Not on Midterm 2

# 61A Lecture 24

Friday, October 22

One more built-in Python container type

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```
>>> s = {3, 2, 1, 4, 4}
>>> s
{1, 2, 3, 4}
```

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- Duplicate elements are removed on construction
- Sets are unordered, just like dictionary entries

```
>>> s = {3, 2, 1, 4, 4}
>>> s
{1, 2, 3, 4}

>>> 3 in s
True
```

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- Sets are unordered, just like dictionary entries

```
>>> s = {3, 2, 1, 4, 4}
>>> s
{1, 2, 3, 4}

>>> len(s)
4
```

- Set literals are enclosed in braces
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- Sets are unordered, just like dictionary entries

```
>>> s = {3, 2, 1, 4, 4}
>>> s
{1, 2, 3, 4}

>>> len(s)
4
>>> s.union({1, 5})
{1, 2, 3, 4, 5}
```

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- Duplicate elements are removed on construction
- Sets are unordered, just like dictionary entries

```
>>> s = {3, 2, 1, 4, 4}
>>> s
{1, 2, 3, 4}

>>> len(s)
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>>> s.union({1, 5})
{1, 2, 3, 4, 5}
>>> s.intersection({6, 5, 4, 3})
{3, 4}
```

The interface for sets

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Membership testing: Is a value an element of a set?

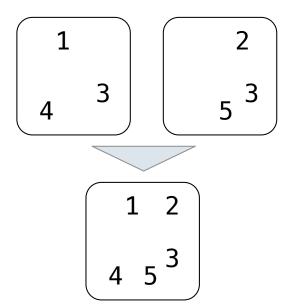
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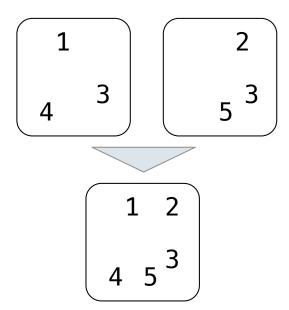
## Union



The interface for sets

- Membership testing: Is a value an element of a set?
- Union: Return a set with all elements in set1 or set2
- Intersection: Return a set with any elements in set1 **and** set2

#### Union



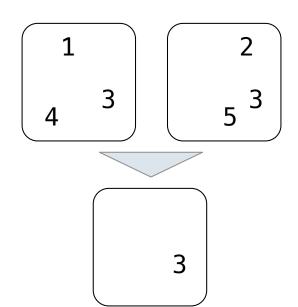
The interface for sets

- Membership testing: Is a value an element of a set?
- Union: Return a set with all elements in set1 or set2
- Intersection: Return a set with any elements in set1 **and** set2

#### Union

 $\begin{array}{c|cccc}
1 & & & & & 2 \\
4 & 3 & & & 5 & & \\
\hline
 & 1 & 2 & & & \\
 & 4 & 5 & & & \\
\end{array}$ 

#### Intersection



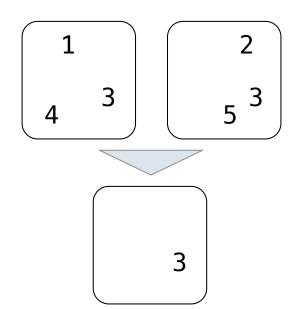
The interface for sets

- Membership testing: Is a value an element of a set?
- Union: Return a set with all elements in set1 or set2
- Intersection: Return a set with any elements in set1 and set2
- Adjunction: Return a set with all elements in s and a value v

#### Union

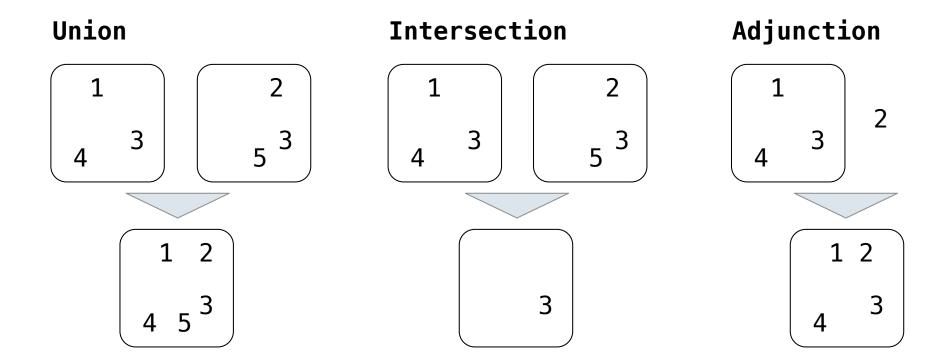
# 1 2 1 5 3

## **Intersection**



The interface for sets

- Membership testing: Is a value an element of a set?
- Union: Return a set with all elements in set1 or set2
- Intersection: Return a set with any elements in set1 **and** set2
- Adjunction: Return a set with all elements in s and a value v



