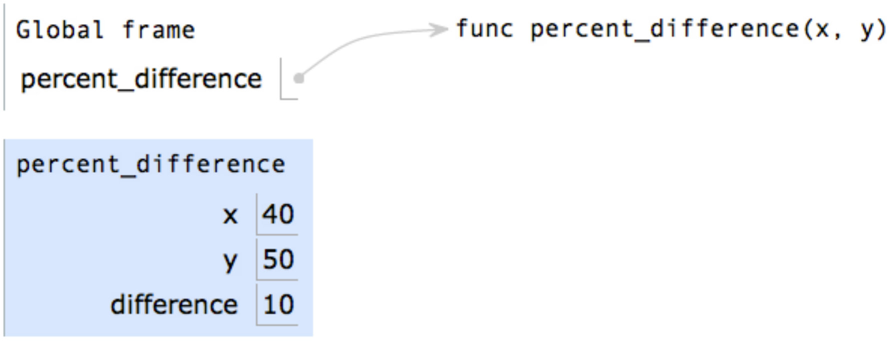

Global frame

percent_difference

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percent_difference

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y 50

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Execution rule for assignment statements:

Example: http://goo.gl/Wxpg5Z

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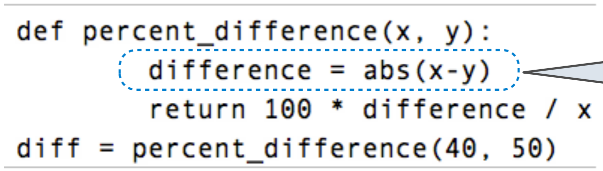

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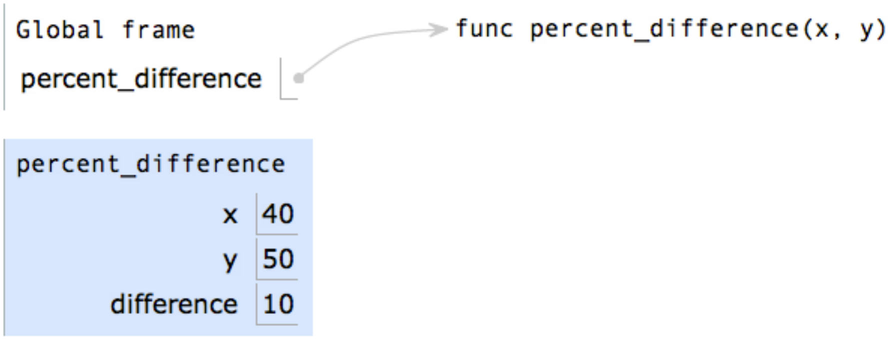

Global frame

percent_difference

func percent_difference(x, y)

percent_difference

x 40

y 50

difference 10

Execution rule for assignment statements:

1. Evaluate all expressions right of =, from left to right.

2. Bind the names on the left the resulting values in the **first frame** of the current environment.

Example: http://goo.gl/Wxpg5Z

Non-Local Assignment & Persistent Local State

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```
def make_withdraw(balance):
```

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def make_withdraw(balance):
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Non-Local Assignment & Persistent Local State

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def make_withdraw(balance):
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    def withdraw(amount):
```

Non-Local Assignment & Persistent Local State

```
def make_withdraw(balance):
    """Return a withdraw function with a starting balance."""
    def withdraw(amount):
        nonlocal balance
```

Non-Local Assignment & Persistent Local State

```
def make_withdraw(balance):
    """Return a withdraw function with a starting balance."""
    def withdraw(amount):
        nonlocal balance
        if amount > balance:
```

Non-Local Assignment & Persistent Local State

```
def make_withdraw(balance):
    """Return a withdraw function with a starting balance."""
    def withdraw(amount):
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        if amount > balance:
            return 'Insufficient funds'
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Non-Local Assignment & Persistent Local State

```python
def make_withdraw(balance):
    """Return a withdraw function with a starting balance."""
    def withdraw(amount):
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        if amount > balance:
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        balance = balance - amount
```

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Declare the name "balance" nonlocal at the top of the body of the function in which it is re-assigned

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Re-bind balance in the first non-local frame in which it was bound previously

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Declare the name "balance" nonlocal at the top of the body of the function in which it is re-assigned

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(Demo)

Non-Local Assignment

The Effect of Nonlocal Statements

```
nonlocal <name>
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nonlocal <name>
```

Effect: Future assignments to that name change its pre-existing binding in the **first non-local frame** of the current environment in which that name is bound.

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Python Docs: an "enclosing scope"

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From the Python 3 language reference:

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From the Python 3 language reference:

Names listed in a nonlocal statement must refer to pre-existing bindings in an enclosing scope.

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Names listed in a nonlocal statement must not collide with pre-existing bindings in the local scope.

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http://docs.python.org/release/3.1.3/reference/simple_stmts.html#the-nonlocal-statement

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http://www.python.org/dev/peps/pep-3104/

The Many Meanings of Assignment Statements

```
x = 2
```

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Status **Effect**

The Many Meanings of Assignment Statements

```
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```

Status	Effect
- No nonlocal statement - "x" **is not** bound locally	

The Many Meanings of Assignment Statements

```
x = 2
```

Status	**Effect**
•No nonlocal statement •"x" **is not** bound locally	Create a new binding from name "x" to object 2 in the first frame of the current environment.

