61A Lecture 4

Monday, September 9

Announcements

- Homework 1 due Tuesday 9/10 at 5pm; Late homework is not accepted!
- Quiz on Wednesday 9/11 released at 1pm, due Thursday 9/12 at 11:59pm
  - *Open-computer*: You can use the Python interpreter, watch course videos, and read the online text (http://composingprograms.com).
  - *No external resources*: Please don't search for answers, talk to your classmates, etc.
  - *Content Covered:* Lectures through last Friday 9/6; Same topics as Homework 1.
- Project 1 due next Thursday 9/19 at 11:59pm

Iteration Example

The Fibonacci Sequence

Example: http://goo.gl/vfymhd

The Fibonacci Sequence

Example: http://goo.gl/vfymhd

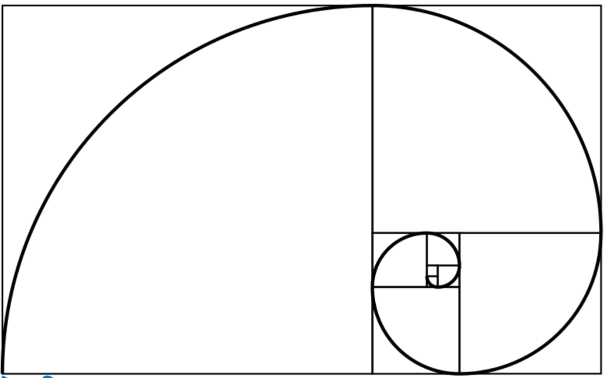
The Fibonacci Sequence

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987

Example: http://goo.gl/vfymhd

The Fibonacci Sequence

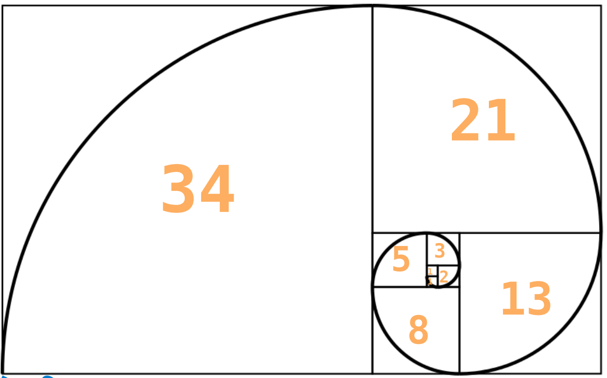
0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987



Example: http://goo.gl/vfymhd

The Fibonacci Sequence

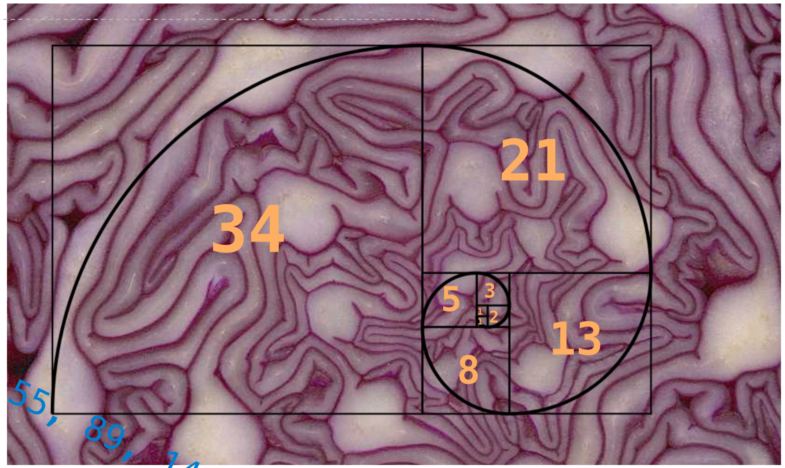
0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987



Example: http://goo.gl/vfymhd

The Fibonacci Sequence

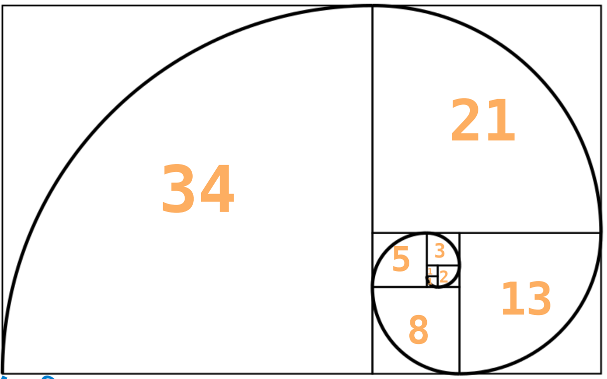
0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987



Example: http://goo.gl/vfymhd

The Fibonacci Sequence

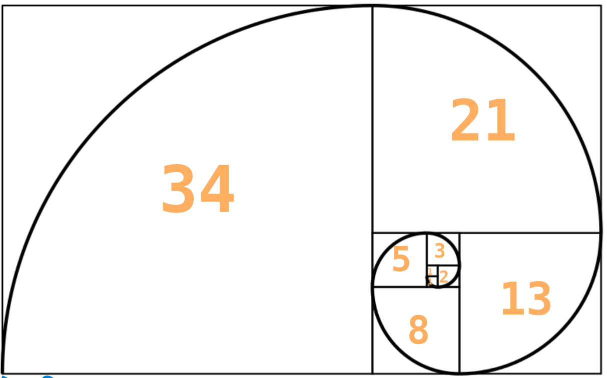
0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987



Example: http://goo.gl/vfymhd

The Fibonacci Sequence

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987

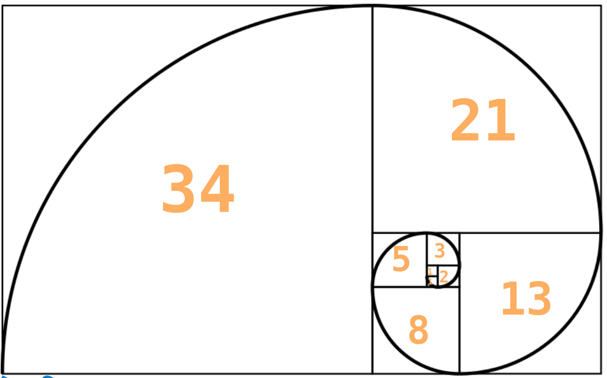


```
def fib(n):
```

Example: http://goo.gl/vfymhd

The Fibonacci Sequence

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987

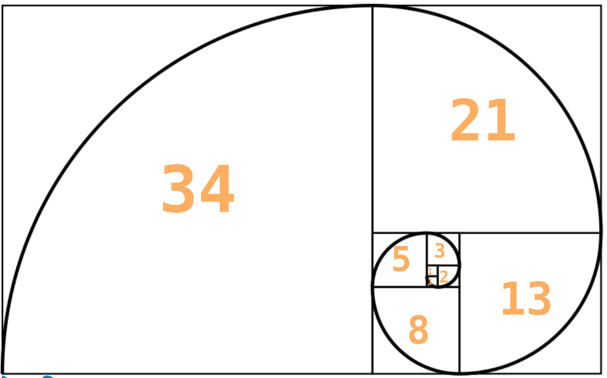


```
def fib(n):
    """Compute the nth Fibonacci number, for n >= 2."""
```

Example: http://goo.gl/vfymhd

The Fibonacci Sequence

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987

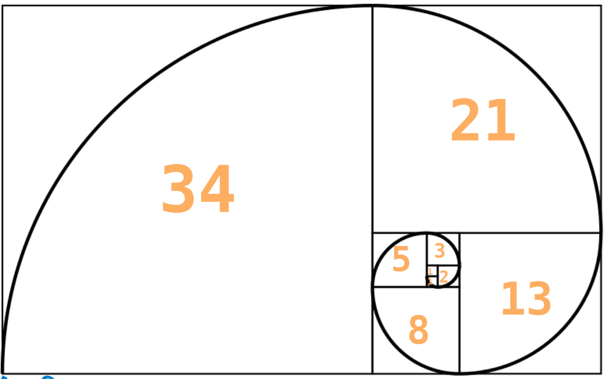


```
def fib(n):
    """Compute the nth Fibonacci number, for n >= 2."""
    predecessor, current = 0, 1   # First two Fibonacci numbers
```

Example: http://goo.gl/vfymhd

The Fibonacci Sequence

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987

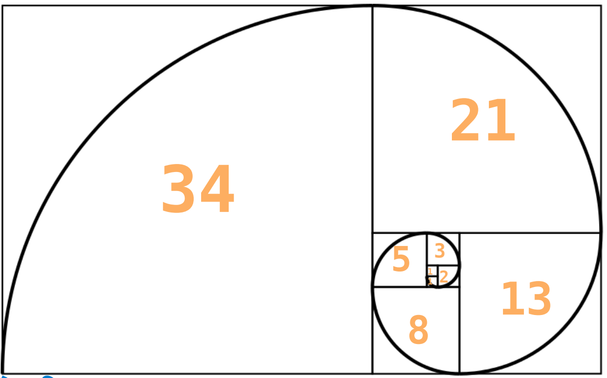


```
def fib(n):
    """Compute the nth Fibonacci number, for n >= 2."""
    predecessor, current = 0, 1   # First two Fibonacci numbers
    k = 2   # Tracks which Fibonacci number is called current
```

Example: http://goo.gl/vfymhd

The Fibonacci Sequence

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987

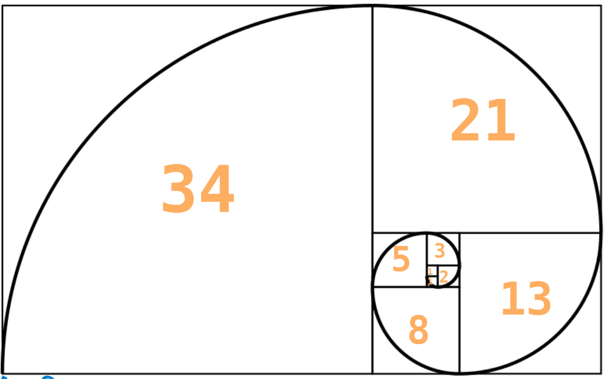


```
def fib(n):
    """Compute the nth Fibonacci number, for n >= 2."""
    predecessor, current = 0, 1   # First two Fibonacci numbers
    k = 2   # Tracks which Fibonacci number is called current
    while k < n:
```

Example: http://goo.gl/vfymhd

The Fibonacci Sequence

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987

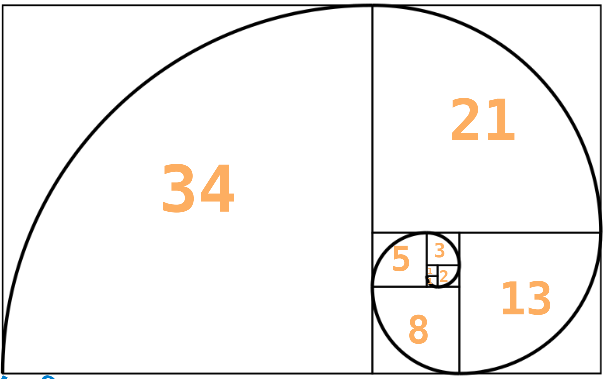


```
def fib(n):
    """Compute the nth Fibonacci number, for n >= 2."""
    predecessor, current = 0, 1   # First two Fibonacci numbers
    k = 2   # Tracks which Fibonacci number is called current
    while k < n:
        predecessor, current = current, predecessor + current
```

Example: http://goo.gl/vfymhd

The Fibonacci Sequence

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987



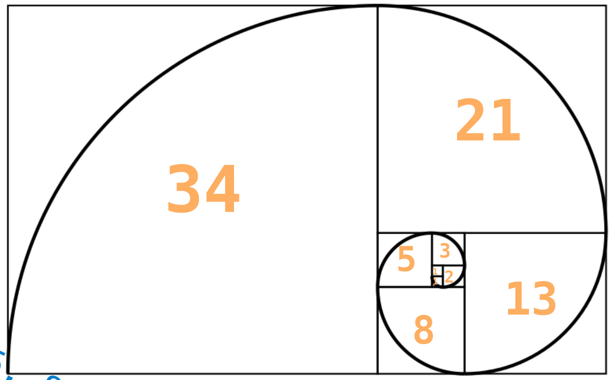
```
def fib(n):
    """Compute the nth Fibonacci number, for n >= 2."""
    predecessor, current = 0, 1   # First two Fibonacci numbers
    k = 2   # Tracks which Fibonacci number is called current
    while k < n:
        predecessor, current = current, predecessor + current
```