```
def empty(s):
    return s is Rlist.empty
```

```
def empty(s):
    return s is Rlist.empty

def set_contains(s, v):
```

```
def empty(s):
    return s is Rlist.empty

def set_contains(s, v):
    if empty(s):
        return False
```

```
def empty(s):
    return s is Rlist.empty

def set_contains(s, v):
    if empty(s):
        return False
    elif s.first == v:
        return True
```

```
def empty(s):
    return s is Rlist.empty

def set_contains(s, v):
    if empty(s):
        return False
    elif s.first == v:
        return True
    return set_contains(s.rest, v)
```

**Proposal 1:** A set is represented by a recursive list that contains no duplicate items

```
def empty(s):
    return s is Rlist.empty

def set_contains(s, v):
    if empty(s):
        return False
    elif s.first == v:
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```

Demo

For a set operation that takes "linear" time, we say that

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Demo

```
def adjoin_set(s, v):
```

```
def adjoin_set(s, v):
    if set_contains(s, v):
```

```
def adjoin_set(s, v):
    if set_contains(s, v):
        return s
```

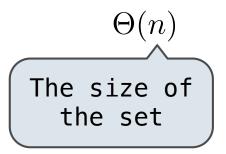
```
def adjoin_set(s, v):
    if set_contains(s, v):
        return s
    return Rlist(v, s)
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```

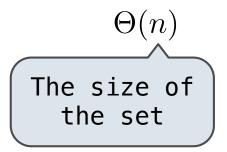
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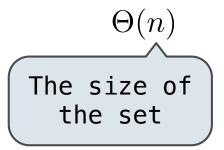
```
def adjoin_set(s, v):
    if set_contains(s, v):
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def intersect_set(set1, set2):
```

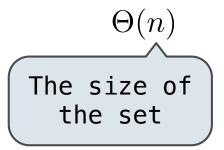


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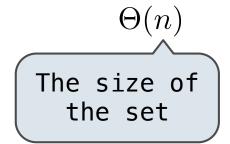
def intersect_set(set1, set2):
    f = lambda v: set_contains(set2, v)
```



# def adjoin\_set(s, v): if set\_contains(s, v): return s return Rlist(v, s) def intersect\_set(set1, set2): f = lambda v: set\_contains(set2, v) return filter\_rlist(set1, f)



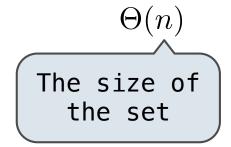
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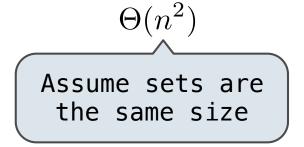


$$\Theta(n^2)$$

```
def adjoin_set(s, v):
    if set_contains(s, v):
        return s
    return Rlist(v, s)

def intersect_set(set1, set2):
    f = lambda v: set_contains(set2, v)
    return filter_rlist(set1, f)
```

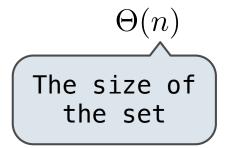


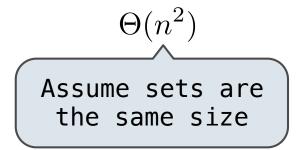


```
def adjoin_set(s, v):
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        return s
    return Rlist(v, s)

def intersect_set(set1, set2):
    f = lambda v: set_contains(set2, v)
    return filter_rlist(set1, f)

def union_set(set1, set2):
```