The Many Meanings of Assignment Statements

```
x = 2
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Status	**Effect**
•No nonlocal statement •"x" **is not** bound locally	Create a new binding from name "x" to object 2 in the first frame of the current environment**.**
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The Many Meanings of Assignment Statements

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•nonlocal x •"x" **is** bound in a non-local frame	

The Many Meanings of Assignment Statements

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•nonlocal x •"x" **is** bound in a non-local frame	Re-bind "x" to 2 in the first non-local frame of the current environment in which it is bound.

The Many Meanings of Assignment Statements

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•nonlocal x •"x" **is** bound in a non-local frame	Re-bind "x" to 2 in the first non-local frame of the current environment in which it is bound.
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The Many Meanings of Assignment Statements

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• nonlocal x • "x" **is** bound in a non-local frame	Re-bind "x" to 2 in the first non-local frame of the current environment in which it is bound.
• nonlocal x • "x" **is not** bound in a non-local frame	SyntaxError: no binding for nonlocal 'x' found

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•nonlocal x •"x" **is not** bound in a non-local frame	SyntaxError: no binding for nonlocal 'x' found
•nonlocal x •"x" **is** bound in a non-local frame •"x" also bound locally	

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•nonlocal x •"x" **is not** bound in a non-local frame	SyntaxError: no binding for nonlocal 'x' found
•nonlocal x •"x" **is** bound in a non-local frame •"x" also bound locally	SyntaxError: name 'x' is parameter and nonlocal

Python Particulars

Example: http://goo.gl/bOVzc6

Python Particulars

Python pre-computes which frame contains each name before executing the body of a function.

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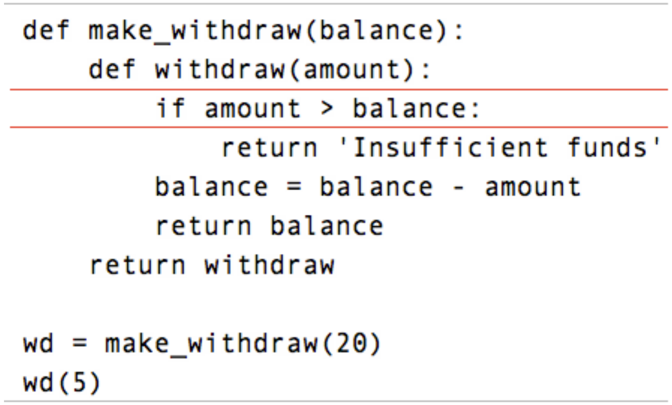

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def make_withdraw(balance):
    def withdraw(amount):
        if amount > balance:
            return 'Insufficient funds'
        balance = balance - amount
        return balance
    return withdraw

wd = make_withdraw(20)
wd(5)
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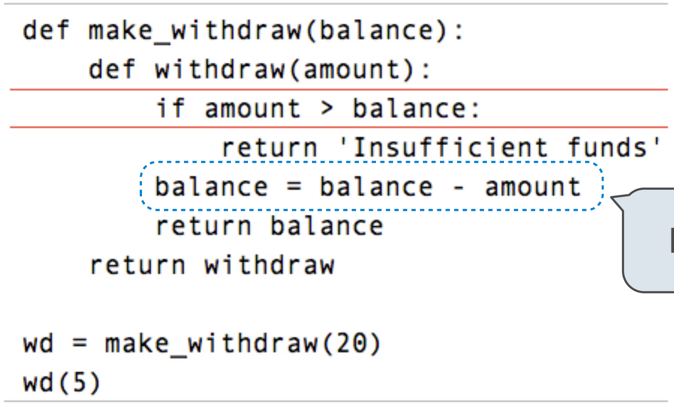

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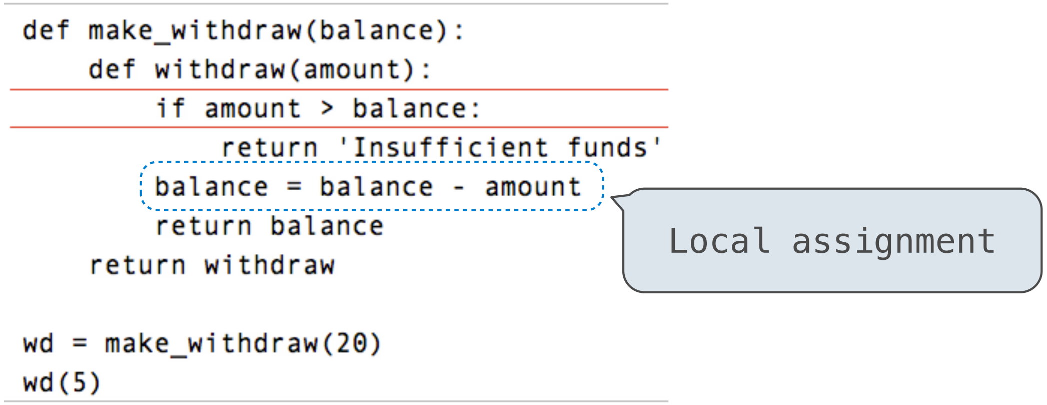