The next Fibonacci number is the sum of the current one and its predecessor

Example: http://goo.gl/vfymhd

The Fibonacci Sequence

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987



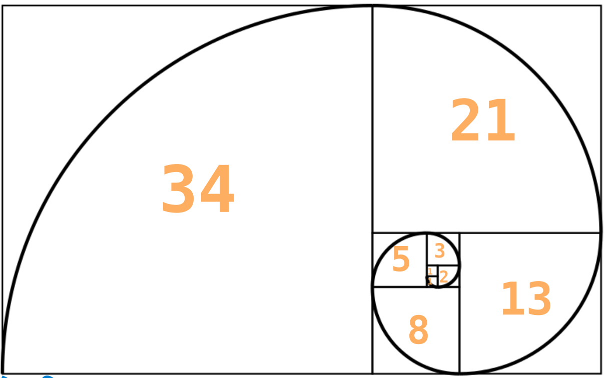
```
def fib(n):
    """Compute the nth Fibonacci number, for n >= 2."""
    predecessor, current = 0, 1   # First two Fibonacci numbers
    k = 2   # Tracks which Fibonacci number is called current
    while k < n:
        predecessor, current = current, predecessor + current
        k = k + 1
```

The next Fibonacci number is the sum of the current one and its predecessor

Example: http://goo.gl/vfymhd

The Fibonacci Sequence

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987

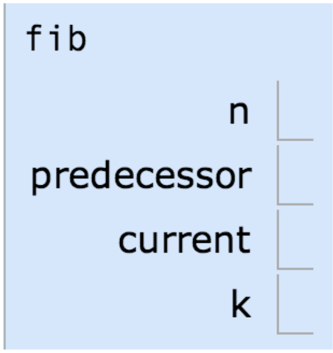


```
def fib(n):
    """Compute the nth Fibonacci number, for n >= 2."""
    predecessor, current = 0, 1   # First two Fibonacci numbers
    k = 2   # Tracks which Fibonacci number is called current
    while k < n:
        predecessor, current = current, predecessor + current
        k = k + 1
    return current
```

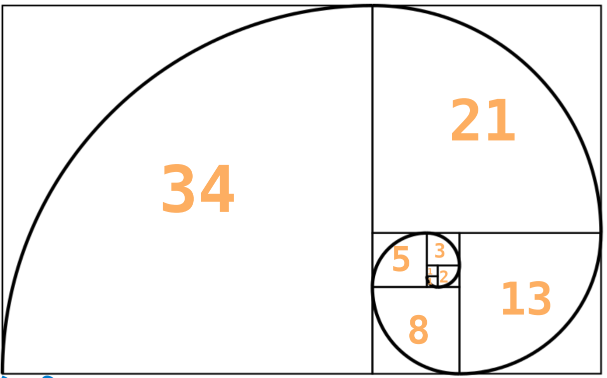
The next Fibonacci number is the sum of the current one and its predecessor

Example: http://goo.gl/vfymhd

The Fibonacci Sequence



0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987

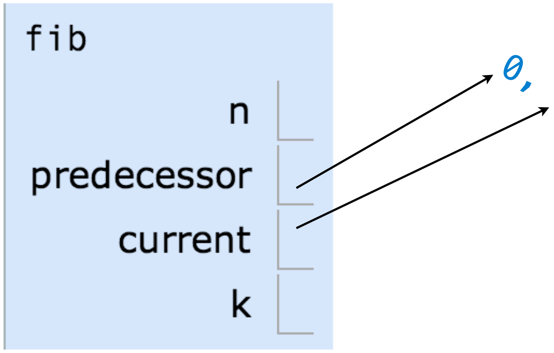


```
def fib(n):
    """Compute the nth Fibonacci number, for n >= 2."""
    predecessor, current = 0, 1   # First two Fibonacci numbers
    k = 2   # Tracks which Fibonacci number is called current
    while k < n:
        predecessor, current = current, predecessor + current
        k = k + 1
    return current
```

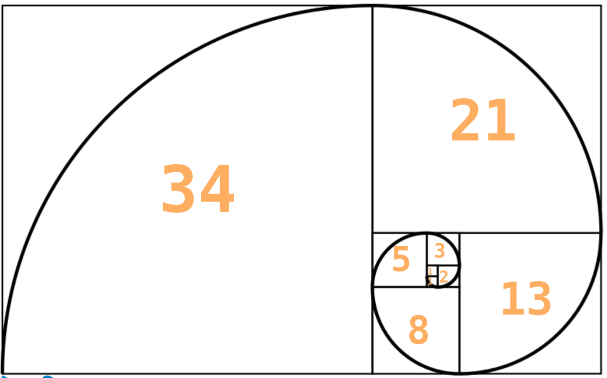
The next Fibonacci number is the sum of the current one and its predecessor

Example: http://goo.gl/vfymhd

The Fibonacci Sequence



0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987

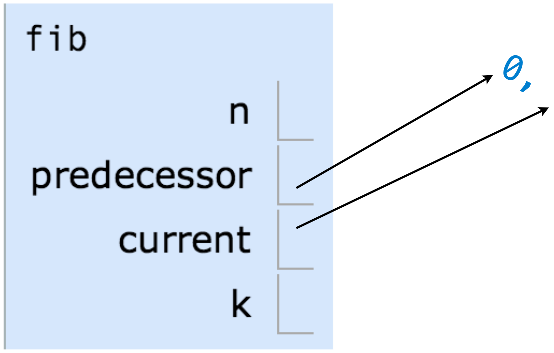


```
def fib(n):
    """Compute the nth Fibonacci number, for n >= 2."""
    predecessor, current = 0, 1   # First two Fibonacci numbers
    k = 2   # Tracks which Fibonacci number is called current
    while k < n:
        predecessor, current = current, predecessor + current
        k = k + 1
    return current
```

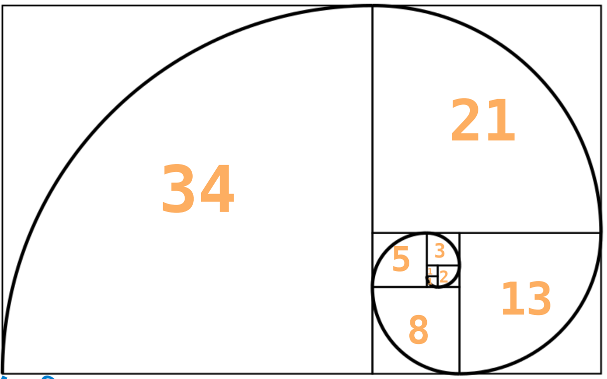
The next Fibonacci number is the sum of the current one and its predecessor

Example: http://goo.gl/vfymhd

The Fibonacci Sequence



0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987

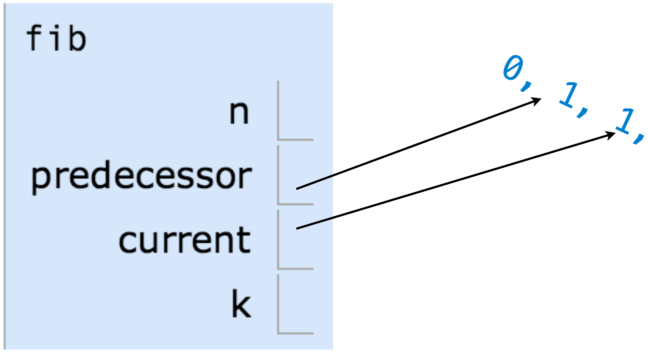


```
def fib(n):
    """Compute the nth Fibonacci number, for n >= 2."""
    predecessor, current = 0, 1   # First two Fibonacci numbers
    k = 2   # Tracks which Fibonacci number is called current
    while k < n:
      ▶ predecessor, current = current, predecessor + current
        k = k + 1
    return current
```

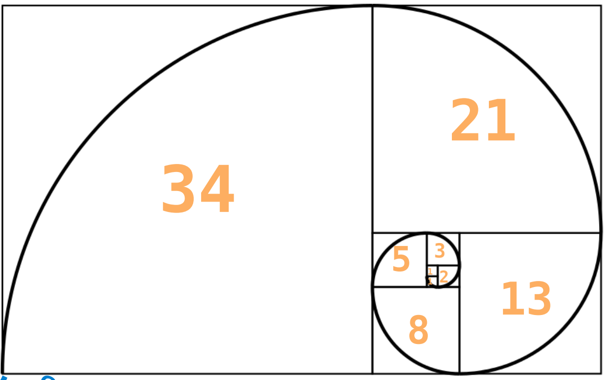
The next Fibonacci number is the sum of the current one and its predecessor

Example: http://goo.gl/vfymhd

The Fibonacci Sequence



0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987

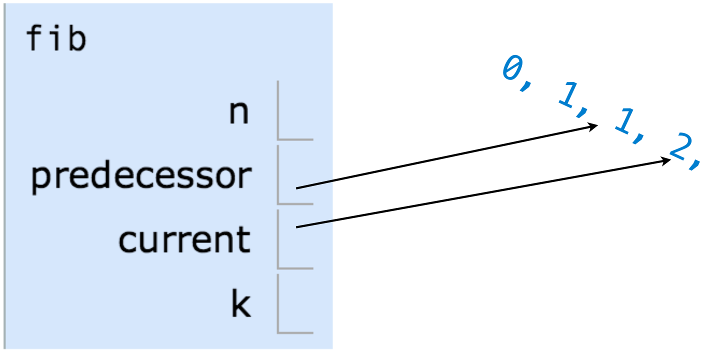


```
def fib(n):
    """Compute the nth Fibonacci number, for n >= 2."""
    predecessor, current = 0, 1   # First two Fibonacci numbers
    k = 2   # Tracks which Fibonacci number is called current
    while k < n:
        predecessor, current = current, predecessor + current
        k = k + 1
    return current
```

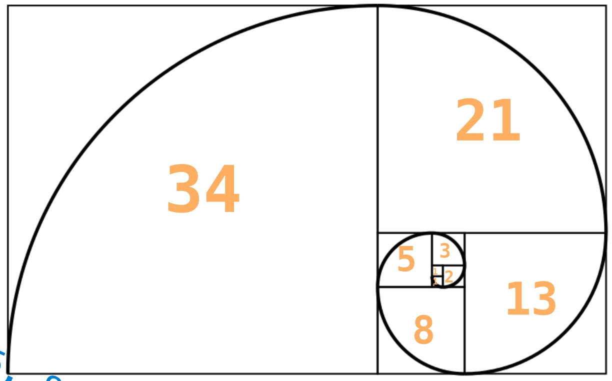
The next Fibonacci number is the sum of the current one and its predecessor

Example: http://goo.gl/vfymhd

The Fibonacci Sequence



0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987

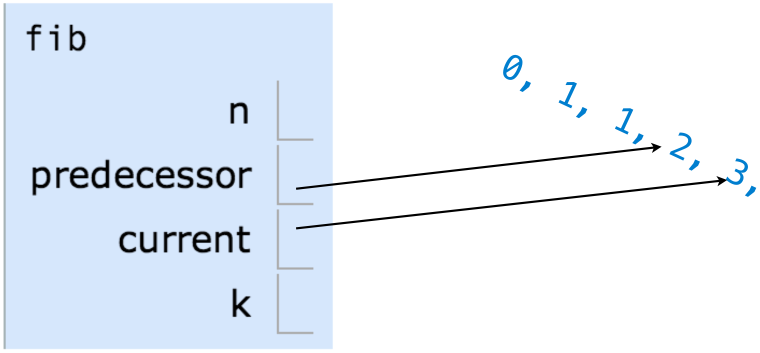


```
def fib(n):
    """Compute the nth Fibonacci number, for n >= 2."""
    predecessor, current = 0, 1   # First two Fibonacci numbers
    k = 2   # Tracks which Fibonacci number is called current
    while k < n:
      ▶ predecessor, current = current, predecessor + current
        k = k + 1
    return current
```

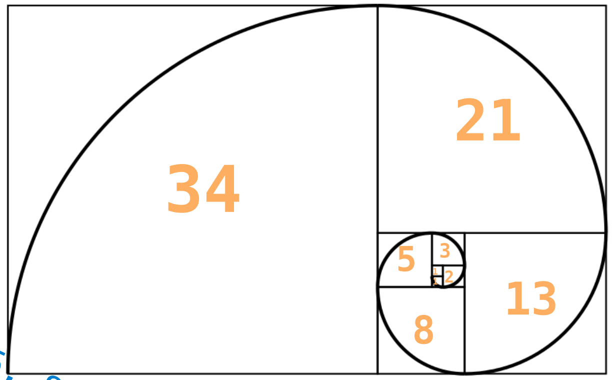
The next Fibonacci number is the sum of the current one and its predecessor