#### $\Theta(n)$ def adjoin\_set(s, v): if set\_contains(s, v): The size of return s the set return Rlist(v, s) $\Theta(n^2)$ def intersect\_set(set1, set2): f = lambda v: set\_contains(set2, v) Assume sets are return filter\_rlist(set1, f) the same size def union\_set(set1, set2): f = lambda v: not set\_contains(set2, v)

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```
def set_contains2(s, v):
```

```
def set_contains2(s, v):
    if empty(s) or s.first > v:
        return False
```

```
def set_contains2(s, v):
    if empty(s) or s.first > v:
        return False
    elif s.first == v:
        return True
```

```
def set_contains2(s, v):
    if empty(s) or s.first > v:
        return False
    elif s.first == v:
        return True
    return set_contains2(s.rest, v)
```

**Proposal 2:** A set is represented by a recursive list with unique elements ordered from least to greatest

```
def set_contains2(s, v):
    if empty(s) or s.first > v:
        return False
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    return set_contains2(s.rest, v)
```

Order of growth?

**Proposal 2:** A set is represented by a recursive list with unique elements ordered from least to greatest

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def set_contains2(s, v):
    if empty(s) or s.first > v:
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```

Order of growth?  $\Theta(n)$ 

```
def intersect_set2(set1, set2):
```

```
def intersect_set2(set1, set2):
    if empty(set1) or empty(set2):
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```

```
def intersect_set2(set1, set2):
    if empty(set1) or empty(set2):
        return Rlist.empty
    e1, e2 = set1.first, set2.first
```

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def intersect_set2(set1, set2):
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    if e1 == e2:
```

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def intersect_set2(set1, set2):
    if empty(set1) or empty(set2):
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    e1, e2 = set1.first, set2.first
    if e1 == e2:
        rest = intersect_set2(set1.rest, set2.rest)
```

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def intersect_set2(set1, set2):
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    e1, e2 = set1.first, set2.first
    if e1 == e2:
        rest = intersect_set2(set1.rest, set2.rest)
        return Rlist(e1, rest)
```

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def intersect_set2(set1, set2):
    if empty(set1) or empty(set2):
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    e1, e2 = set1.first, set2.first
    if e1 == e2:
        rest = intersect_set2(set1.rest, set2.rest)
        return Rlist(e1, rest)
    elif e1 < e2:</pre>
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        return Rlist(e1, rest)
    elif e1 < e2:
        return intersect_set2(set1.rest, set2)
    elif e2 < e1:</pre>
```

This algorithm assumes that elements are in order.

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def intersect_set2(set1, set2):
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Demo

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def intersect_set2(set1, set2):
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        return Rlist(e1, rest)
    elif e1 < e2:
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Order of growth?

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        rest = intersect_set2(set1.rest, set2.rest)
        return Rlist(e1, rest)
    elif e1 < e2:
        return intersect set2(set1.rest, set2)
    elif e2 < e1:
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Order of growth?  $\Theta(n)$ 

Proposal 3: A set is represented as a Tree. Each entry is:

9

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Larger than all entries in its left branch and

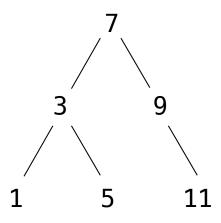
Proposal 3: A set is represented as a Tree. Each entry is:

- Larger than all entries in its left branch and
- Smaller than all entries in its right branch

