61A Lecture 7

Monday, September 16

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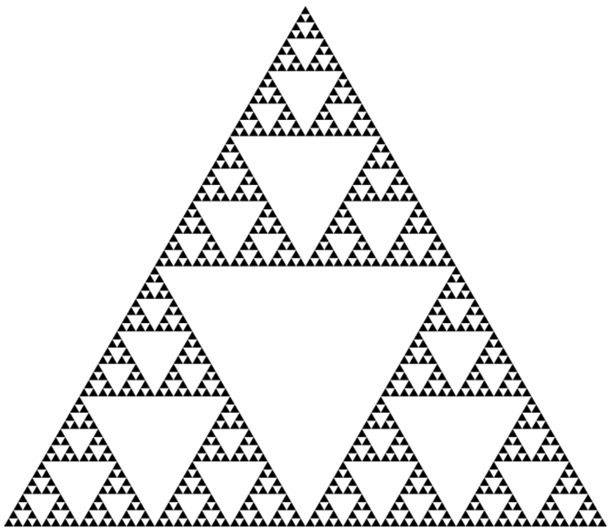

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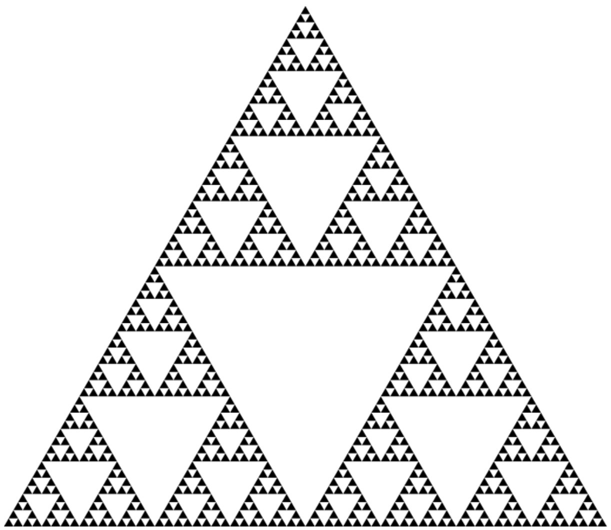

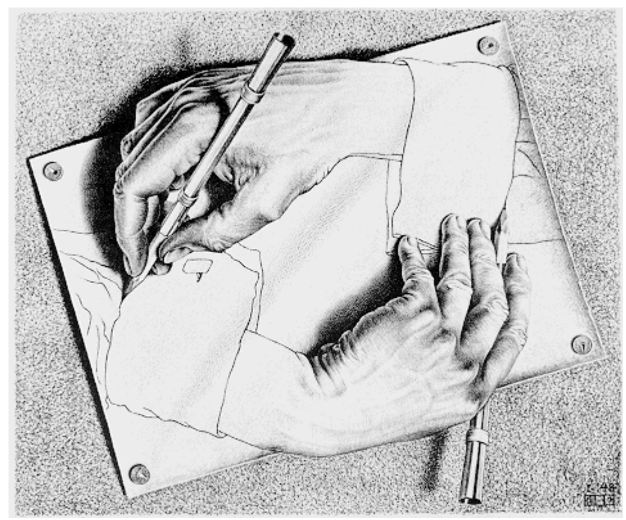

Drawing Hands, by M. C. Escher (lithograph, 1948)

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A checksum digit is a function of all the other digits; It can be computed to detect typos

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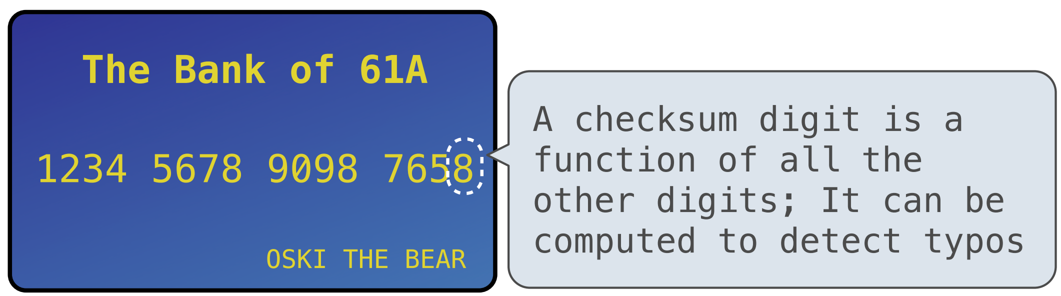

- Credit cards actually use the Luhn algorithm, which we'll implement after digit_sum.