Example: http://goo.gl/vfymhd

The Fibonacci Sequence



0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987

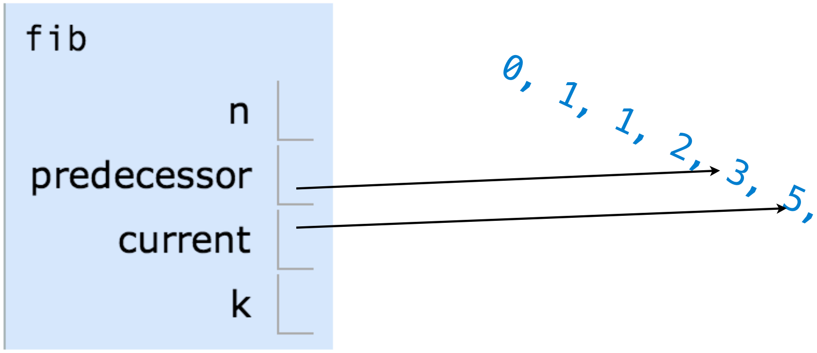


```
def fib(n):
    """Compute the nth Fibonacci number, for n >= 2."""
    predecessor, current = 0, 1   # First two Fibonacci numbers
    k = 2   # Tracks which Fibonacci number is called current
    while k < n:
        predecessor, current = current, predecessor + current
        k = k + 1
    return current
```

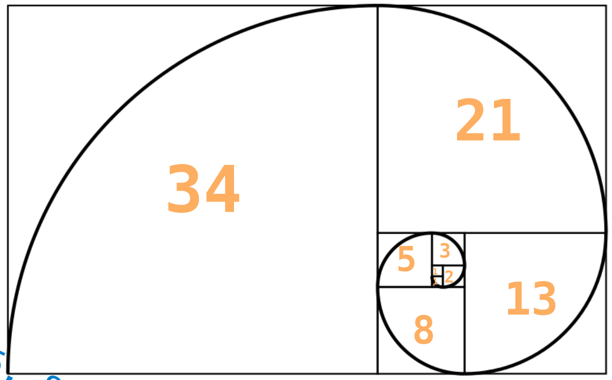
The next Fibonacci number is the sum of the current one and its predecessor

Example: http://goo.gl/vfymhd

The Fibonacci Sequence



0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987



```
def fib(n):
    """Compute the nth Fibonacci number, for n >= 2."""
    predecessor, current = 0, 1   # First two Fibonacci numbers
    k = 2   # Tracks which Fibonacci number is called current
    while k < n:
      ▶ predecessor, current = current, predecessor + current
        k = k + 1
    return current
```

The next Fibonacci number is the sum of the current one and its predecessor

Example: http://goo.gl/vfymhd

Discussion Question

Complete the following definition by placing an expression in ____________________.

Example: http://goo.gl/38ch3o

Discussion Question

Complete the following definition by placing an expression in ______________________________.

```
def choose(total, selection):
```

Example: http://goo.gl/38ch3o

Discussion Question

Complete the following definition by placing an expression in ______________________.

```
def choose(total, selection):
    """Return the number of ways to choose SELECTION items from TOTAL.
```

Example: http://goo.gl/38ch3o

Discussion Question

Complete the following definition by placing an expression in ______________________.

```
def choose(total, selection):
    """Return the number of ways to choose SELECTION items from TOTAL.

    choose(n, k) is typically defined in math as:  n! / (n-k)! / k!
```

Example: http://goo.gl/38ch3o

Discussion Question

Complete the following definition by placing an expression in ______________________.

```
def choose(total, selection):
    """Return the number of ways to choose SELECTION items from TOTAL.

    choose(n, k) is typically defined in math as:  n! / (n-k)! / k!
```

$$\frac{n \cdot (n-1) \cdot (n-2) \cdot \ \ldots \ \cdot (n-k+1)}{k \cdot (k-1) \cdot (k-2) \cdot \ \ldots \ \cdot 2 \cdot 1}$$

Example: http://goo.gl/38ch3o

Discussion Question

Complete the following definition by placing an expression in _________________________.

```
def choose(total, selection):
    """Return the number of ways to choose SELECTION items from TOTAL.

    choose(n, k) is typically defined in math as:  n! / (n-k)! / k!

    >>> choose(5, 2)
    10
```

$$\frac{n \cdot (n-1) \cdot (n-2) \cdot \ \ldots \ \cdot (n-k+1)}{k \cdot (k-1) \cdot (k-2) \cdot \ \ldots \ \cdot 2 \cdot 1}$$

Example: http://goo.gl/38ch3o

Discussion Question

Complete the following definition by placing an expression in ________________________.

```
def choose(total, selection):
    """Return the number of ways to choose SELECTION items from TOTAL.

    choose(n, k) is typically defined in math as:  n! / (n-k)! / k!

    >>> choose(5, 2)
    10
    >>> choose(20, 6)
    38760
    """
```

$$\frac{n \cdot (n-1) \cdot (n-2) \cdot \ \ldots \ \cdot (n-k+1)}{k \cdot (k-1) \cdot (k-2) \cdot \ \ldots \ \cdot 2 \cdot 1}$$

Example: http://goo.gl/38ch3o

Discussion Question

Complete the following definition by placing an expression in ________________________.

```
def choose(total, selection):
    """Return the number of ways to choose SELECTION items from TOTAL.

    choose(n, k) is typically defined in math as:  n! / (n-k)! / k!

    >>> choose(5, 2)
    10
    >>> choose(20, 6)
    38760
    """
    ways = 1
```

$$\frac{n \cdot (n-1) \cdot (n-2) \cdot \ \ldots \ \cdot (n-k+1)}{k \cdot (k-1) \cdot (k-2) \cdot \ \ldots \ \cdot 2 \cdot 1}$$

Example: http://goo.gl/38ch3o

Discussion Question

Complete the following definition by placing an expression in ________________________.

```
def choose(total, selection):
    """Return the number of ways to choose SELECTION items from TOTAL.

    choose(n, k) is typically defined in math as:  n! / (n-k)! / k!

    >>> choose(5, 2)
    10
    >>> choose(20, 6)
    38760
    """
    ways = 1
    selected = 0
```

$$\frac{n \cdot (n-1) \cdot (n-2) \cdot \ \ldots \ \cdot (n-k+1)}{k \cdot (k-1) \cdot (k-2) \cdot \ \ldots \ \cdot 2 \cdot 1}$$

Example: http://goo.gl/38ch3o

Discussion Question

Complete the following definition by placing an expression in ______________________.

```
def choose(total, selection):
    """Return the number of ways to choose SELECTION items from TOTAL.

    choose(n, k) is typically defined in math as:  n! / (n-k)! / k!

    >>> choose(5, 2)
    10
    >>> choose(20, 6)
    38760
    """
    ways = 1
    selected = 0
    while selected < selection:
```

$$\frac{n \cdot (n-1) \cdot (n-2) \cdot \ \ldots \ \cdot (n-k+1)}{k \cdot (k-1) \cdot (k-2) \cdot \ \ldots \ \cdot 2 \cdot 1}$$

Example: http://goo.gl/38ch3o

Discussion Question

Complete the following definition by placing an expression in ________________________.

```
def choose(total, selection):
    """Return the number of ways to choose SELECTION items from TOTAL.

    choose(n, k) is typically defined in math as:  n! / (n-k)! / k!

    >>> choose(5, 2)
    10
    >>> choose(20, 6)
    38760
    """
    ways = 1
    selected = 0
    while selected < selection:
        selected = selected + 1
```

$$\frac{n \cdot (n-1) \cdot (n-2) \cdot \ \ldots \ \cdot (n-k+1)}{k \cdot (k-1) \cdot (k-2) \cdot \ \ldots \ \cdot 2 \cdot 1}$$

Example: http://goo.gl/38ch3o

Discussion Question

Complete the following definition by placing an expression in ______________________.

```
def choose(total, selection):
    """Return the number of ways to choose SELECTION items from TOTAL.

    choose(n, k) is typically defined in math as:  n! / (n-k)! / k!

    >>> choose(5, 2)
    10
    >>> choose(20, 6)
    38760
    """
    ways = 1
    selected = 0
    while selected < selection:
        selected = selected + 1
        ways, total = ways * ______________________, total - 1
    return ways
```

$$\frac{n \cdot (n-1) \cdot (n-2) \cdot \ \ldots \ \cdot (n-k+1)}{k \cdot (k-1) \cdot (k-2) \cdot \ \ldots \ \cdot 2 \cdot 1}$$

Example: http://goo.gl/38ch3o

Discussion Question

Complete the following definition by placing an expression in ______________________.

```
def choose(total, selection):
    """Return the number of ways to choose SELECTION items from TOTAL.

    choose(n, k) is typically defined in math as:  n! / (n-k)! / k!

    >>> choose(5, 2)
    10
    >>> choose(20, 6)
    38760
    """
    ways = 1
    selected = 0
    while selected < selection:
        selected = selected + 1
        ways, total = ways *  total // selected    , total - 1
    return ways
```

$$\frac{n\cdot(n-1)\cdot(n-2)\cdot\ \ldots\ \cdot(n-k+1)}{k\cdot(k-1)\cdot(k-2)\cdot\ \ldots\ \cdot 2\cdot 1}$$

Example: http://goo.gl/38ch3o

Discussion Question

Complete the following definition by placing an expression in ________________.

```
def choose(total, selection):
    """Return the number of ways to choose SELECTION items from TOTAL.

    choose(n, k) is typically defined in math as:  n! / (n-k)! / k!

    >>> choose(5, 2)
    10
    >>> choose(20, 6)
    38760
    """
    ways = 1
    selected = 0
    while selected < selection:
        selected = selected + 1
        ways, total = ways * total // selected, total - 1
    return ways
```

$$\frac{n \cdot (n-1) \cdot (n-2) \cdot \ \ldots \ \cdot (n-k+1)}{k \cdot (k-1) \cdot (k-2) \cdot \ \ldots \ \cdot 2 \cdot 1}$$

Example: http://goo.gl/38ch3o

Discussion Question

Complete the following definition by placing an expression in ________________________.

```
def choose(total, selection):
    """Return the number of ways to choose SELECTION items from TOTAL.

    choose(n, k) is typically defined in math as:  n! / (n-k)! / k!

    >>> choose(5, 2)
    10
    >>> choose(20, 6)
    38760
    """
    ways = 1
    selected = 0
    while selected < selection:
        selected = selected + 1
        ways, total = ways *   total // selected   , total - 1
    return ways
```

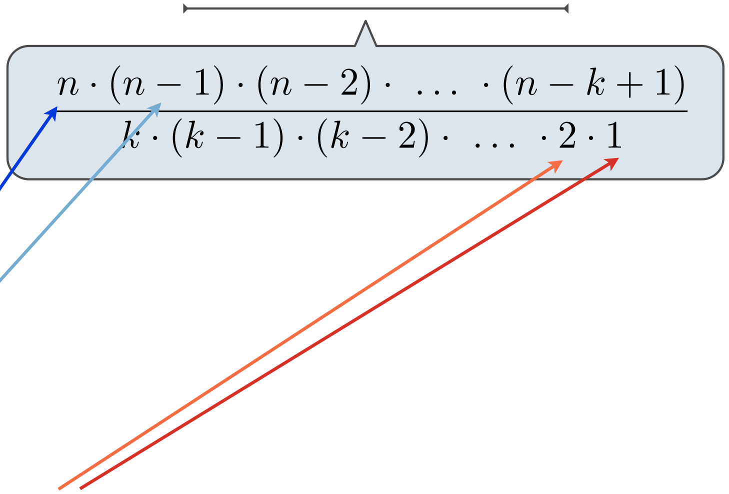
$$\frac{n \cdot (n-1) \cdot (n-2) \cdot \ \ldots \ \cdot (n-k+1)}{k \cdot (k-1) \cdot (k-2) \cdot \ \ldots \ \cdot 2 \cdot 1}$$

Example: http://goo.gl/38ch3o

Discussion Question

Complete the following definition by placing an expression in ______________________.

```
def choose(total, selection):
    """Return the number of ways to choose SELECTION items from TOTAL.

    choose(n, k) is typically defined in math as:  n! / (n-k)! / k!

    >>> choose(5, 2)
    10
    >>> choose(20, 6)
    38760
    """
    ways = 1
    selected = 0
    while selected < selection:
        selected = selected + 1
        ways, total = ways * __total // selected__, total - 1
    return ways
```