9

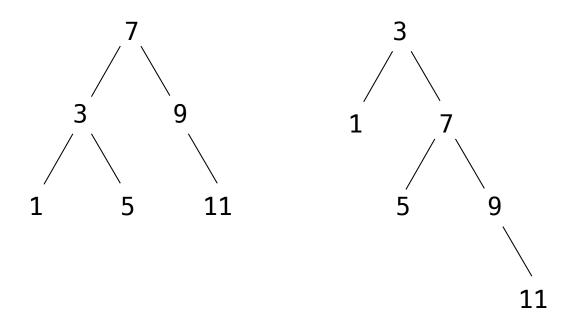
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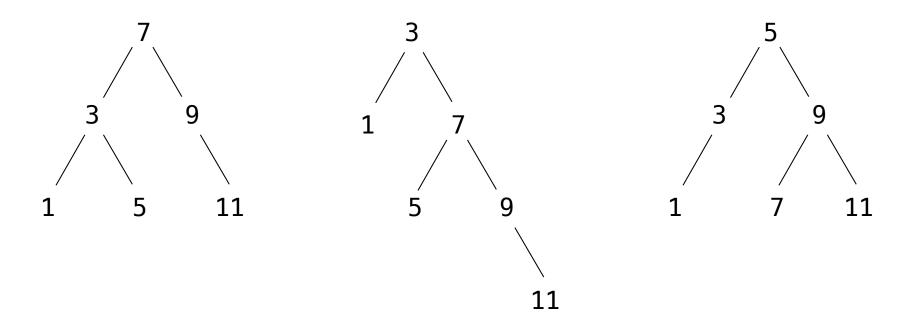
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9

Set membership tests traverse the tree

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```
def set_contains3(s, v):
    if s is None:
        return False
```

- The element is either in the left or right sub-branch
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def set_contains3(s, v):
    if s is None:
        return False
    elif s.entry == v:
        return True
```

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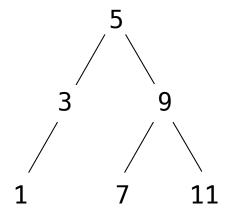
```
def set_contains3(s, v):
    if s is None:
        return False
    elif s.entry == v:
        return True
    elif s.entry < v:
        return set_contains3(s.right, v)</pre>
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    if s is None:
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    elif s.entry < v:
        return set_contains3(s.right, v)
    elif s.entry > v:
        return set contains3(s.left, v)
```

