Last day of Midterm 2 Material

61A Lecture 23

Friday, October 19

Trees with Internal Node Values

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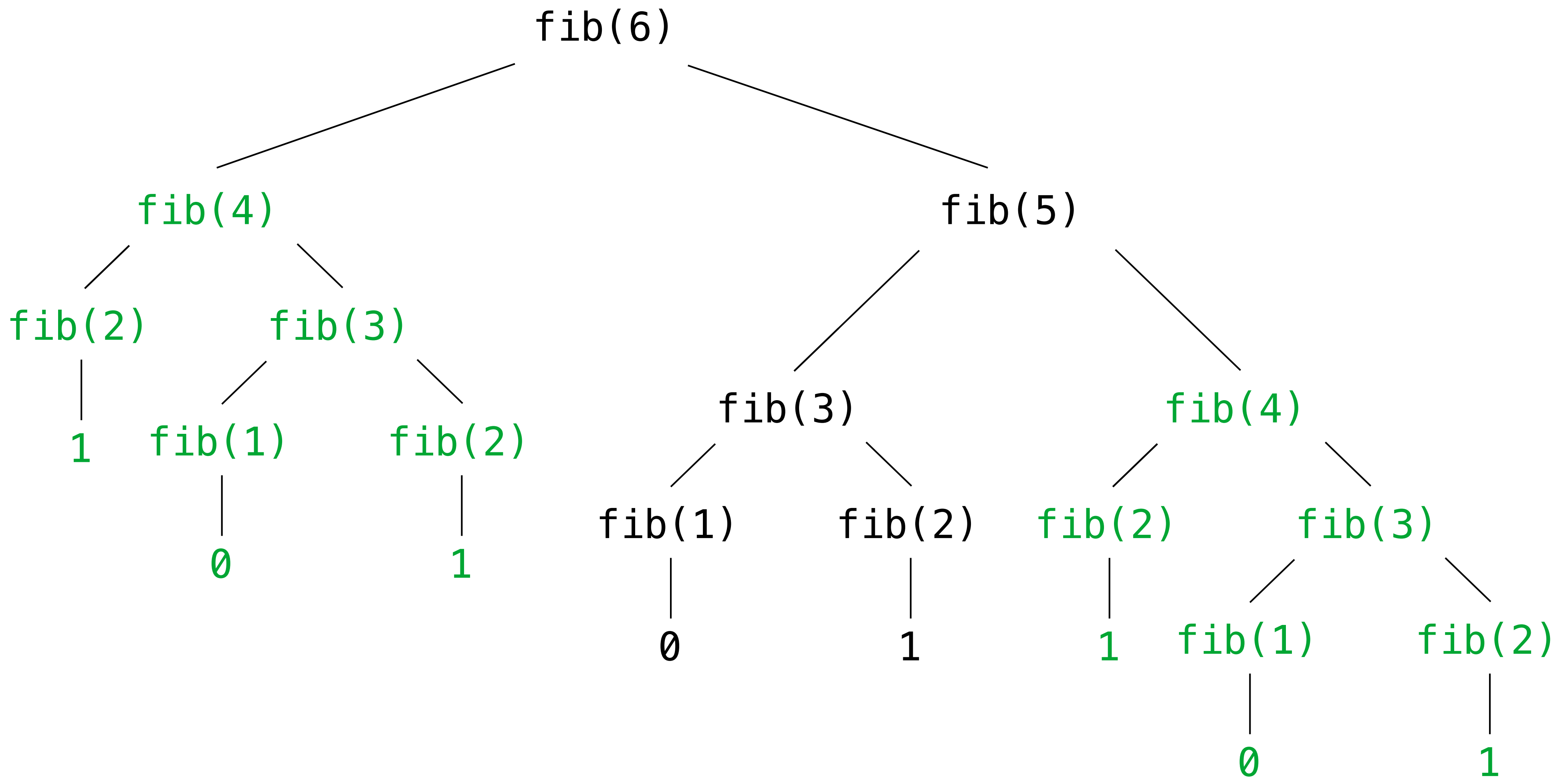

Trees with Internal Node Values (Entries)

Trees need not only have values at their leaves.