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Example: http://goo.gl/XOP9ps

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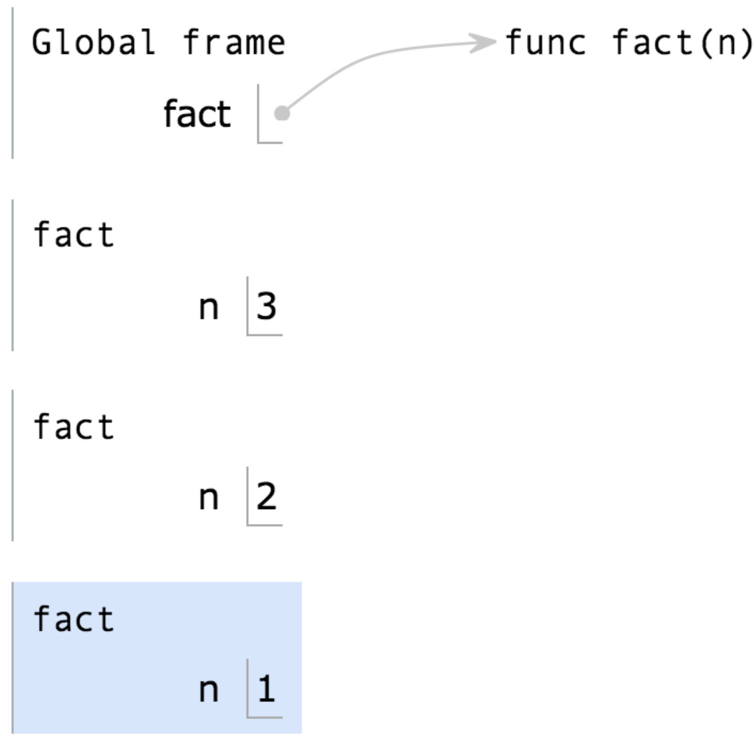

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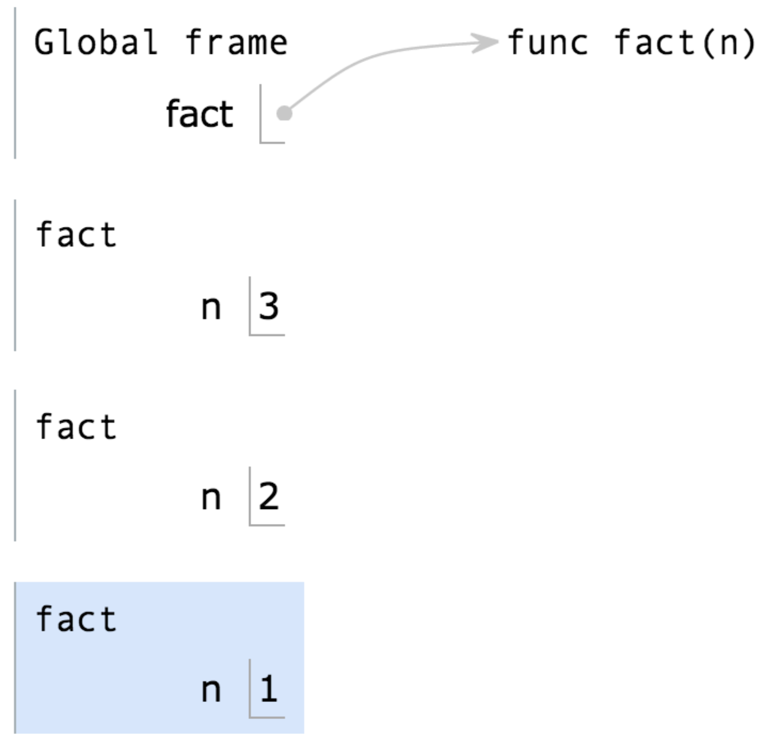