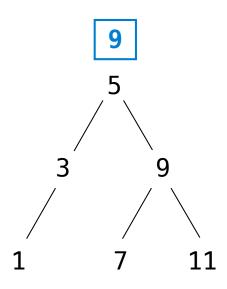
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def set_contains3(s, v):
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        return set_contains3(s.left, v)
```



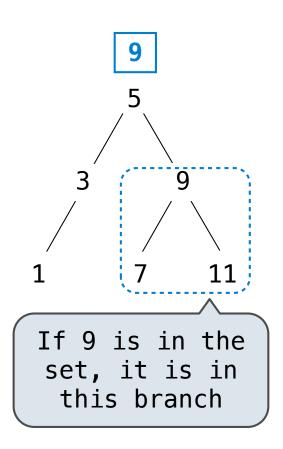
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def set_contains3(s, v):
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```



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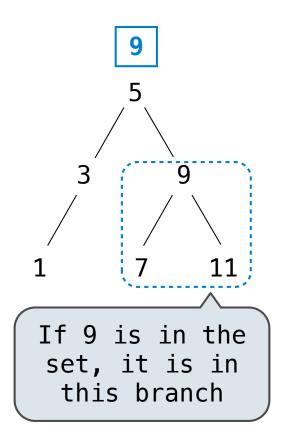
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```



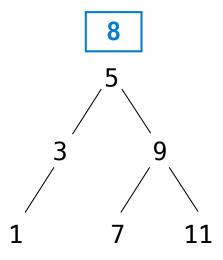
Set membership tests traverse the tree