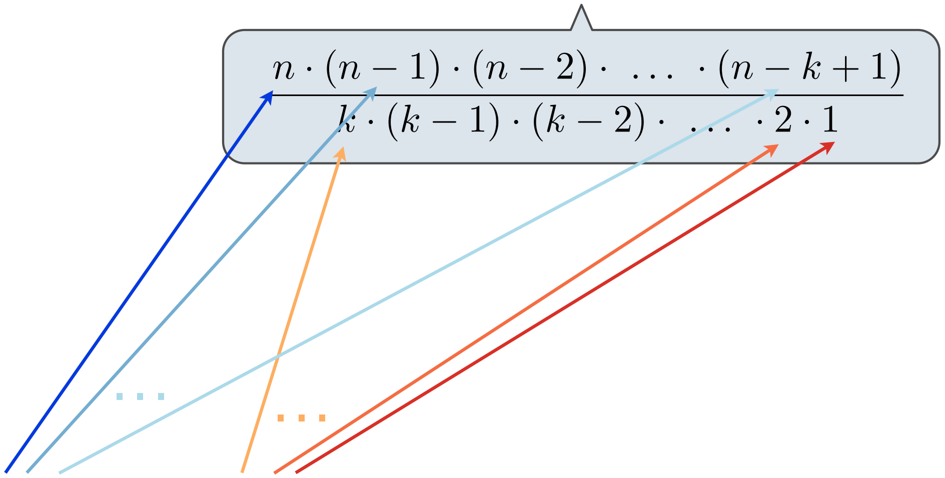
$$\frac{n \cdot (n-1) \cdot (n-2) \cdot \ \ldots \ \cdot (n-k+1)}{k \cdot (k-1) \cdot (k-2) \cdot \ \ldots \ \cdot 2 \cdot 1}$$

Example: http://goo.gl/38ch3o

Discussion Question

Complete the following definition by placing an expression in ______________________.

```
def choose(total, selection):
    """Return the number of ways to choose SELECTION items from TOTAL.

    choose(n, k) is typically defined in math as:  n! / (n-k)! / k!

    >>> choose(5, 2)
    10
    >>> choose(20, 6)
    38760
    """
    ways = 1
    selected = 0
    while selected < selection:
        selected = selected + 1
        ways, total = ways *   total // selected  , total - 1
    return ways
```



$$\frac{n\cdot(n-1)\cdot(n-2)\cdot\ \ldots\ \cdot(n-k+1)}{k\cdot(k-1)\cdot(k-2)\cdot\ \ldots\ \cdot 2\cdot 1}$$

Example: http://goo.gl/38ch3o

Default Arguments

(Demo)

Designing Functions

Characteristics of Functions

Characteristics of Functions

A function's ***domain*** is the set of all inputs it might possibly take as arguments.

Characteristics of Functions

A function's ***domain*** is the set of all inputs it might possibly take as arguments.

A function's ***range*** is the set of output values it might possibly return.

Characteristics of Functions

A function's ***domain*** is the set of all inputs it might possibly take as arguments.

A function's ***range*** is the set of output values it might possibly return.

A pure function's ***behavior*** is the relationship it creates between input and output.

Characteristics of Functions

```
def square(x):
    """Return X * X."""
```

A function's ***domain*** is the set of all inputs it might possibly take as arguments.

A function's ***range*** is the set of output values it might possibly return.

A pure function's ***behavior*** is the relationship it creates between input and output.

Characteristics of Functions

```
def square(x):
    """Return X * X."""
```

```
def choose(n, d):
    """Return the number of ways to choose D of N items."""
```

A function's ***domain*** is the set of all inputs it might possibly take as arguments.

A function's ***range*** is the set of output values it might possibly return.

A pure function's ***behavior*** is the relationship it creates between input and output.

Characteristics of Functions

```
def square(x):
    """Return X * X."""
```

```
def choose(n, d):
    """Return the number of ways to choose D of N items."""
```

A function's ***domain*** is the set of all inputs it might possibly take as arguments.

x is a number

A function's ***range*** is the set of output values it might possibly return.

A pure function's ***behavior*** is the relationship it creates between input and output.

Characteristics of Functions

```
def square(x):
    """Return X * X."""
```

```
def choose(n, d):
    """Return the number of ways to choose D of N items."""
```

A function's ***domain*** is the set of all inputs it might possibly take as arguments.

x is a number

n and d are positive integers with n greater than or equal to d.

A function's ***range*** is the set of output values it might possibly return.

A pure function's ***behavior*** is the relationship it creates between input and output.

Characteristics of Functions

```
def square(x):
    """Return X * X."""
```

```
def choose(n, d):
    """Return the number of ways to choose D of N items."""
```

A function's ***domain*** is the set of all inputs it might possibly take as arguments.

x is a number

n and d are positive integers with n greater than or equal to d.

A function's ***range*** is the set of output values it might possibly return.

return value is a positive number

A pure function's ***behavior*** is the relationship it creates between input and output.

Characteristics of Functions

```
def square(x):
    """Return X * X."""
```

```
def choose(n, d):
    """Return the number of ways to choose D of N items."""
```

A function's ***domain*** is the set of all inputs it might possibly take as arguments.

x is a number

n and d are positive integers with n greater than or equal to d.

A function's ***range*** is the set of output values it might possibly return.

return value is a positive number

return value is a positive integer

A pure function's ***behavior*** is the relationship it creates between input and output.

Characteristics of Functions

```
def square(x):
    """Return X * X."""
```

```
def choose(n, d):
    """Return the number of ways to choose D of N items."""
```

A function's ***domain*** is the set of all inputs it might possibly take as arguments.

x is a number

n and d are positive integers with n greater than or equal to d.

A function's ***range*** is the set of output values it might possibly return.

return value is a positive number

return value is a positive integer

A pure function's ***behavior*** is the relationship it creates between input and output.

return value is the square of the input

Characteristics of Functions

```
def square(x):
    """Return X * X."""
```

```
def choose(n, d):
    """Return the number of ways to choose D of N items."""
```

A function's ***domain*** is the set of all inputs it might possibly take as arguments.

x is a number

n and d are positive integers with n greater than or equal to d.

A function's ***range*** is the set of output values it might possibly return.

return value is a positive number

return value is a positive integer

A pure function's ***behavior*** is the relationship it creates between input and output.

return value is the square of the input

return value is the number of ways to choose d of n items.

A Guide to Designing Function

A Guide to Designing Function

```
Give each function exactly one job.
```

A Guide to Designing Function

Give each function exactly one job.

Don’t repeat yourself (DRY). Implement a process just once, but execute it many times.

A Guide to Designing Function

Give each function exactly one job.

Don’t repeat yourself (DRY). Implement a process just once, but execute it many times.

Define functions generally.

A Guide to Designing Function

Give each function exactly one job.

Don’t repeat yourself (DRY). Implement a process just once, but execute it many times.

Define functions generally.

A Guide to Designing Function

Give each function exactly one job.

not

Don’t repeat yourself (DRY). Implement a process just once, but execute it many times.

Define functions generally.

A Guide to Designing Function

Give each function exactly one job.

not

Don’t repeat yourself (DRY). Implement a process just once, but execute it many times.

Define functions generally.

A Guide to Designing Function

Give each function exactly one job.

not

Don’t repeat yourself (DRY). Implement a process just once, but execute it many times.

Define functions generally.

Generalization

Generalizing Patterns with Arguments

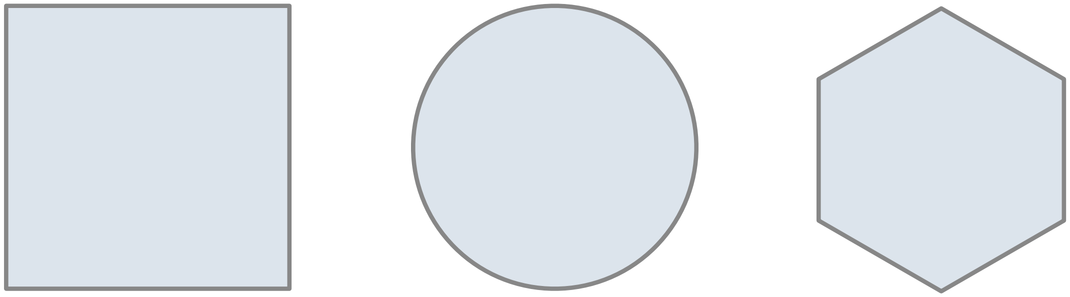
Generalizing Patterns with Arguments

```
Regular geometric shapes relate length and area.
```

Generalizing Patterns with Arguments

Regular geometric shapes relate length and area.

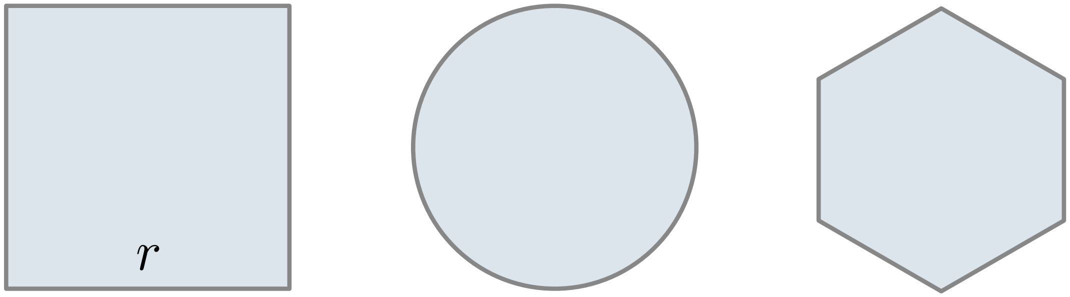
Shape:



Generalizing Patterns with Arguments

Regular geometric shapes relate length and area.

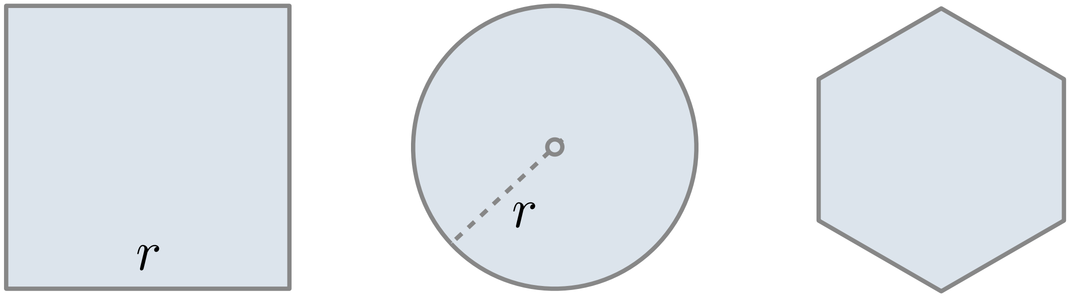
Shape:



Generalizing Patterns with Arguments

Regular geometric shapes relate length and area.

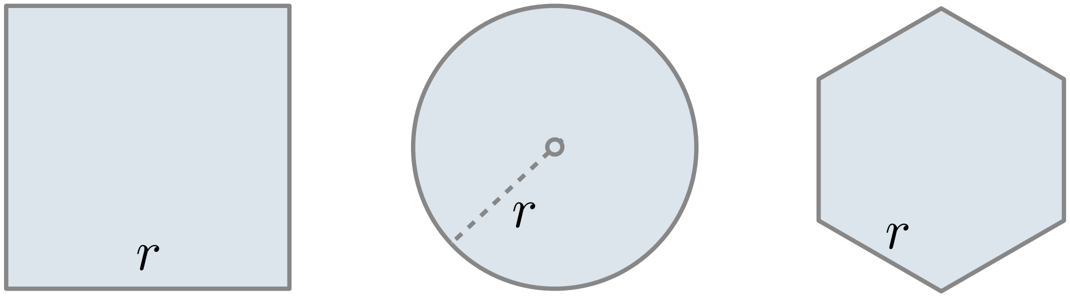
Shape:



Generalizing Patterns with Arguments

Regular geometric shapes relate length and area.

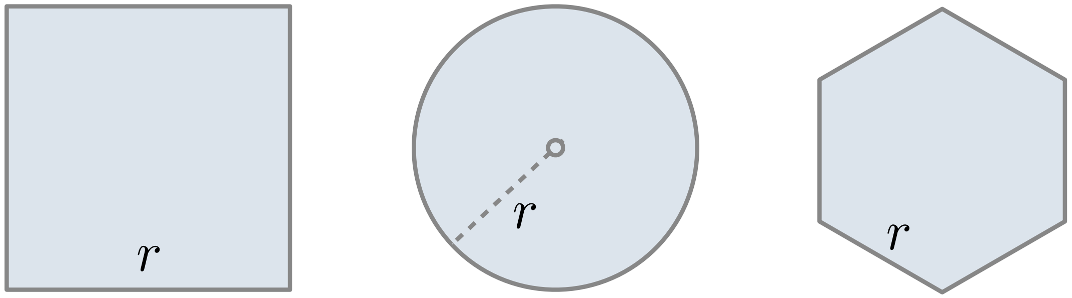
Shape:



Generalizing Patterns with Arguments

Regular geometric shapes relate length and area.