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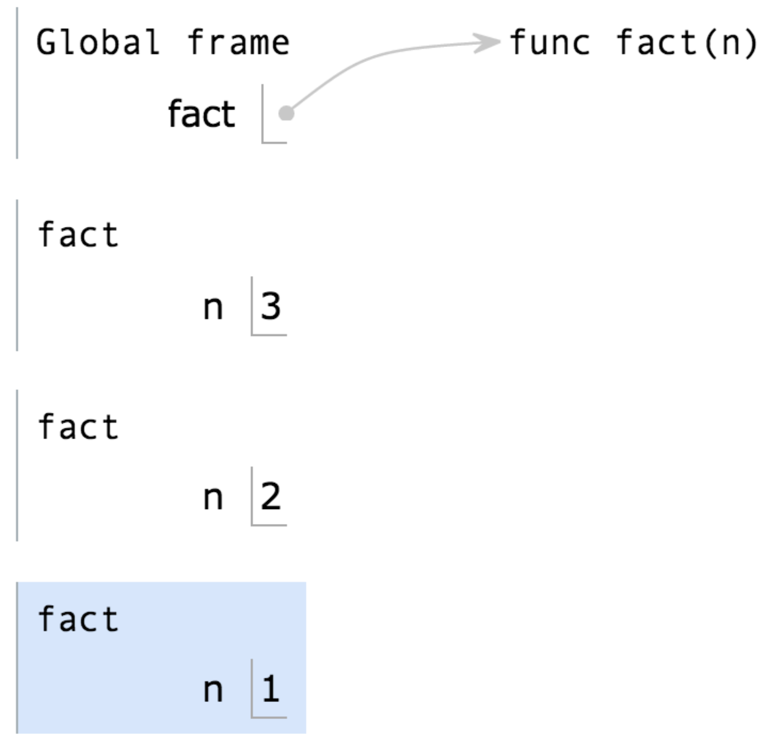

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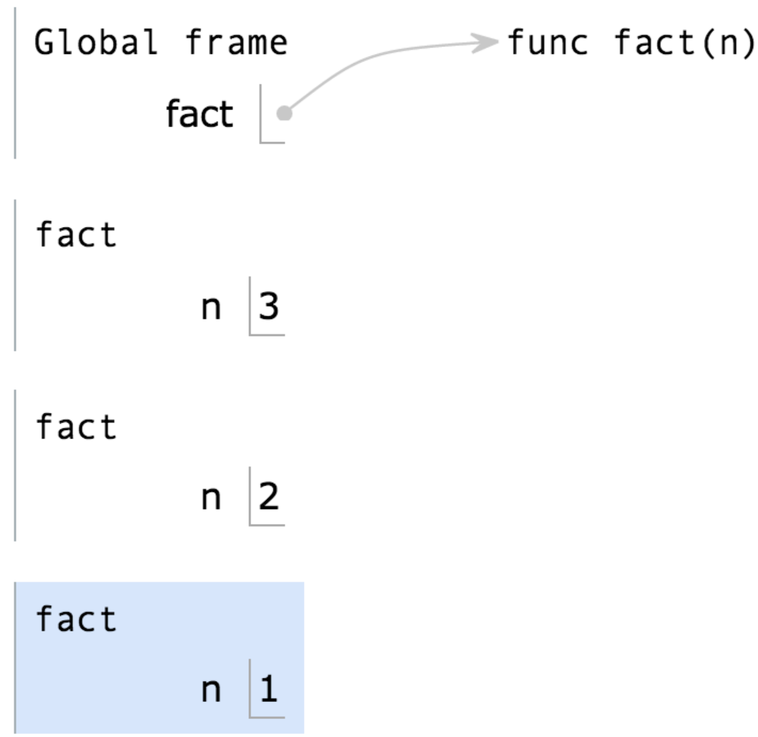