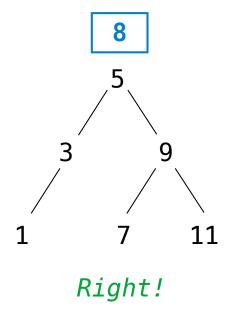
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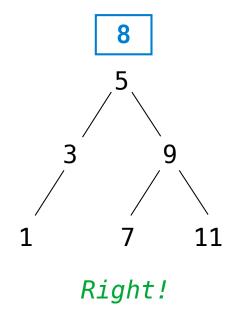
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def set_contains3(s, v):
    if s is None:
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        return True
    elif s.entry < v:
        return set_contains3(s.right, v)
    elif s.entry > v:
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```

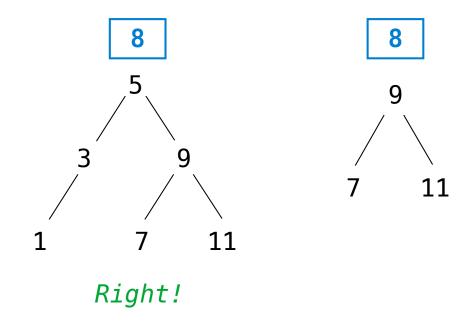


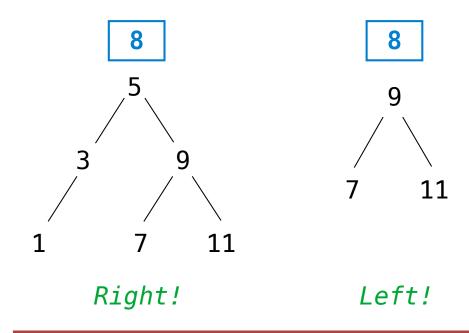
Order of growth?

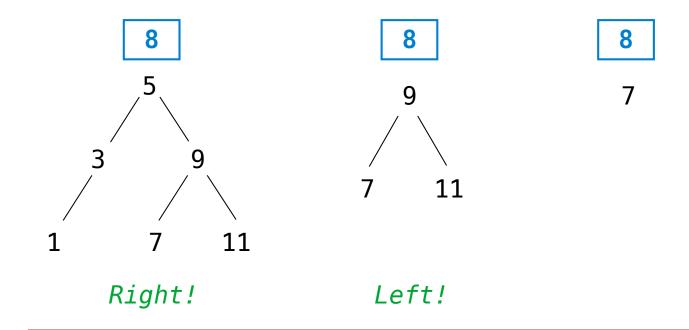


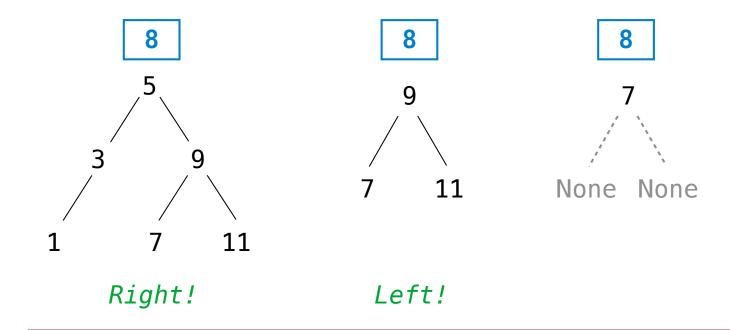


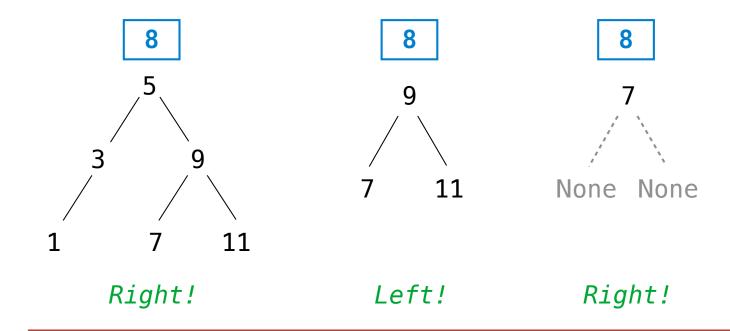