A valid tree cannot
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    def __init__(self, entry, left=None, right=None):
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class Tree(object):
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class Tree(object):
    def __init__(self, entry, left=None, right=None):
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```

Valid if left and right are each either None or a Tree instance

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class Tree(object):
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def fib_tree(n):
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def fib_tree(n):
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def fib_tree(n):
    if n == 1:
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    if n == 2:
        return Tree(1)
    left = fib_tree(n-2)
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Demo

The Consumption of Time

Implementations of the same functional abstraction can require different amounts of time to compute their result.

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Time (remainders)

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```
def count_factors(n):

    factors = 0
    for k in range(1, n+1):
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(Demo)

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n

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    sqrt_n = sqrt(n)
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    while k < sqrt_n:
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            factors += 2
        k += 1
    if k * k == n:
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    return factors
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$\lfloor \sqrt{n} \rfloor$

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Active environments:

The Consumption of Space

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Active environments:

- Environments for any statements currently being executed

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Values and frames in active environments consume memory.

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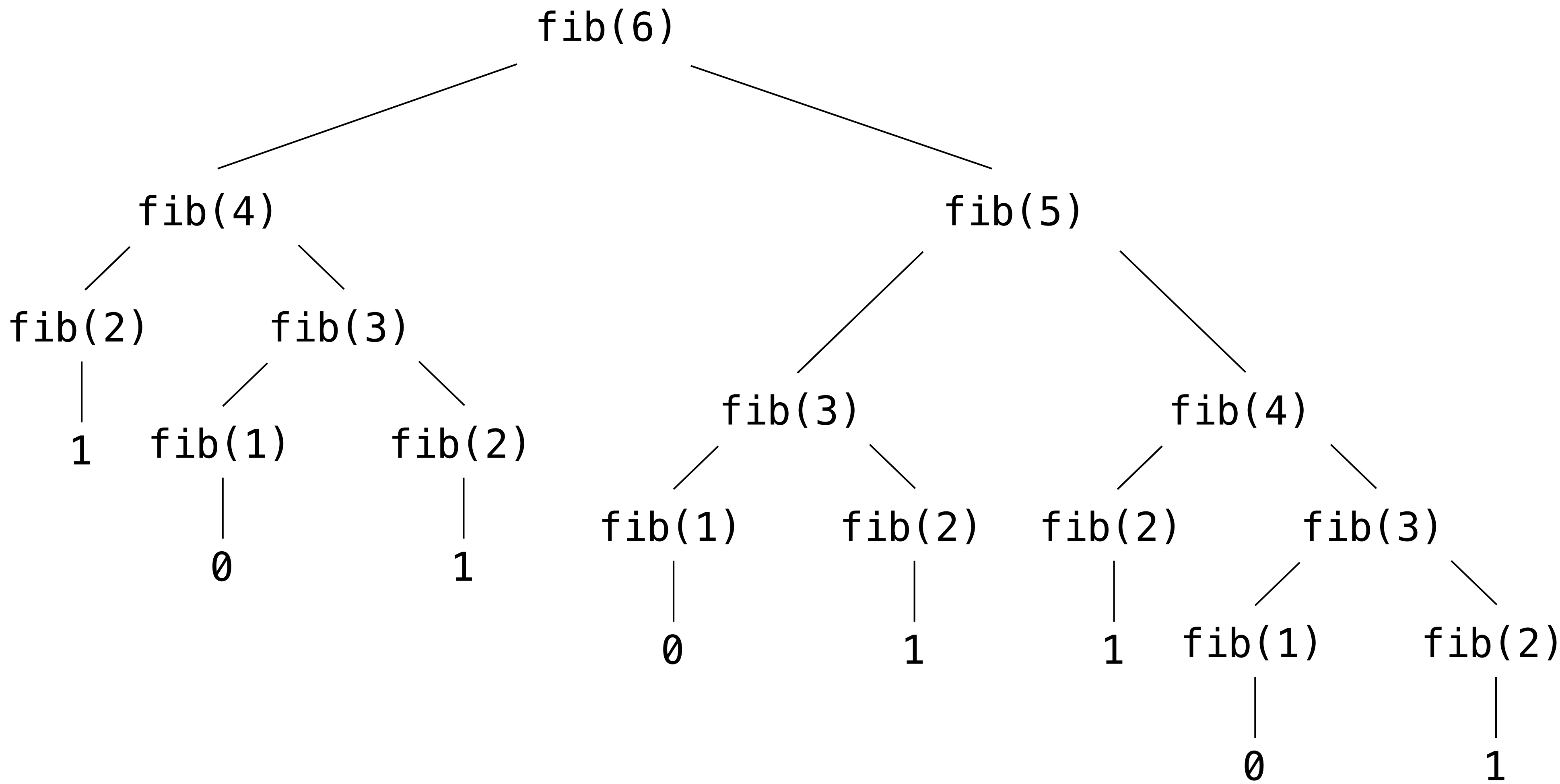
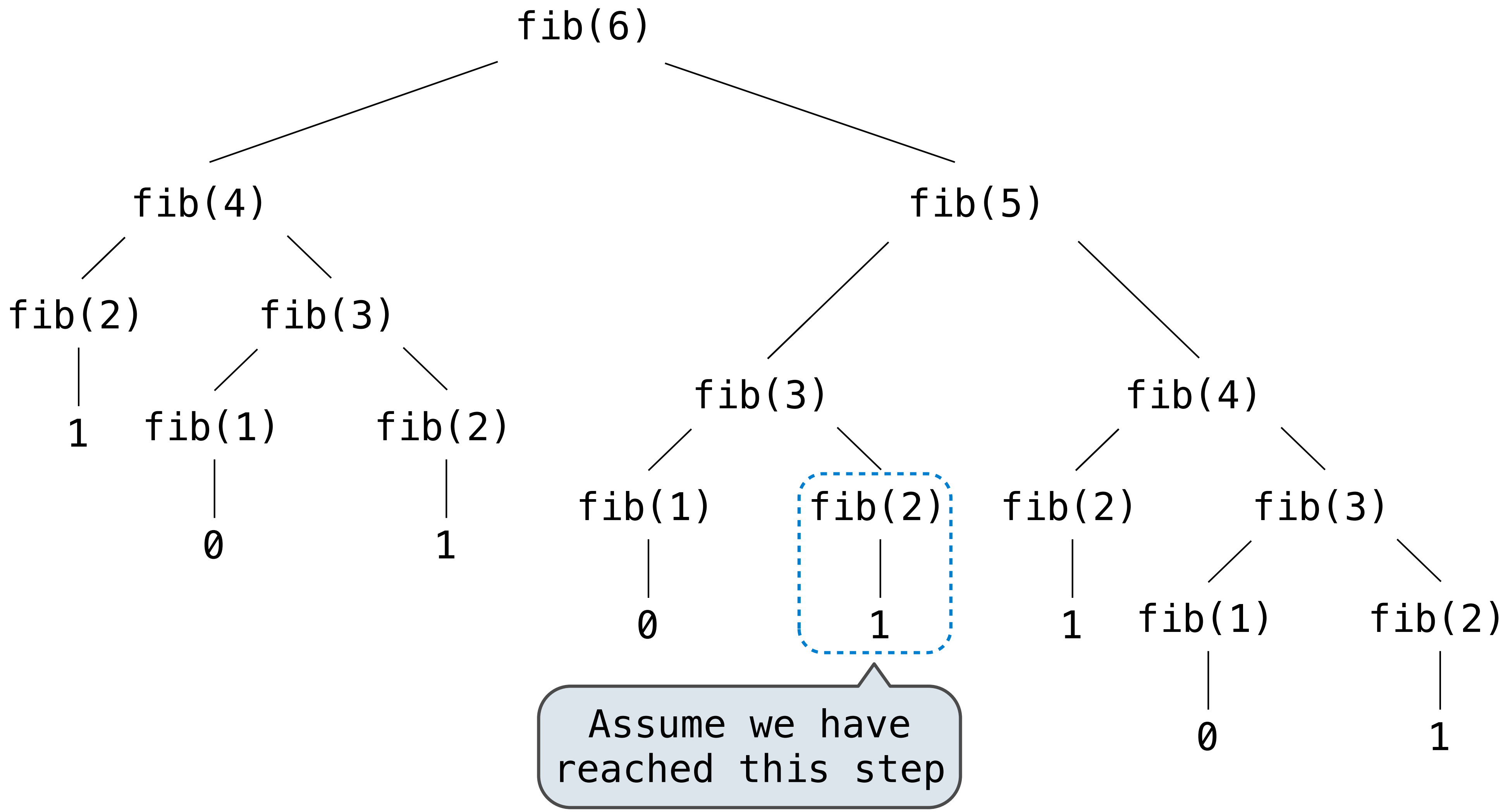
Active environments:

- Environments for any statements currently being executed
- Parent environments of functions named in active environments

Fibonacci Memory Consumption



Fibonacci Memory Consumption



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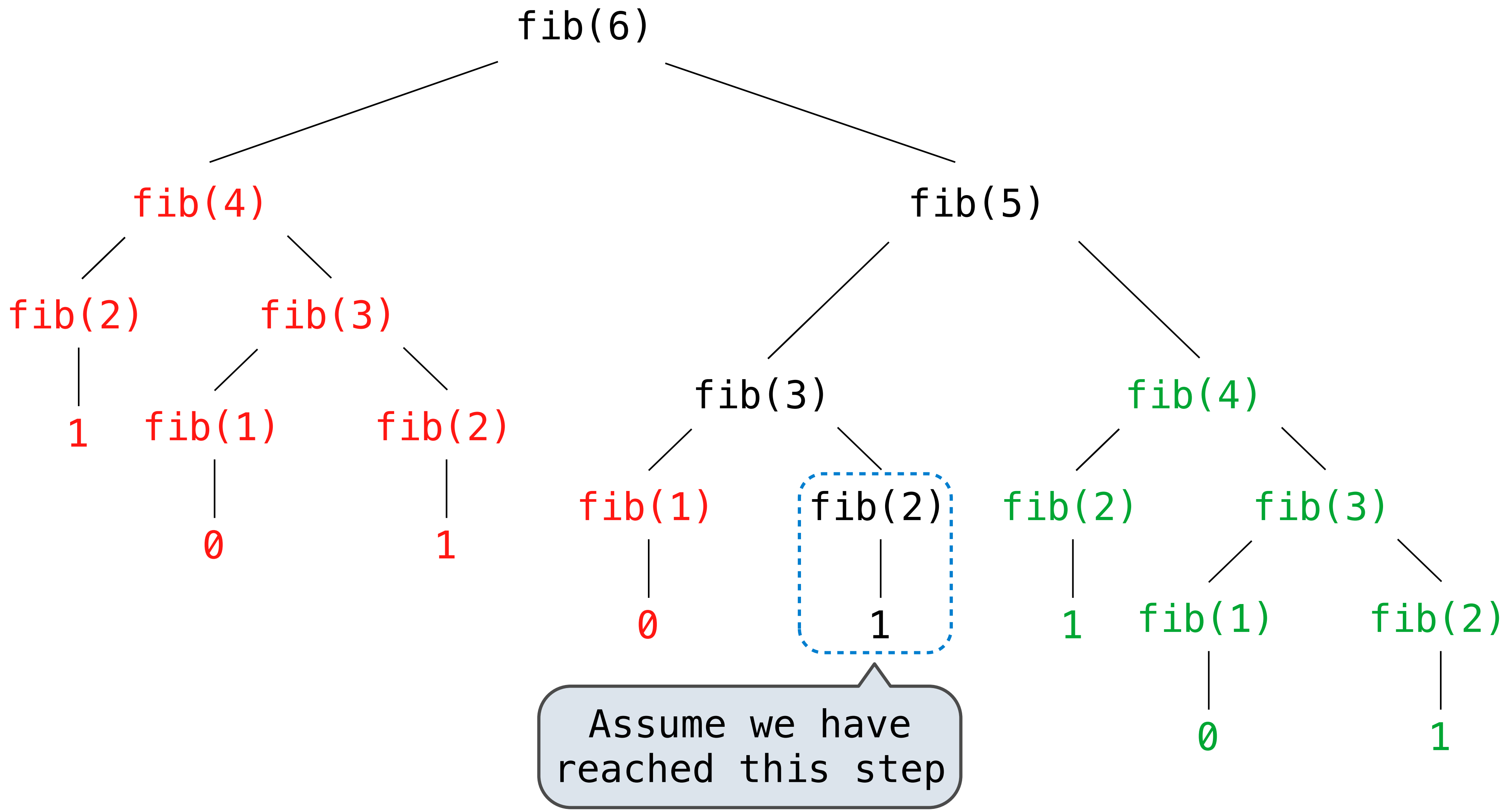