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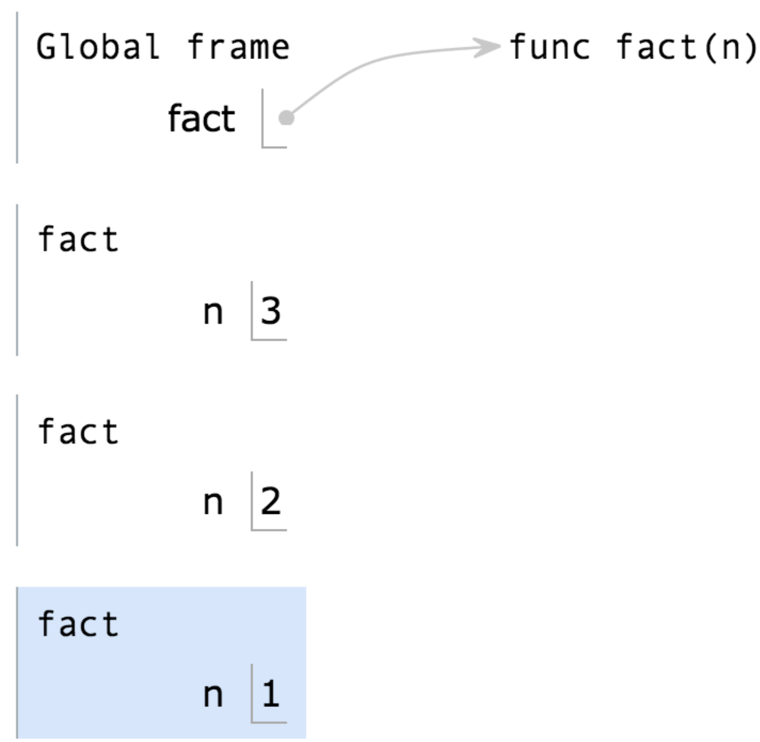

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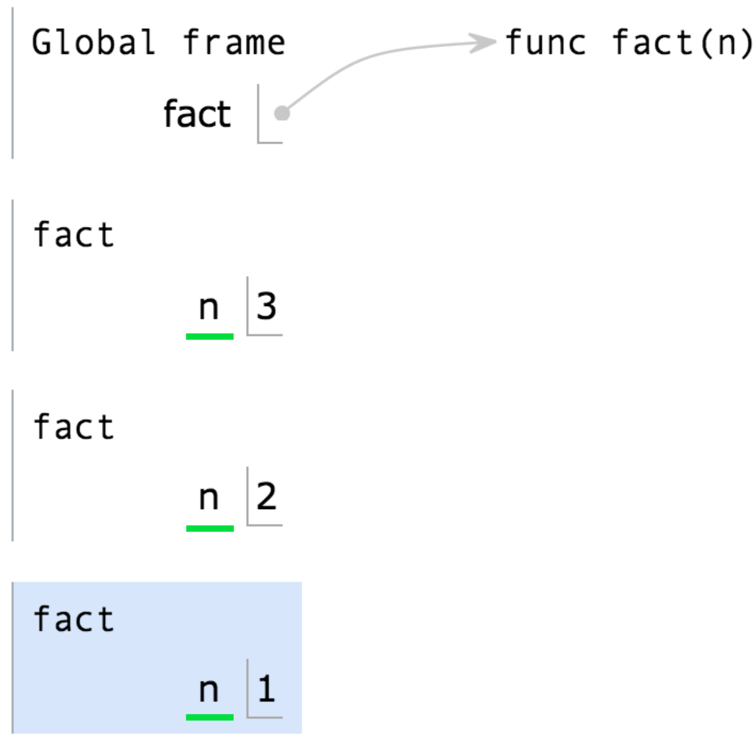