Shape:

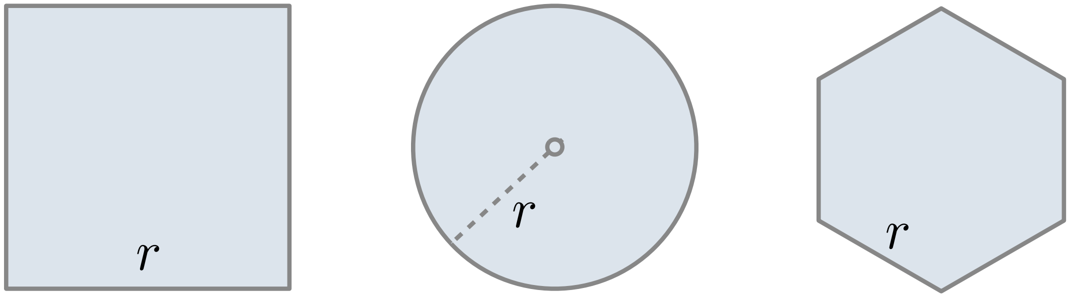


Area:

Generalizing Patterns with Arguments

Regular geometric shapes relate length and area.

Shape:

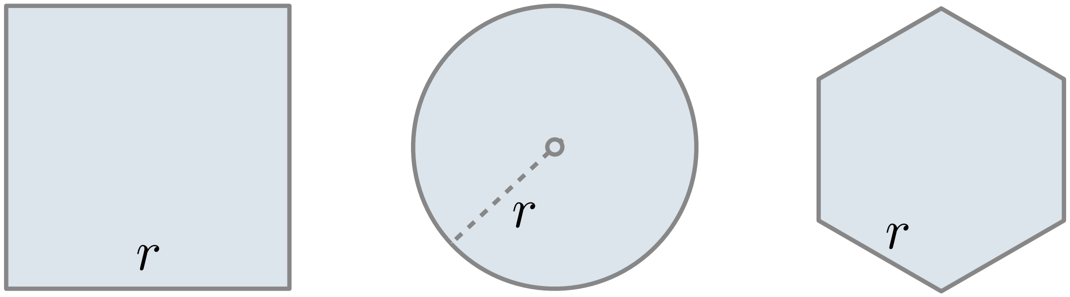


Area: r^2

Generalizing Patterns with Arguments

Regular geometric shapes relate length and area.

Shape:

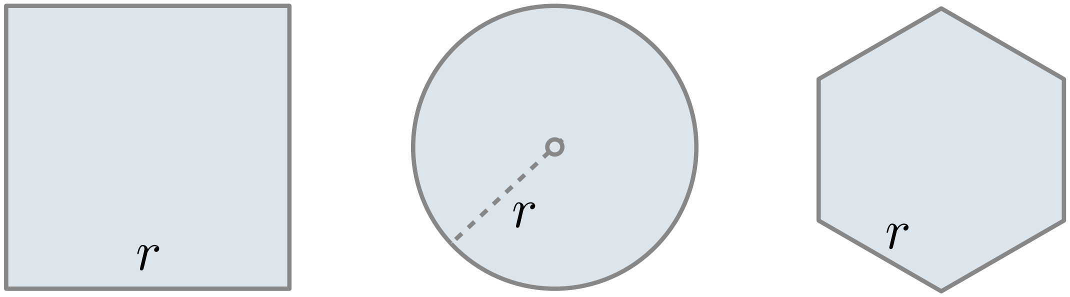


Area: r^2 $\pi \cdot r^2$

Generalizing Patterns with Arguments

Regular geometric shapes relate length and area.

Shape:

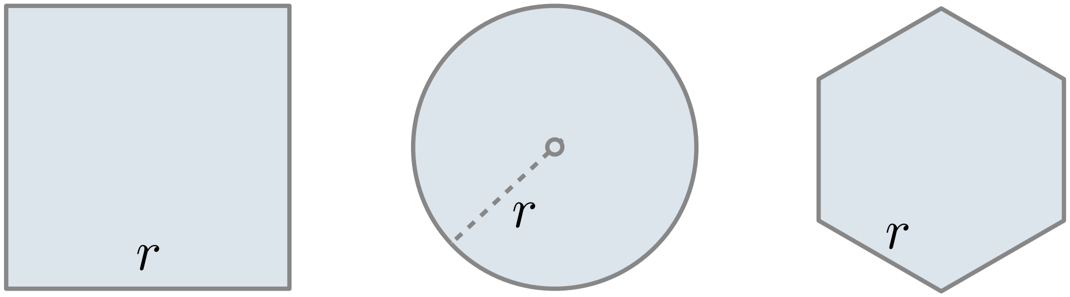


Area: r^2 $\pi \cdot r^2$ $\frac{3\sqrt{3}}{2} \cdot r^2$

Generalizing Patterns with Arguments

Regular geometric shapes relate length and area.

Shape:



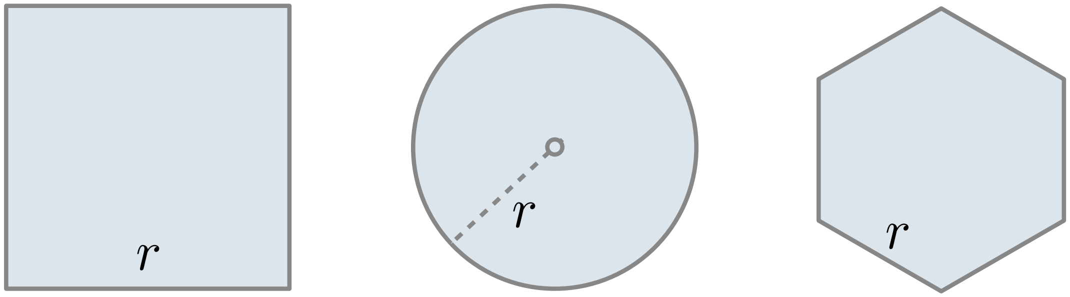
Area:

$1 \cdot \boldsymbol{r^2}$ $\pi \cdot r^2$ $\frac{3\sqrt{3}}{2} \cdot r^2$

Generalizing Patterns with Arguments

Regular geometric shapes relate length and area.

Shape:

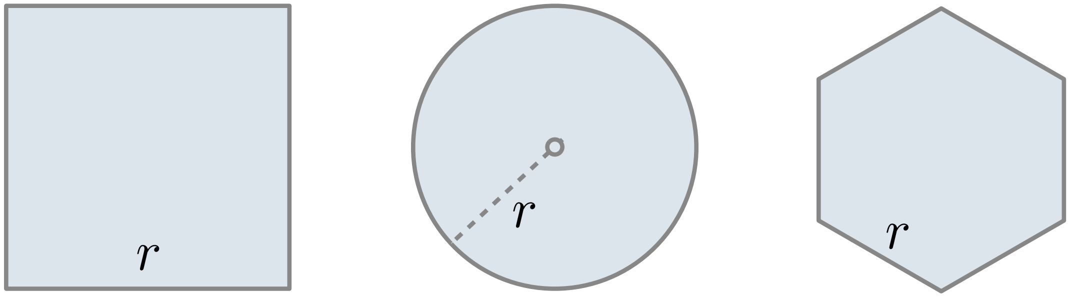


Area: $\boxed{1} \cdot \boldsymbol{r^2}$ $\quad \pi \cdot r^2$ $\quad \frac{3\sqrt{3}}{2} \cdot r^2$

Generalizing Patterns with Arguments

Regular geometric shapes relate length and area.

Shape:



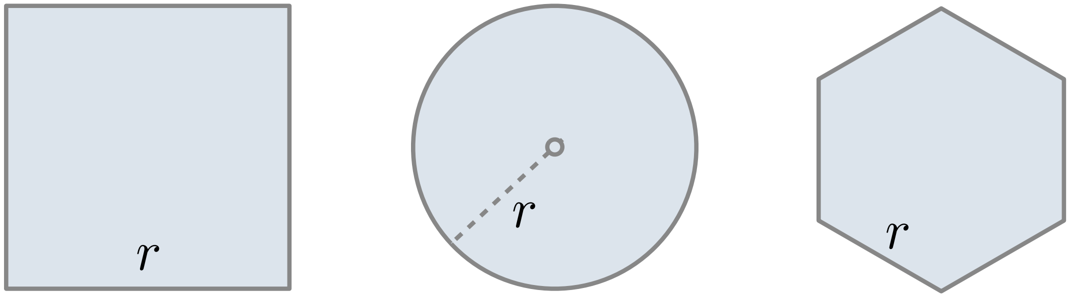
Area:

$\boxed{1} \cdot \boldsymbol{r^2}$ $\qquad$ $\boxed{\pi} \cdot r^2$ $\qquad$ $\dfrac{3\sqrt{3}}{2} \cdot r^2$

Generalizing Patterns with Arguments

Regular geometric shapes relate length and area.

Shape:



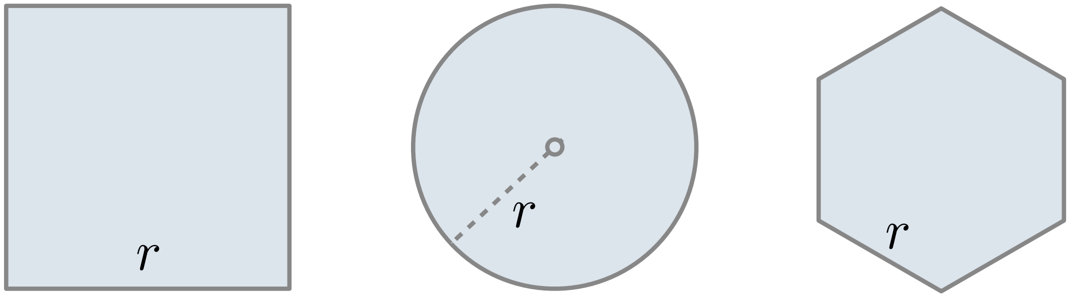
Area:

$\boxed{1} \cdot \boldsymbol{r^2}$ $\qquad$ $\boxed{\pi} \cdot r^2$ $\qquad$ $\boxed{\frac{3\sqrt{3}}{2}} \cdot r^2$

Generalizing Patterns with Arguments

Regular geometric shapes relate length and area.

Shape:



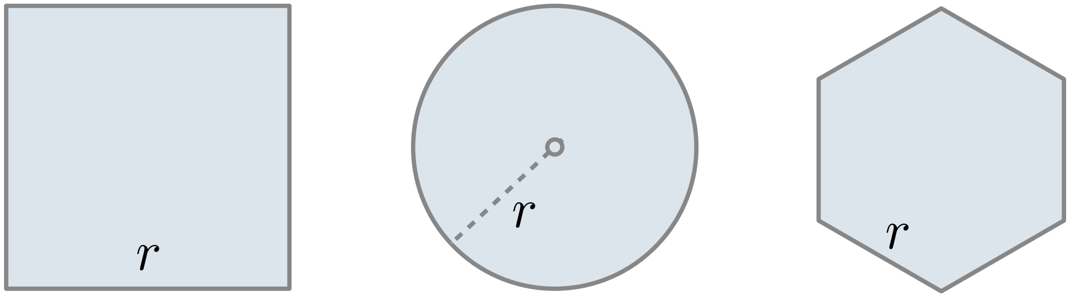
Area: $\boxed{1} \cdot \boldsymbol{r^2}$ $\boxed{\pi} \cdot r^2$ $\boxed{\frac{3\sqrt{3}}{2}} \cdot r^2$

Finding common structure allows for shared implementation

Generalizing Patterns with Arguments

Regular geometric shapes relate length and area.

Shape:



Area: $\boxed{1} \cdot \boldsymbol{r^2}$ $\boxed{\pi} \cdot r^2$ $\boxed{\frac{3\sqrt{3}}{2}} \cdot r^2$

Finding common structure allows for shared implementation

(Demo)

Higher-Order Functions

Generalizing Over Computational Processes

Generalizing Over Computational Processes

The common structure among functions may be a computational process, rather than a number.