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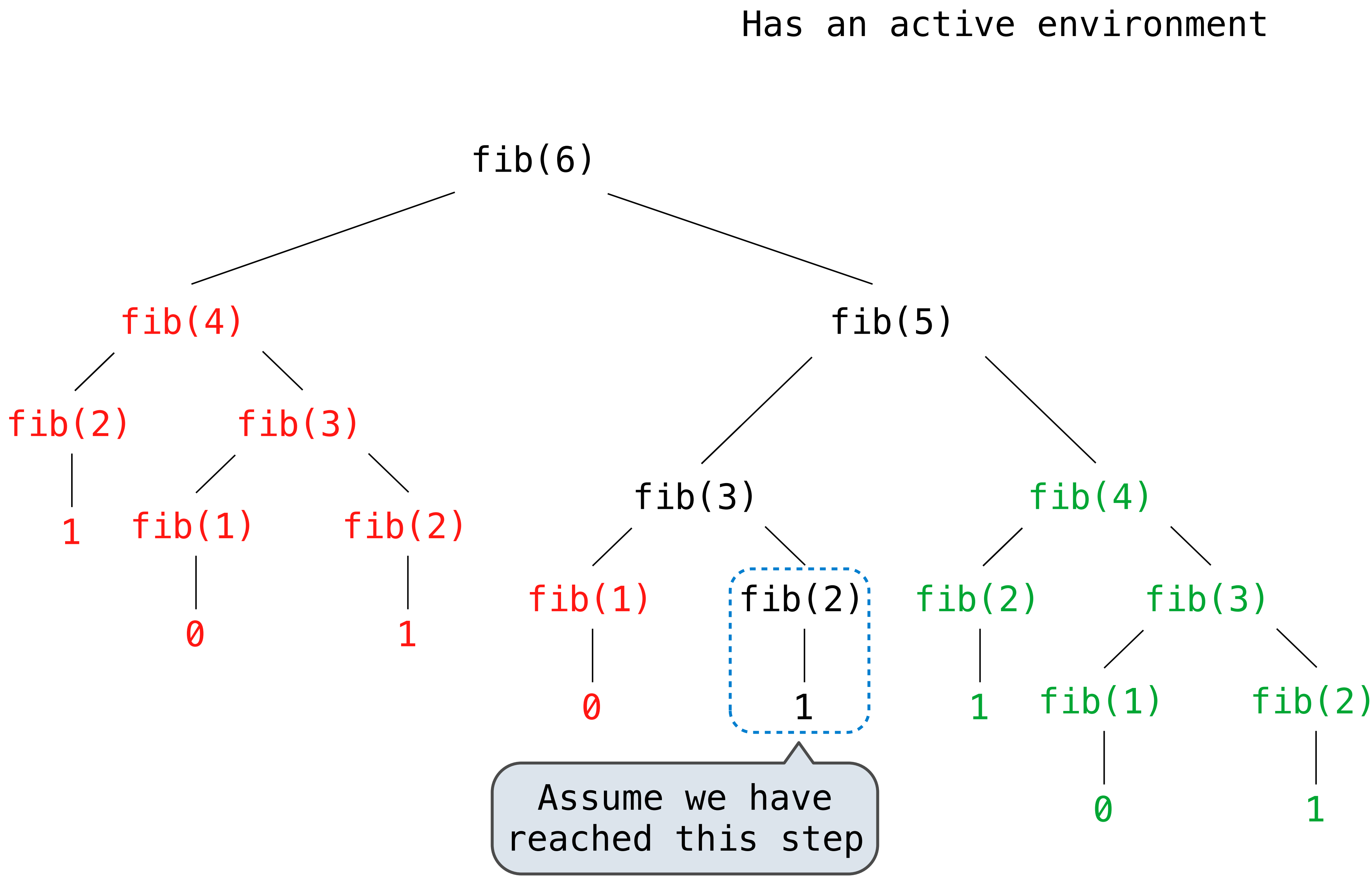