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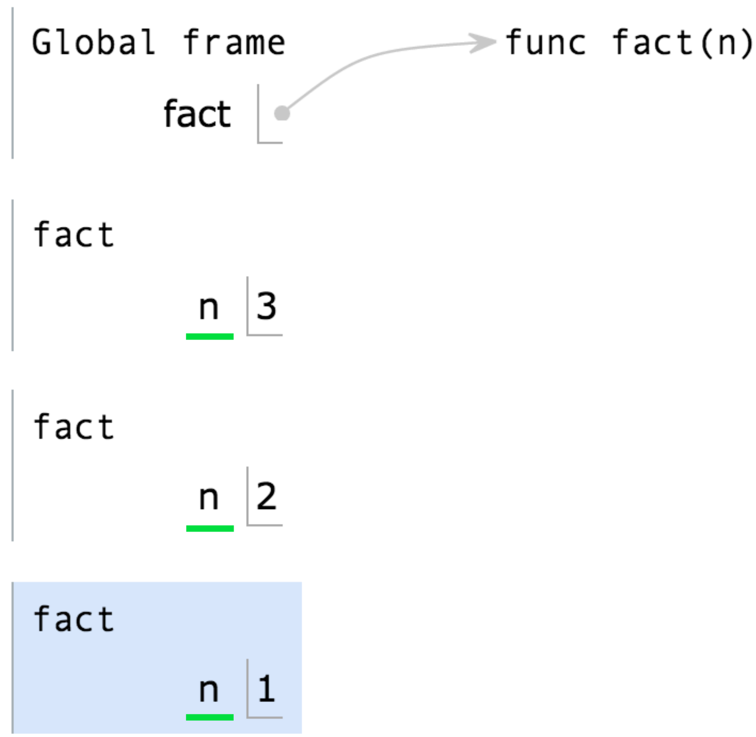

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- What **n** evaluates to depends upon which is the current environment.
- Each call to **fact** solves a simpler problem than the last: smaller **n**.

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Iteration vs Recursion

Example: http://goo.gl/NgH3Lf

Iteration vs Recursion

`Iteration is a special case of recursion`

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Verifying Recursive Functions

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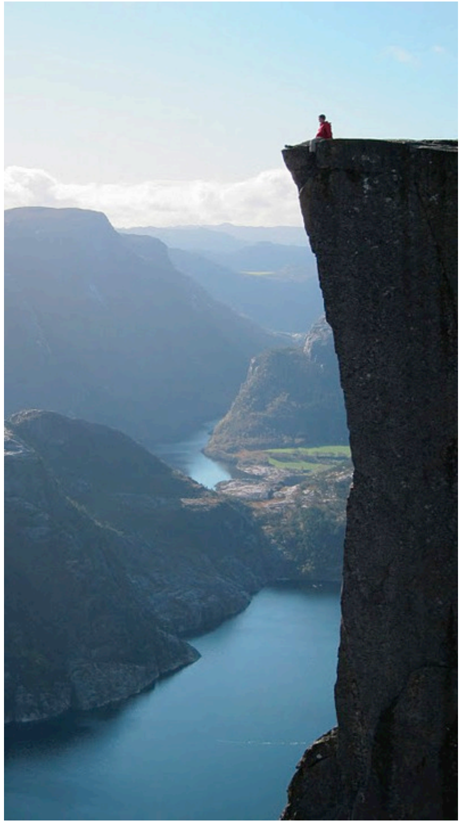

Photo by Kevin Lee, Preikestolen, Norway

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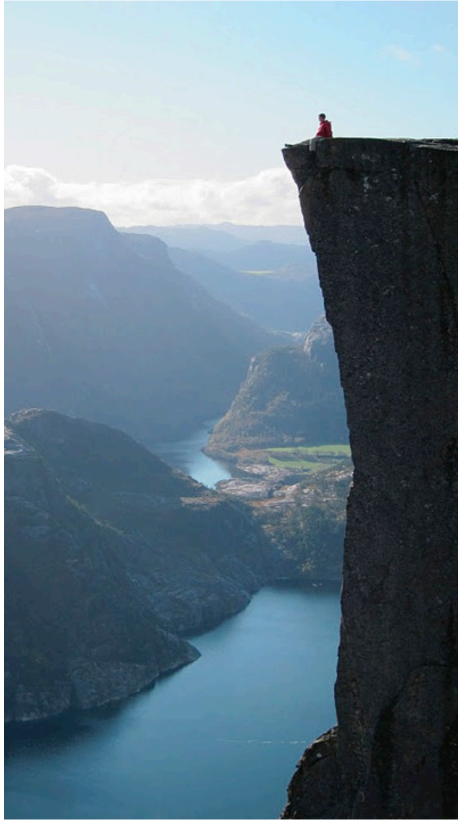