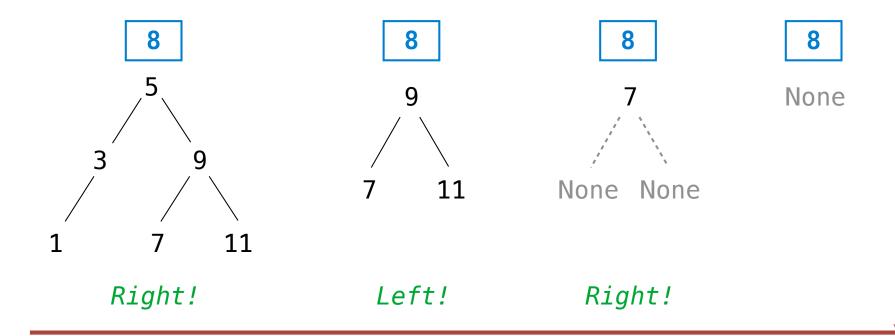


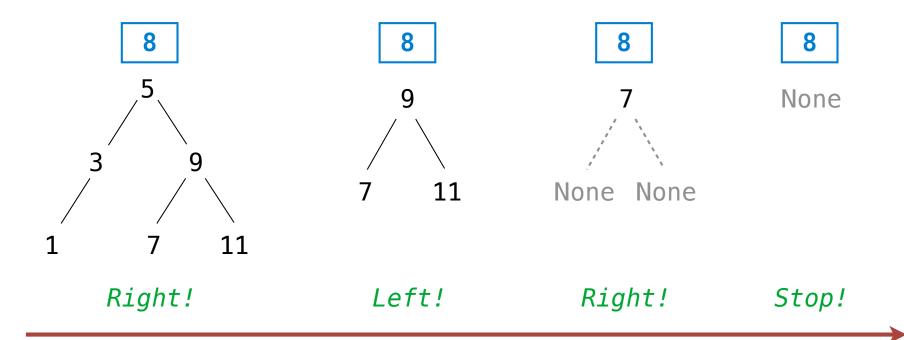


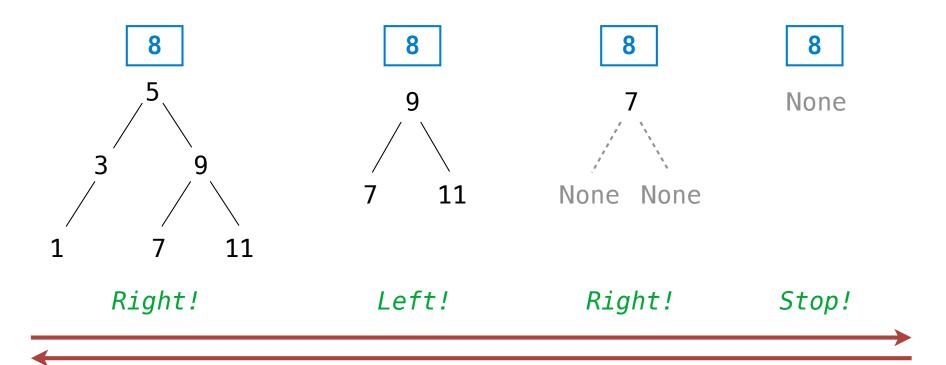


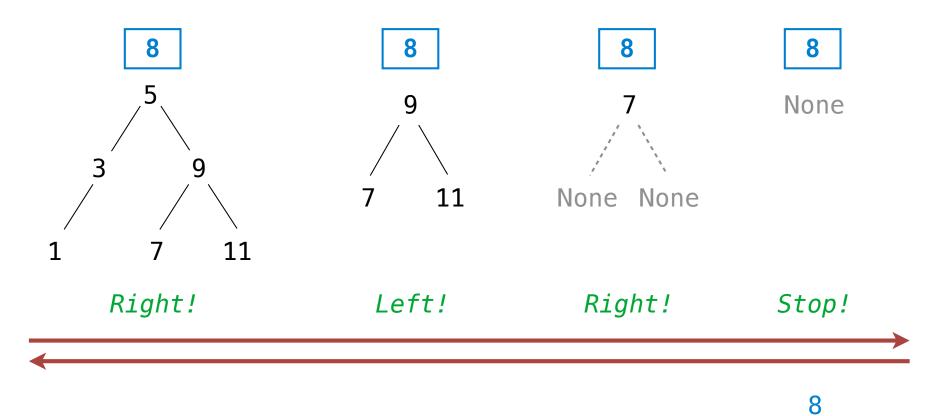


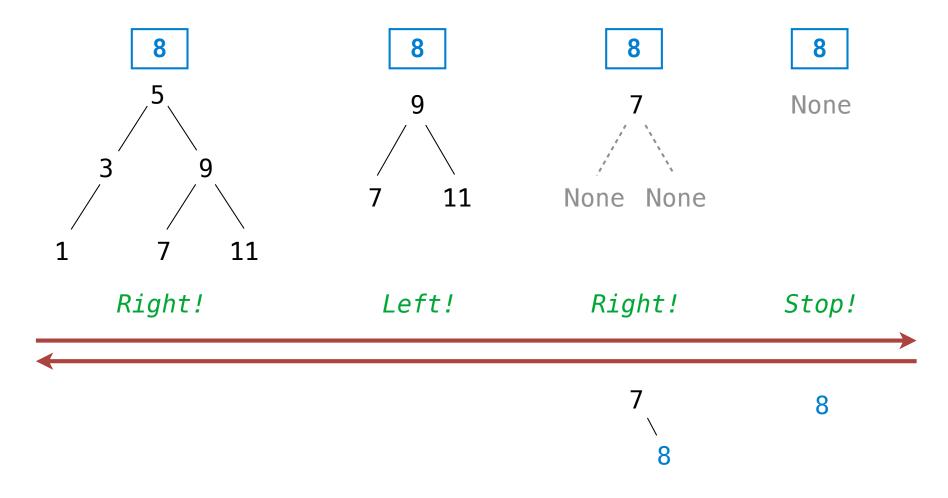


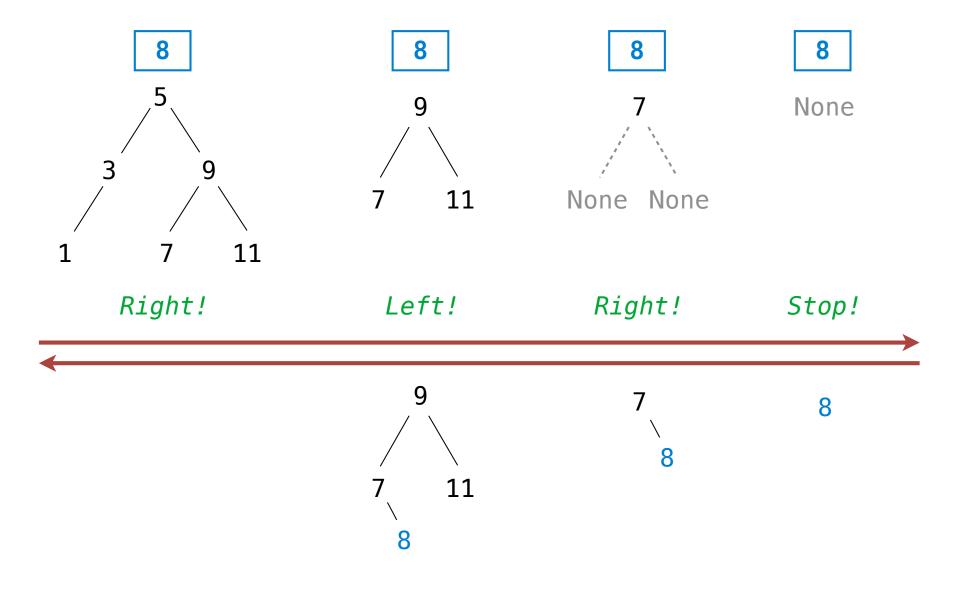


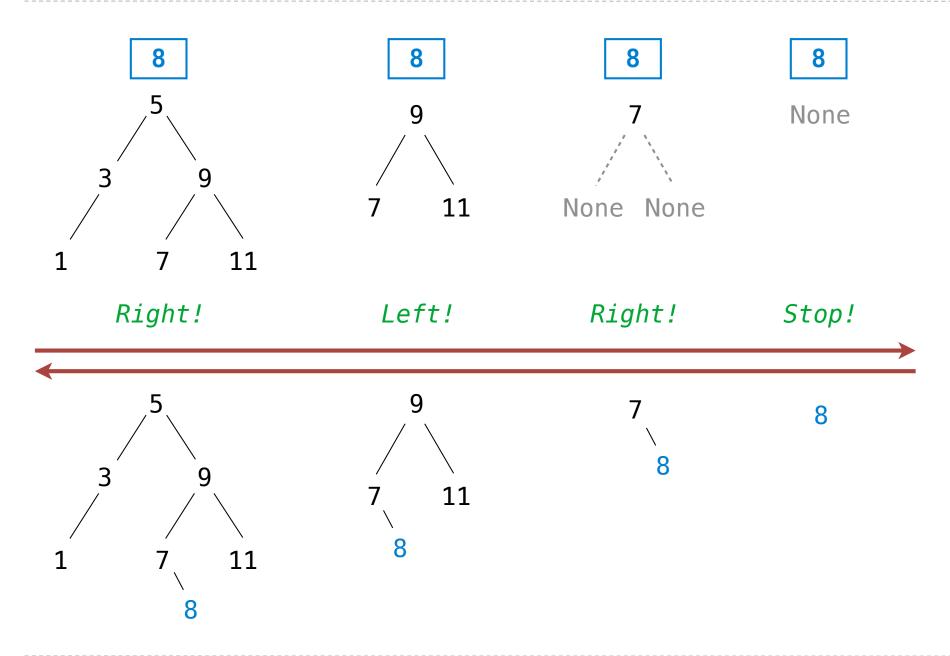


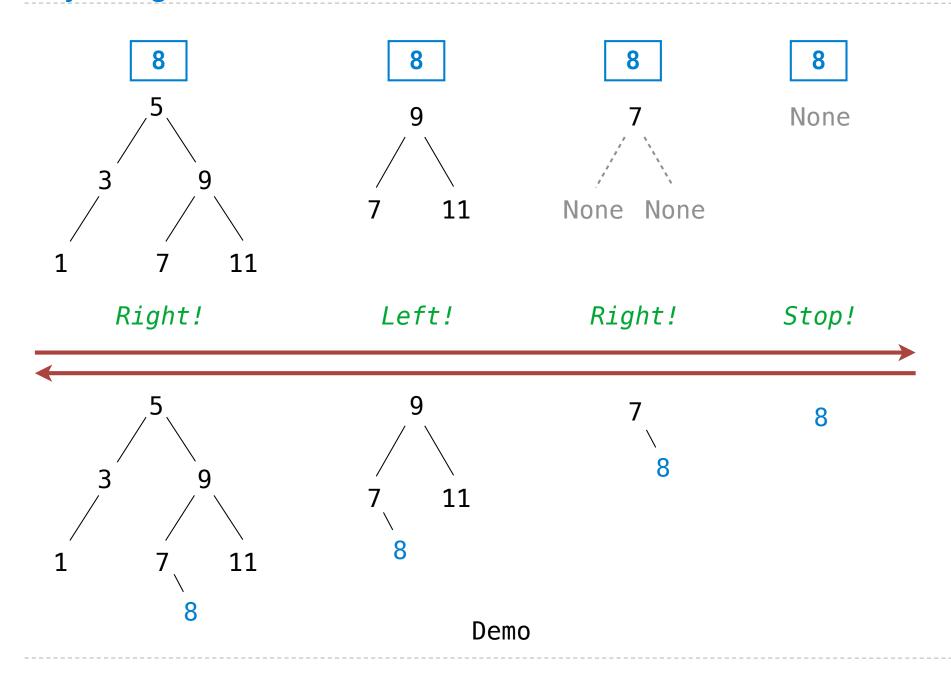












Sets as ordered sequences:

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Adjoining an element to a set

Sets as ordered sequences:

- Adjoining an element to a set
- Union of two sets

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Sets as binary trees:

Sets as ordered sequences:

- Adjoining an element to a set
- Union of two sets

Sets as binary trees:

Intersection of two sets

#### Sets as ordered sequences:

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That's homework 8!

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No lecture on Wednesday

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No lecture on Wednesday
Midterm 2 tomorrow, 7pm-9pm

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- Union of two sets

Sets as binary trees:

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That's homework 8!

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Good luck!