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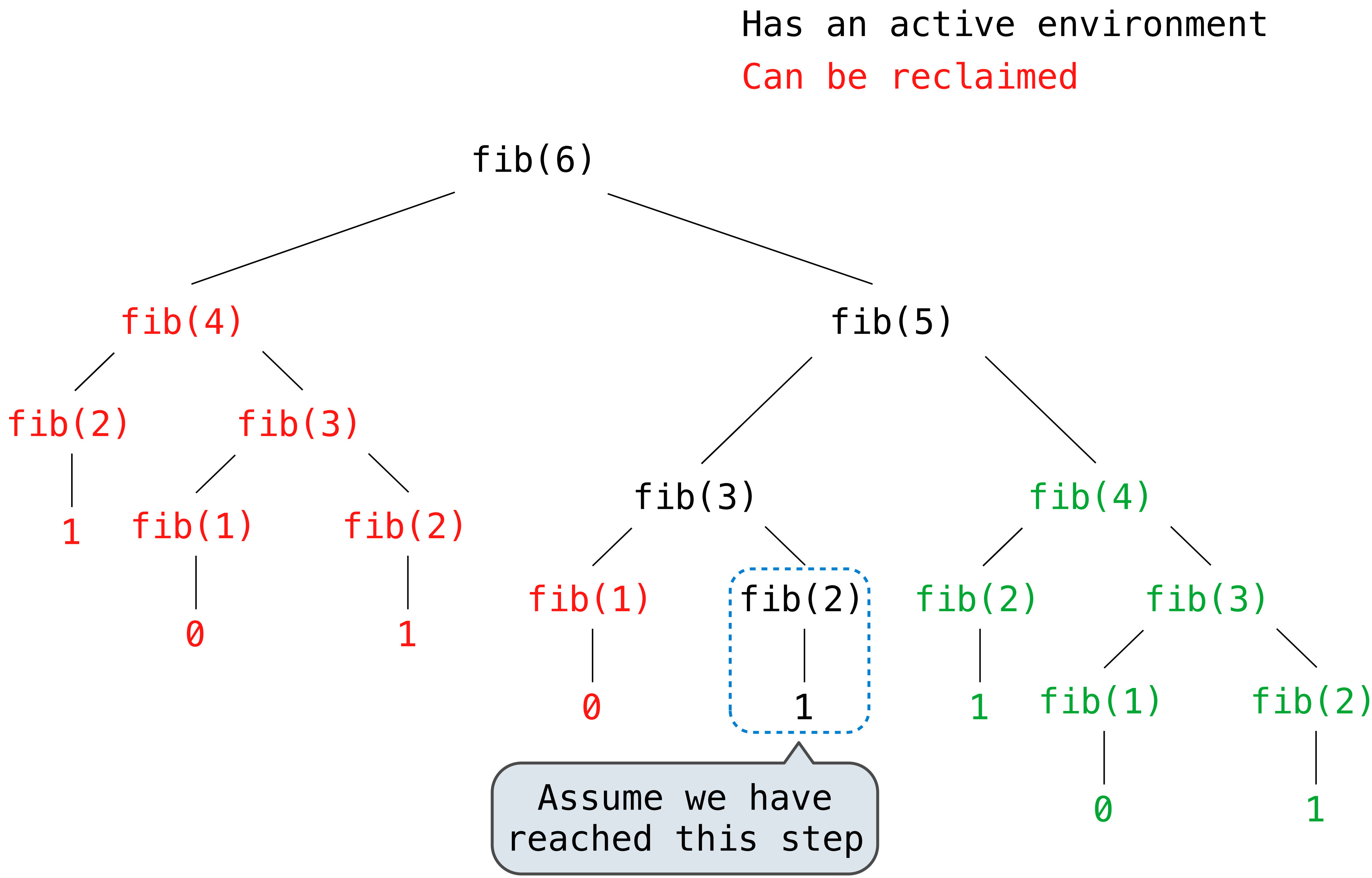