Photo by Kevin Lee, Preikestolen, Norway

The Recursive Leap of Faith

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def fact(n):
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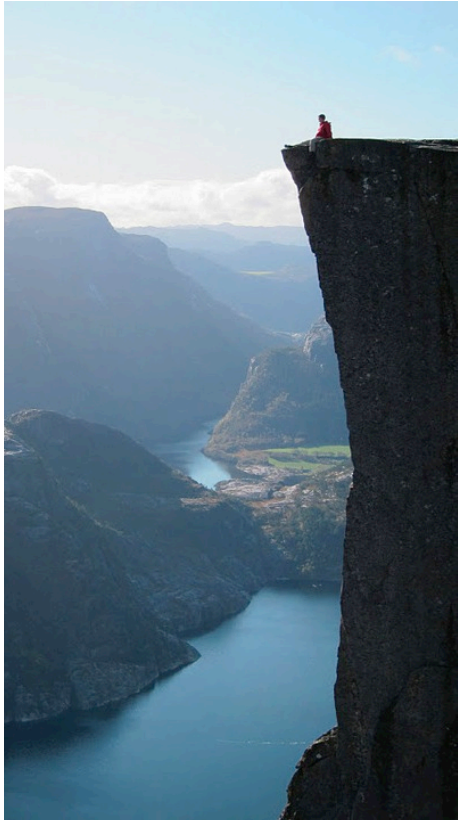
Is fact implemented correctly?



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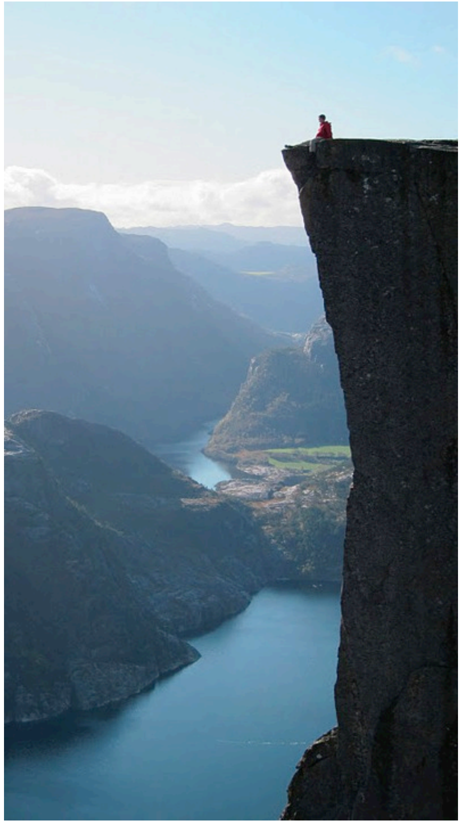

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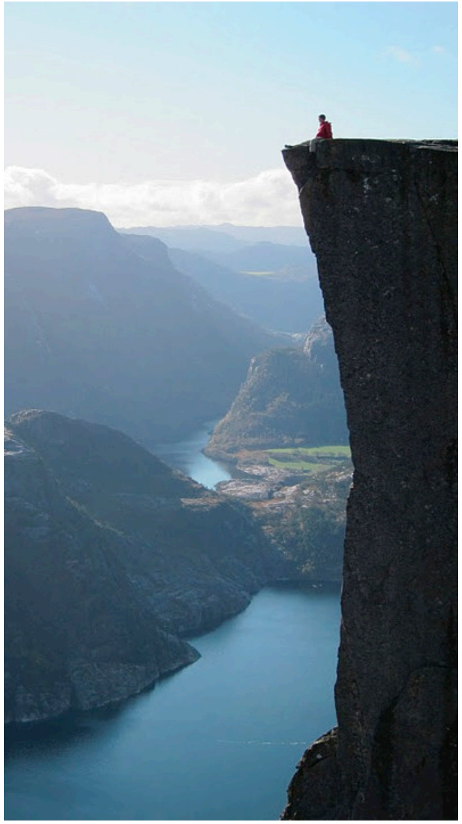