Generalizing Over Computational Processes

The common structure among functions may be a computational process, rather than a number.

$$\sum_{k=1}^{5} k = 1 + 2 + 3 + 4 + 5 \qquad = 15$$

$$\sum_{k=1}^{5} k^3 = 1^3 + 2^3 + 3^3 + 4^3 + 5^3 \qquad = 225$$

$$\sum_{k=1}^{5} \frac{8}{(4k-3)\cdot(4k-1)} = \frac{8}{3} + \frac{8}{35} + \frac{8}{99} + \frac{8}{195} + \frac{8}{323} \qquad = 3.04$$

Generalizing Over Computational Processes

The common structure among functions may be a computational process, rather than a number.

$$\sum_{k=1}^{5} \boxed{k} = 1 + 2 + 3 + 4 + 5 \qquad = 15$$

$$\sum_{k=1}^{5} k^3 = 1^3 + 2^3 + 3^3 + 4^3 + 5^3 \qquad = 225$$

$$\sum_{k=1}^{5} \frac{8}{(4k-3)\cdot(4k-1)} = \frac{8}{3} + \frac{8}{35} + \frac{8}{99} + \frac{8}{195} + \frac{8}{323} \qquad = 3.04$$

Generalizing Over Computational Processes

`The common structure among functions may be a computational process, rather than a number.`

$$\sum_{k=1}^{5} \boxed{k} = 1 + 2 + 3 + 4 + 5 \qquad = 15$$

$$\sum_{k=1}^{5} \boxed{k^3} = 1^3 + 2^3 + 3^3 + 4^3 + 5^3 \qquad = 225$$

$$\sum_{k=1}^{5} \frac{8}{(4k-3)\cdot(4k-1)} = \frac{8}{3} + \frac{8}{35} + \frac{8}{99} + \frac{8}{195} + \frac{8}{323} \qquad = 3.04$$

Generalizing Over Computational Processes

`The common structure among functions may be a computational process, rather than a number.`

$$\sum_{k=1}^{5} \boxed{k} = 1 + 2 + 3 + 4 + 5 \qquad = 15$$

$$\sum_{k=1}^{5} \boxed{k^3} = 1^3 + 2^3 + 3^3 + 4^3 + 5^3 \qquad = 225$$

$$\sum_{k=1}^{5} \boxed{\frac{8}{(4k-3)\cdot(4k-1)}} = \frac{8}{3} + \frac{8}{35} + \frac{8}{99} + \frac{8}{195} + \frac{8}{323} \qquad = 3.04$$

Generalizing Over Computational Processes

The common structure among functions may be a computational process, rather than a number.

$$\sum_{k=1}^{5} \boxed{k} = 1 + 2 + 3 + 4 + 5 \qquad = 15$$

$$\sum_{k=1}^{5} \boxed{k^3} = 1^3 + 2^3 + 3^3 + 4^3 + 5^3 \qquad = 225$$

$$\sum_{k=1}^{5} \boxed{\frac{8}{(4k-3)\cdot(4k-1)}} = \frac{8}{3} + \frac{8}{35} + \frac{8}{99} + \frac{8}{195} + \frac{8}{323} \qquad = 3.04$$

(Demo)

Summation Example

```python
def cube(k):
    return pow(k, 3)

def summation(n, term):
    """Sum the first n terms of a sequence.

    >>> summation(5, cube)
    225
    """
    total, k = 0, 1
    while k <= n:
        total, k = total + term(k), k + 1
    return total
```

Summation Example

```
def cube(k):
    return pow(k, 3)
```

Function of a single argument
(not called term)

```
def summation(n, term):
    """Sum the first n terms of a sequence.

    >>> summation(5, cube)
    225
    """
    total, k = 0, 1
    while k <= n:
        total, k = total + term(k), k + 1
    return total
```

Summation Example

```
def cube(k):
    return pow(k, 3)

def summation(n, term):
    """Sum the first n terms of a sequence.

    >>> summation(5, cube)
    225
    """
    total, k = 0, 1
    while k <= n:
        total, k = total + term(k), k + 1
    return total
```

Function of a single argument
(not called term)

A formal parameter that will
be bound to a function

Summation Example

```
def cube(k):
    return pow(k, 3)

def summation(n, term):
    """Sum the first n terms of a sequence.

    >>> summation(5, cube)
    225
    """
    total, k = 0, 1
    while k <= n:
        total, k = total + term(k), k + 1
    return total
```

Function of a single argument (not called term)

A formal parameter that will be bound to a function

The function bound to term gets called here

Summation Example

```
def cube(k):
    return pow(k, 3)

def summation(n, term):
    """Sum the first n terms of a sequence.

    >>> summation(5, cube)
    225
    """
    total, k = 0, 1
    while k <= n:
        total, k = total + term(k), k + 1
    return total
```

Function of a single argument (not called term)

A formal parameter that will be bound to a function

The cube function is passed as an argument value

The function bound to term gets called here

Summation Example

```
def cube(k):
    return pow(k, 3)

def summation(n, term):
    """Sum the first n terms of a sequence.

    >>> summation(5, cube)
    225
    """
    total, k = 0, 1
    while k <= n:
        total, k = total + term(k), k + 1
    return total
```

Function of a single argument (not called term)

A formal parameter that will be bound to a function

The cube function is passed as an argument value

$0 + 1^3 + 2^3 + 3^3 + 4^3 + 5^3$

The function bound to term gets called here

Functions as Return Values

(Demo)

Locally Defined Functions

Locally Defined Functions

Functions defined within other function bodies are bound to names in a *local* frame

Locally Defined Functions

Functions defined within other function bodies are bound to names in a *local* frame

```
def make_adder(n):
    """Return a function that takes one argument k and returns k + n.

    >>> add_three = make_adder(3)
    >>> add_three(4)
    7
    """
    def adder(k):
        return k + n
    return adder
```

Locally Defined Functions

Functions defined within other function bodies are bound to names in a *local* frame

A function that returns a function

```
def make_adder(n):
    """Return a function that takes one argument k and returns k + n.

    >>> add_three = make_adder(3)
    >>> add_three(4)
    7
    """
    def adder(k):
        return k + n
    return adder
```

Locally Defined Functions

Functions defined within other function bodies are bound to names in a *local* frame

A function that returns a function

```
def make_adder(n):
    """Return a function that takes one argument k and returns k + n.

    >>> add_three = make_adder(3)
    >>> add_three(4)
    7
    """
    def adder(k):
        return k + n
    return adder
```

The name add_three is bound to a function

Locally Defined Functions

Functions defined within other function bodies are bound to names in a *local* frame

A function that returns a function

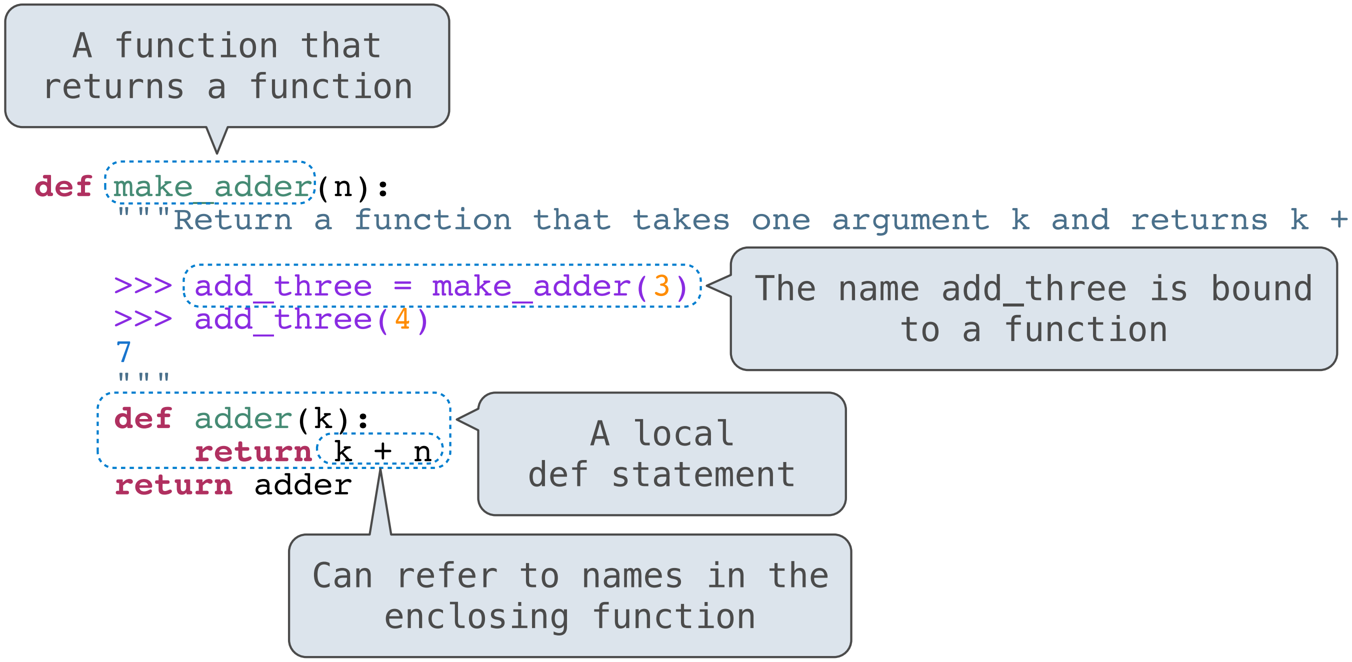
```
def make_adder(n):
    """Return a function that takes one argument k and returns k + n.

    >>> add_three = make_adder(3)
    >>> add_three(4)
    7
    """
    def adder(k):
        return k + n
    return adder
```

The name add_three is bound to a function

A local def statement

Locally Defined Functions

Functions defined within other function bodies are bound to names in a *local* frame



A function that returns a function

```
def make_adder(n):
    """Return a function that takes one argument k and returns k + n.

    >>> add_three = make_adder(3)
    >>> add_three(4)
    7
    """
    def adder(k):
        return k + n
    return adder
```

The name add_three is bound to a function

A local def statement

Can refer to names in the enclosing function

Call Expressions as Operator Expressions

```
def make_adder(n):
    def adder(k):
        return k + n
    return adder
```

Call Expressions as Operator Expressions

```
make_adder(1)    (        2        )
```

```
def make_adder(n):
    def adder(k):
        return k + n
    return adder
```

Call Expressions as Operator Expressions

Operator

```
make_adder(1)    (        2        )
```

```
def make_adder(n):
    def adder(k):
        return k + n
    return adder
```

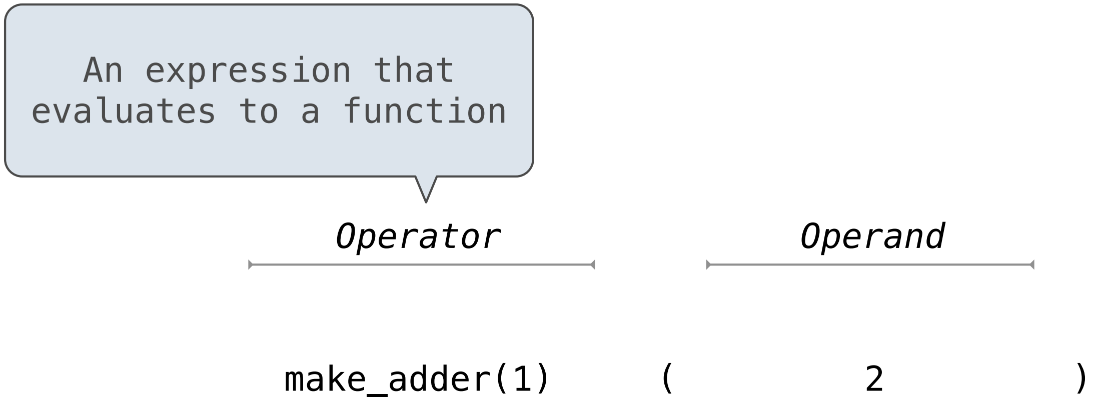
Call Expressions as Operator Expressions

Operator *Operand*

```
make_adder(1)    (      2      )
```

```
def make_adder(n):
    def adder(k):
        return k + n
    return adder
```

Call Expressions as Operator Expressions



An expression that evaluates to a function

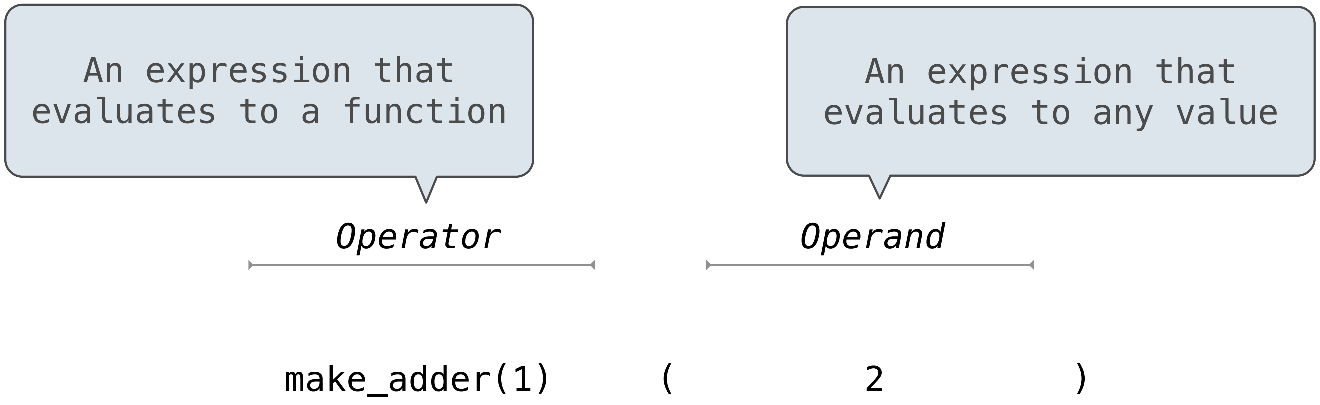
Operator

Operand

make_adder(1)   (2   )

```
def make_adder(n):
    def adder(k):
        return k + n
    return adder
```