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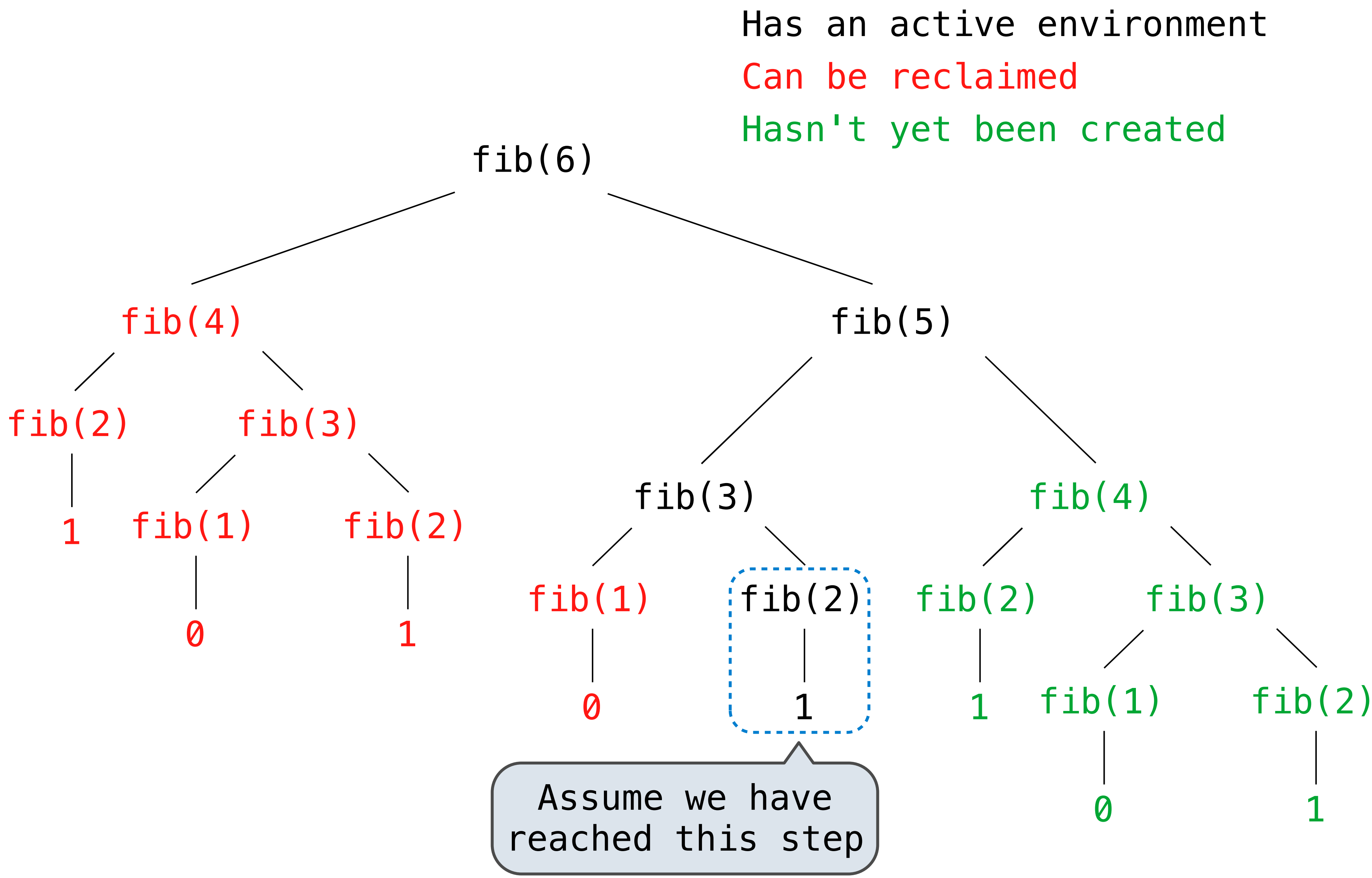