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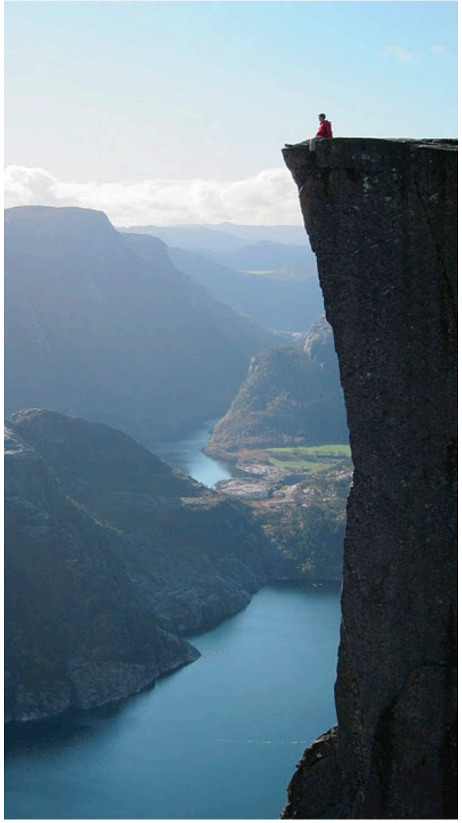

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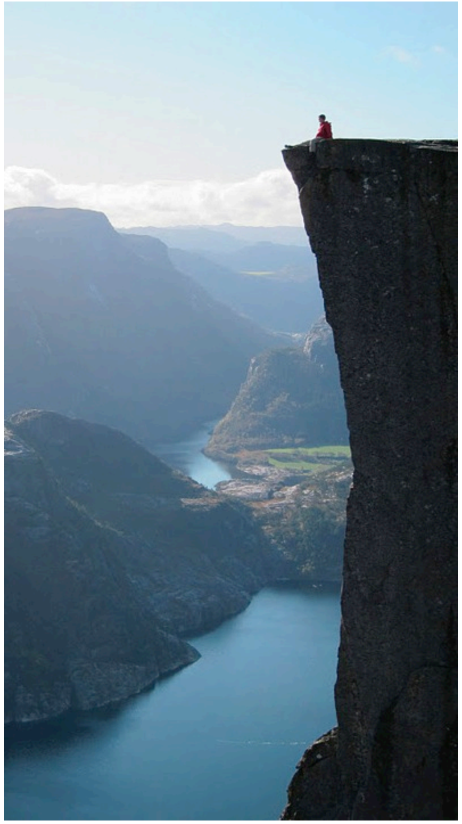

Is fact implemented correctly?

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2. Treat fact as a functional abstraction!

3. Assume that fact(n-1) is correct.

4. Verify that fact(n) is correct, assuming that fact(n-1) correct.

Photo by Kevin Lee, Preikestolen, Norway

Mutual Recursion

The Luhn Algorithm

The Luhn Algorithm

Used to verify credit card numbers

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From Wikipedia: http://en.wikipedia.org/wiki/Luhn_algorithm

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1. From the rightmost digit, which is the check digit, moving left, double the value of every second digit; if product of this doubling operation is greater than 9 (e.g., 7 * 2 = 14), then sum the digits of the products (e.g., 10: 1 + 0 = 1, 14: 1 + 4 = 5).

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(Demo)

Recursion and Iteration

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def sum_digits(n):
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    if n < 10:
        return n
    else:
        all_but_last, last = split(n)
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...arguments to a recursive call