Call Expressions as Operator Expressions



An expression that evaluates to a function

An expression that evaluates to any value

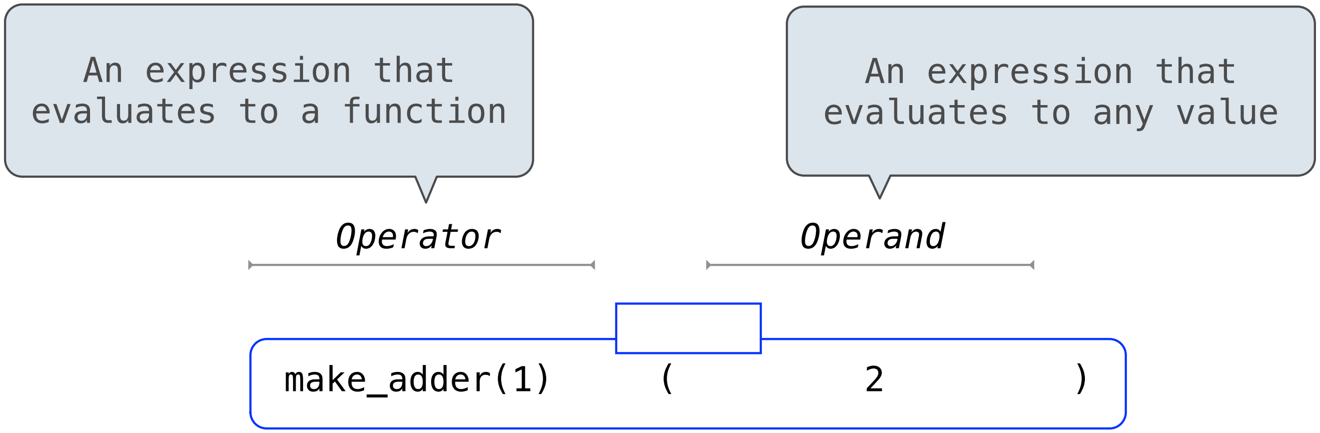
Operator

Operand

make_adder(1) (2)

```
def make_adder(n):
    def adder(k):
        return k + n
    return adder
```

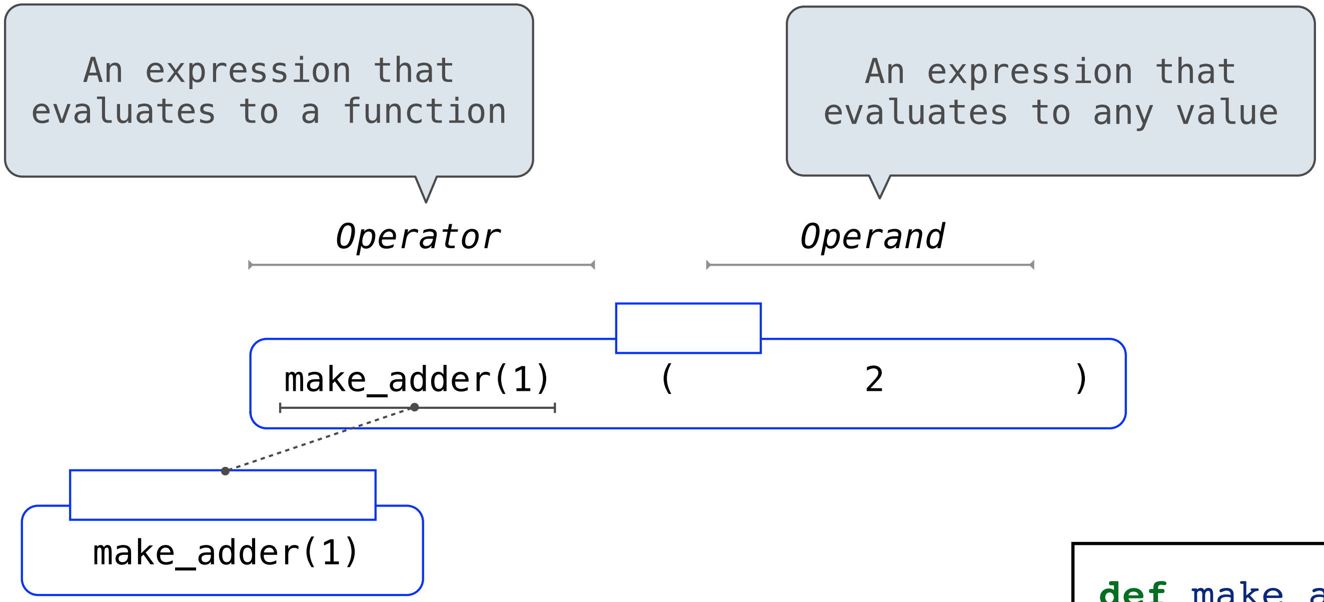
Call Expressions as Operator Expressions



An expression that evaluates to a function
An expression that evaluates to any value
Operator
Operand
make_adder(1)
(
2
)

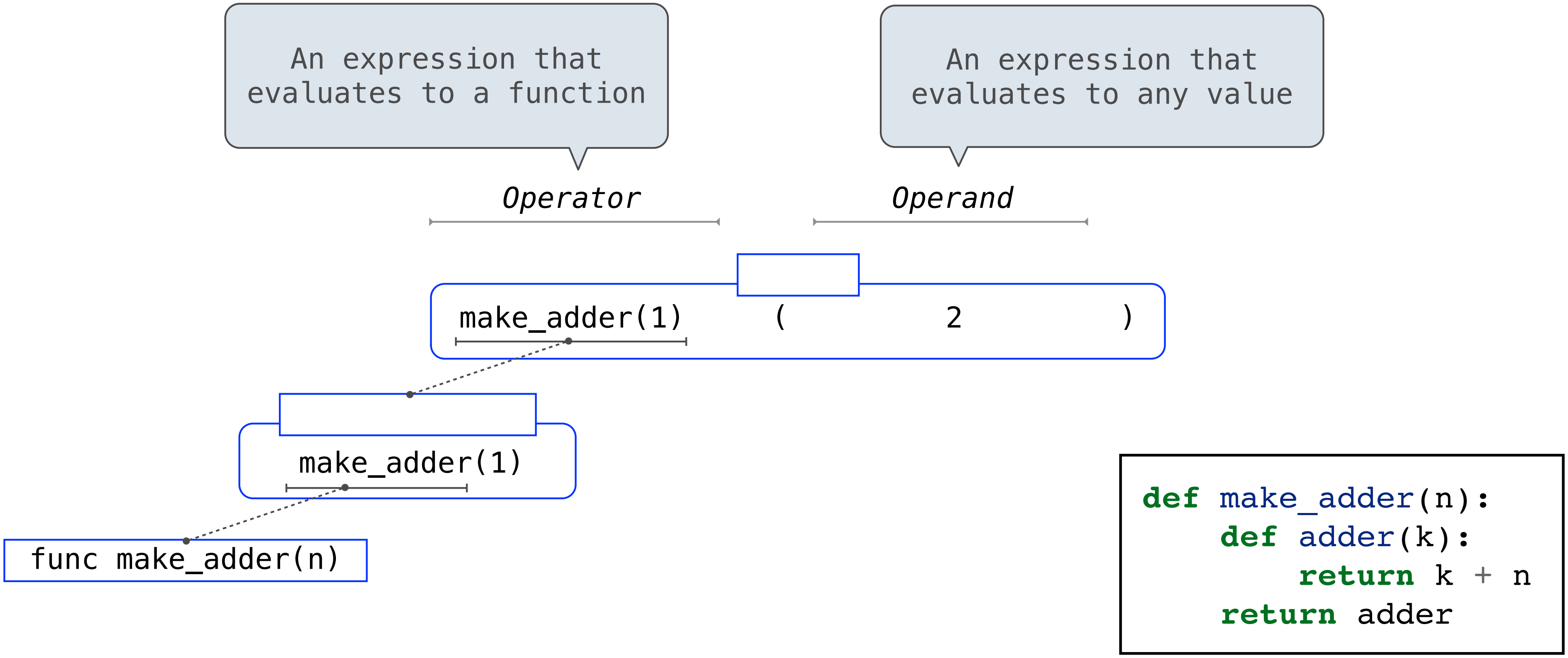
```
def make_adder(n):
    def adder(k):
        return k + n
    return adder
```

Call Expressions as Operator Expressions


An expression that evaluates to a function
An expression that evaluates to any value
Operator
Operand
make_adder(1) (2)
make_adder(1)

```
def make_adder(n):
    def adder(k):
        return k + n
    return adder
```

Call Expressions as Operator Expressions



An expression that evaluates to a function

An expression that evaluates to any value

Operator

Operand

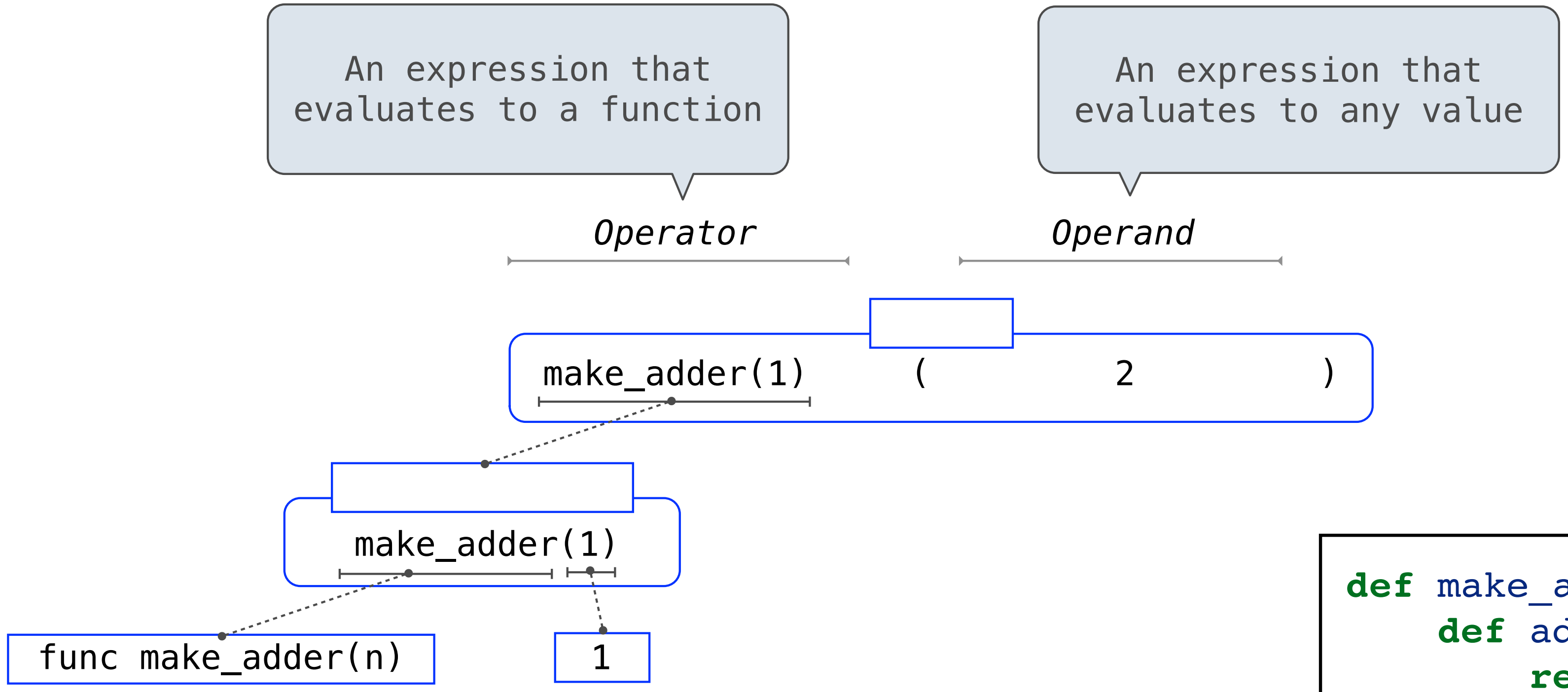
make_adder(1) (2)

make_adder(1)

func make_adder(n)