Order of Growth

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Order of Growth

A method for bounding the resources used by a function as the "size" of a problem increases

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$$R(n) = \Theta(f(n))$$

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***R*(*n*):** Measurement of some resource used (time or space)

$$R(n) = \Theta(f(n))$$

means that there are positive constants k_1 and k_2 such that

$$k_1 \cdot f(n) \leq R(n) \leq k_2 \cdot f(n)$$

for sufficiently large values of ***n*.**

Iteration vs Memoized Tree Recursion

Iterative and memoized implementations are not the same.

	Time	Space

```
def fib_iter(n):
    prev, curr = 1, 0
    for _ in range(n-1):
         prev, curr = curr, prev + curr
    return curr
```

```
@memo
def fib(n):
    if n == 1:
        return 0
    if n == 2:
        return 1
    return fib(n-2) + fib(n-1)
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fib_iter	$\Theta(n)$	
fib (memoized)		

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`fib_iter`	$\Theta(n)$	$\Theta(1)$

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`def fib_iter(n): ...`	$\Theta(n)$	$\Theta(1)$
`@memo def fib(n): ...`	$\Theta(n)$	

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def count_factors(n):


    factors = 0
    for k in range(1, n+1):
        if n % k == 0:
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    return factors
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```
    sqrt_n = sqrt(n)
    k, factors = 1, 0
    while k < sqrt_n:
        if n % k == 0:
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        k += 1
    if k * k == n:
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Time	**Space**

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Time	Space
$\Theta(n)$	$\Theta(1)$
$\Theta(\sqrt{n})$	$\Theta(1)$

Exponentiation

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Goal: one more multiplication lets us double the problem size.

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def square(x):
    return x*x
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def square(x):
    return x*x

def fast_exp(b, n):
```

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```
def square(x):
    return x*x

def fast_exp(b, n):
    if n == 0:
        return 1
    if n % 2 == 0:
        return square(fast_exp(b, n//2))
```

$$b^n = \begin{cases} 1 & \text{if } n = 0 \\ (b^{\frac{1}{2}n})^2 & \text{if } n \text{ is even} \\ b \cdot b^{n-1} & \text{if } n \text{ is odd} \end{cases}$$

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    else:
        return b * fast_exp(b, n-1)
```

$$b^n = \begin{cases} 1 & \text{if } n = 0 \\ (b^{\frac{1}{2}n})^2 & \text{if } n \text{ is even} \\ b \cdot b^{n-1} & \text{if } n \text{ is odd} \end{cases}$$

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	Time	Space

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def exp(b, n):
    if n == 0:
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def square(x):
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def fast_exp(b, n):
    if n == 0:
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Exponentiation

Goal: one more multiplication lets us double the problem size.

	Time	**Space**
`exp`	$\Theta(n)$	$\Theta(n)$

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def exp(b, n):
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Goal: one more multiplication lets us double the problem size.

	Time	**Space**
`exp`	$\Theta(n)$	$\Theta(n)$
`fast_exp`	$\Theta(\log n)$	$\Theta(\log n)$

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def exp(b, n):
    if n == 0:
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    return b * exp(b, n-1)
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```
def square(x):
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def fast_exp(b, n):
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        return square(fast_exp(b, n//2))
    else:
        return b * fast_exp(b, n-1)
```

Comparing orders of growth (n is the problem size)

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$\Theta(b^n)$

Comparing orders of growth (n is the problem size)

$\Theta(b^n)$ Exponential growth! Recursive fib takes

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$\Theta(n^2)$ Quadratic growth. E.g., operations on all pairs.

Incrementing n increases R(n) by the problem size n.

$\Theta(n)$ Linear growth. Resources scale with the problem.

$\Theta(\sqrt{n})$

$\Theta(\log n)$ Logarithmic growth. These processes scale well.

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