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Local assignment

UnboundLocalError: local variable 'balance' referenced before assignment

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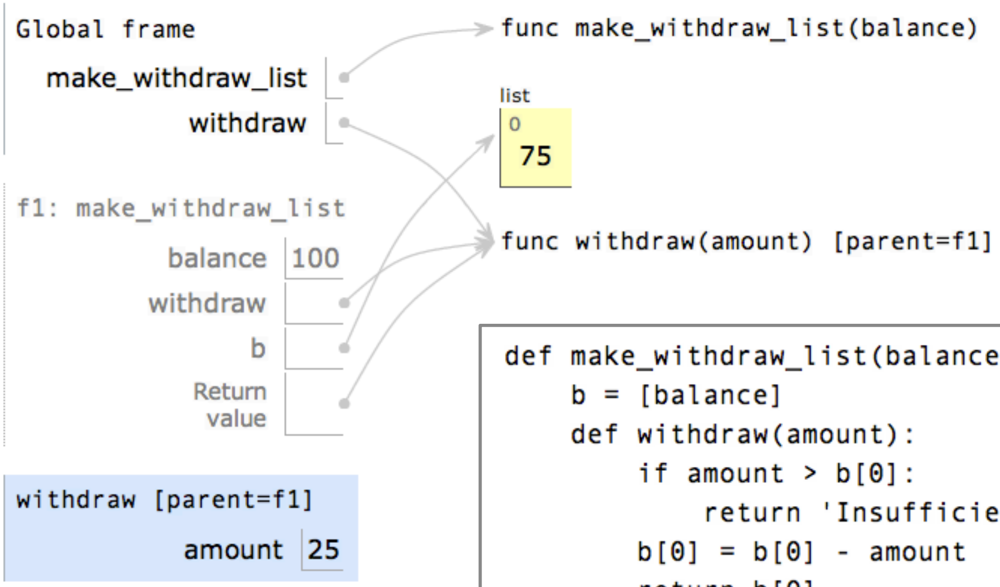
Mutable Values & Persistent Local State

Mutable values can be changed *without* a nonlocal statement.



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def make_withdraw_list(balance):
    b = [balance]
    def withdraw(amount):
        if amount > b[0]:
            return 'Insufficient funds'
        b[0] = b[0] - amount
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withdraw = make_withdraw_list(100)
withdraw(25)
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Example: http://goo.gl/y4TyFZ

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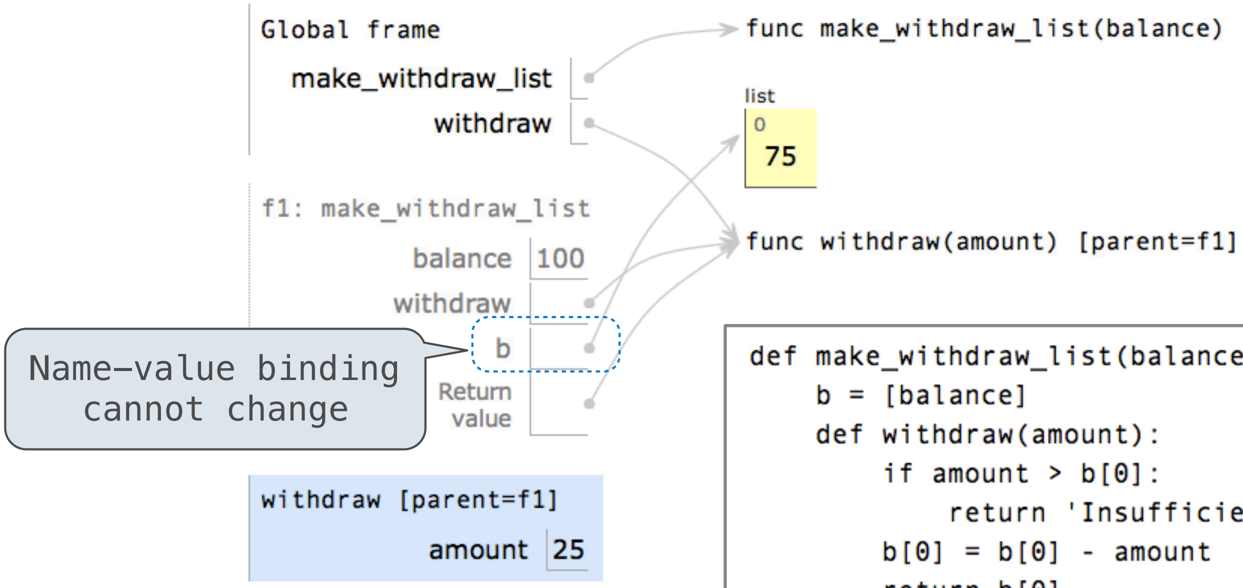

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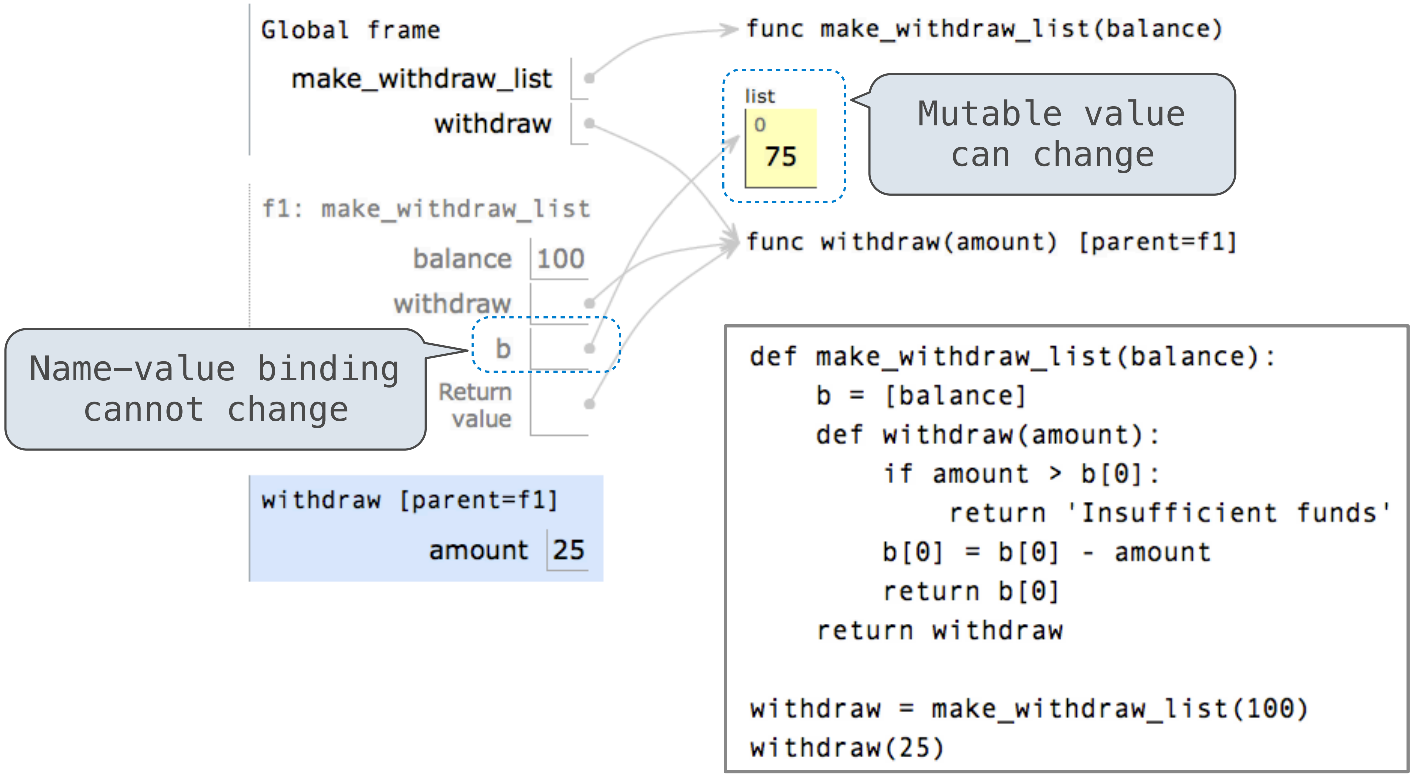

Mutable value can change

Name-value binding cannot change

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Multiple Mutable Functions

(Demo)

Sameness and Change

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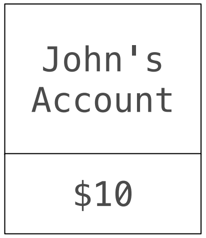

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