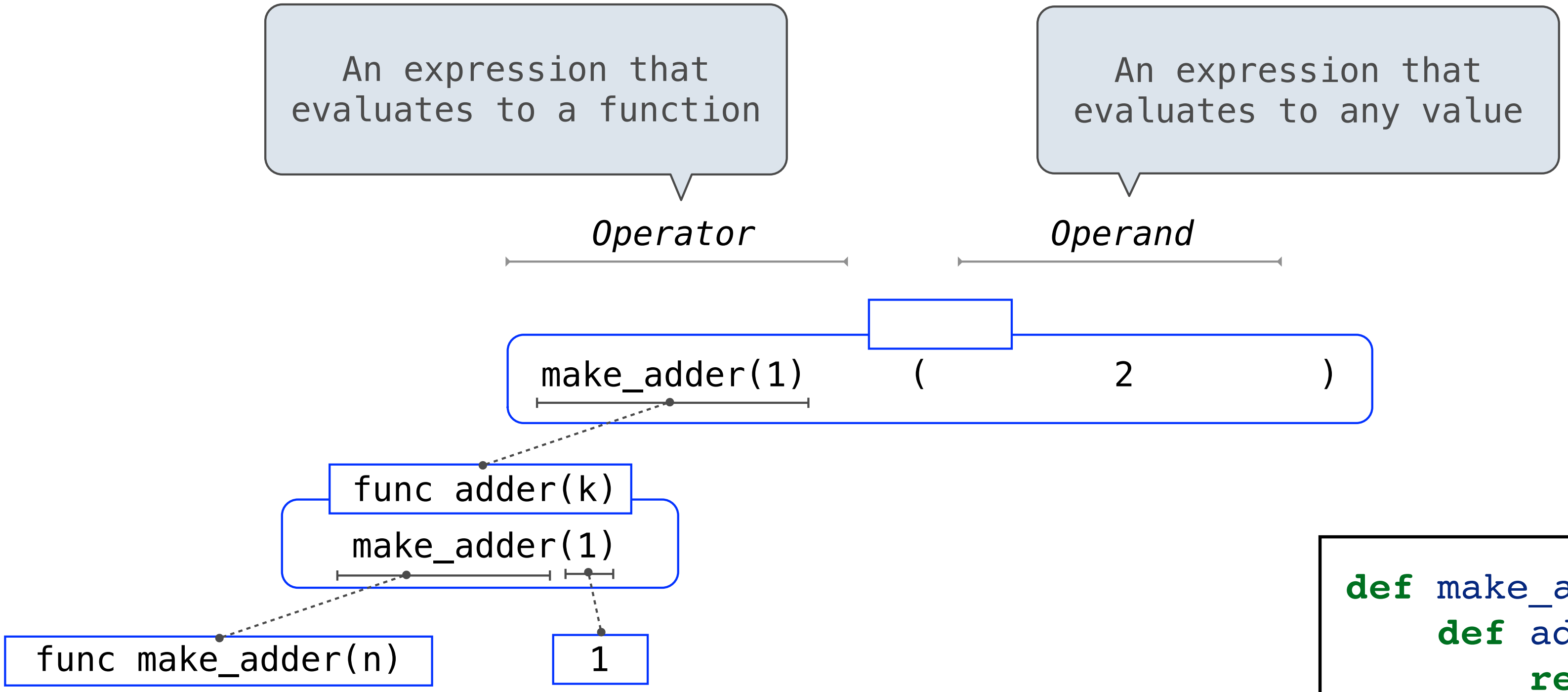
```
def make_adder(n):
    def adder(k):
        return k + n
    return adder
```

Call Expressions as Operator Expressions


An expression that evaluates to a function
An expression that evaluates to any value
Operator
Operand
make_adder(1)   (2   )
make_adder(1)
func make_adder(n)
1

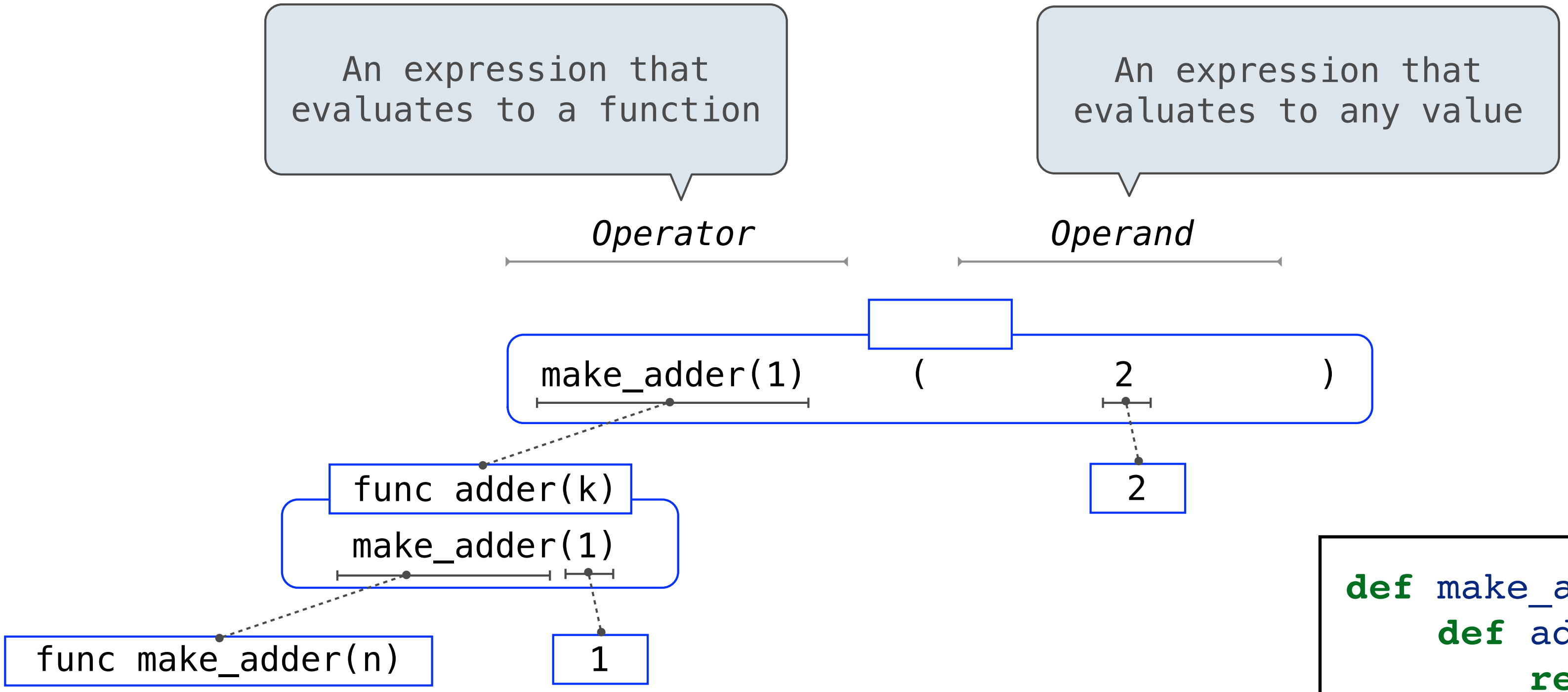
```
def make_adder(n):
    def adder(k):
        return k + n
    return adder
```

Call Expressions as Operator Expressions


An expression that evaluates to a function
An expression that evaluates to any value
Operator
Operand
make_adder(1)
(
2
)
func adder(k)
make_adder(1)
func make_adder(n)
1

```
def make_adder(n):
    def adder(k):
        return k + n
    return adder
```

Call Expressions as Operator Expressions



An expression that evaluates to a function

An expression that evaluates to any value

Operator

Operand

make_adder(1) (2)

func adder(k)

make_adder(1)

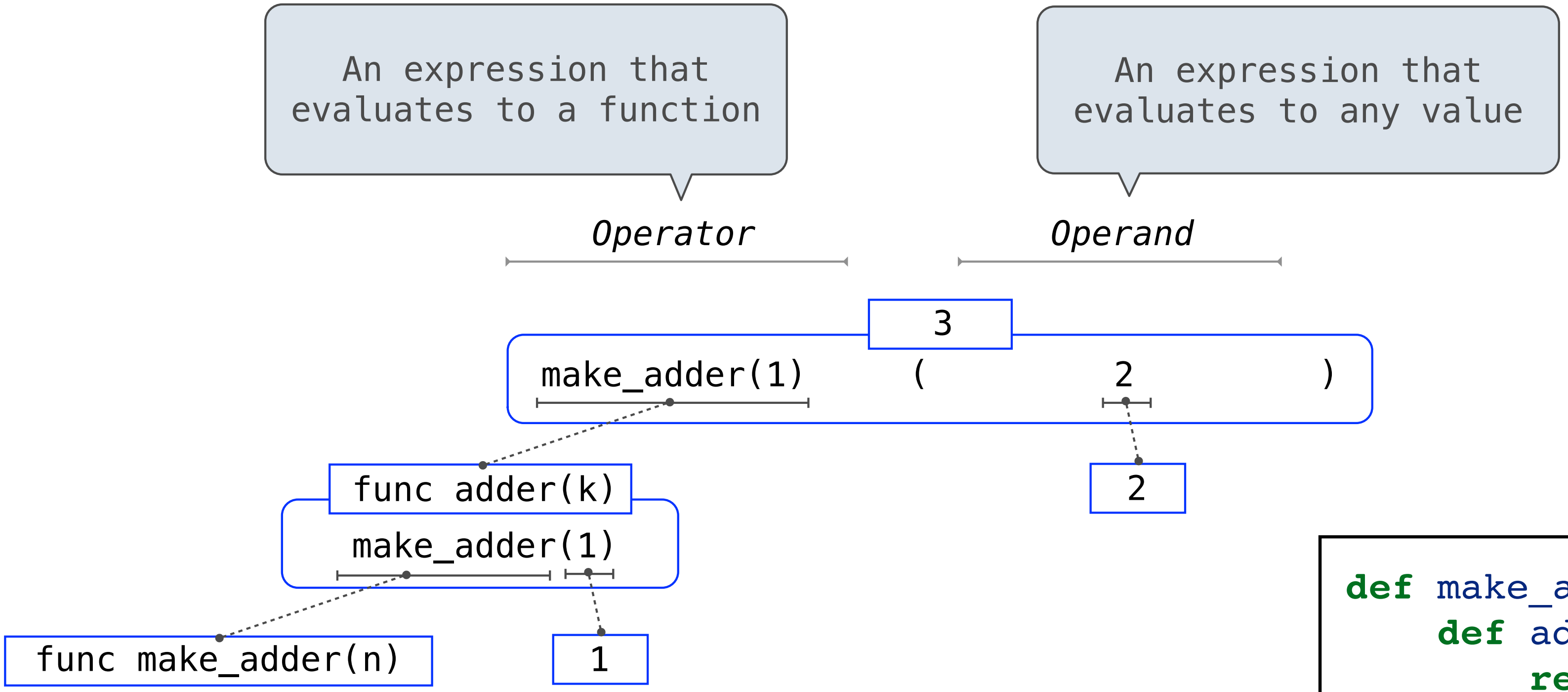
2

func make_adder(n)

1

```
def make_adder(n):
    def adder(k):
        return k + n
    return adder
```

Call Expressions as Operator Expressions



An expression that evaluates to a function

An expression that evaluates to any value

Operator

Operand

```
def make_adder(n):
    def adder(k):
        return k + n
    return adder
```

The Purpose of Higher-Order Functions

The Purpose of Higher-Order Functions

Functions are first-class: Functions can be manipulated as values in our programming language.

The Purpose of Higher-Order Functions

Functions are first-class: Functions can be manipulated as values in our programming language.

Higher-order function: A function that takes a function as an argument value or returns a function as a return value

The Purpose of Higher-Order Functions

Functions are first-class: Functions can be manipulated as values in our programming language.

Higher-order function: A function that takes a function as an argument value or returns a function as a return value

Higher-order functions:

The Purpose of Higher-Order Functions

Functions are first-class: Functions can be manipulated as values in our programming language.

Higher-order function: A function that takes a function as an argument value or returns a function as a return value

Higher-order functions:

- Express general methods of computation

The Purpose of Higher-Order Functions

Functions are first-class: Functions can be manipulated as values in our programming language.

Higher-order function: A function that takes a function as an argument value or returns a function as a return value

Higher-order functions:

- Express general methods of computation
- Remove repetition from programs

The Purpose of Higher-Order Functions

Functions are first-class: Functions can be manipulated as values in our programming language.

Higher-order function: A function that takes a function as an argument value or returns a function as a return value

Higher-order functions:

- Express general methods of computation
- Remove repetition from programs
- Separate concerns among functions

The Game of Hog

(Demo)