61A Lecture 21

Monday, October 15

Tree Recursion

Tree-shaped processes arise whenever executing the body of a function entails making **more than one** call to that function.

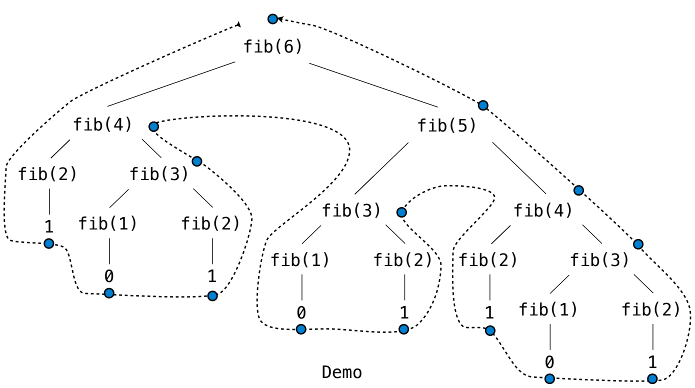
n:	1,	2,	3,	4,	5,	6,	7,	8,	9,	... ,	35
fib(n):	0,	1,	1,	2,	3,	5,	8,	13,	21,	... ,	5,702,887

```
def fib(n):
    if n == 1:
        return 0
    if n == 2:
        return 1
    return fib(n-2) + fib(n-1)
```

http://en.wikipedia.org/wiki/File:Fibonacci.jpg

A Tree-Recursive Process

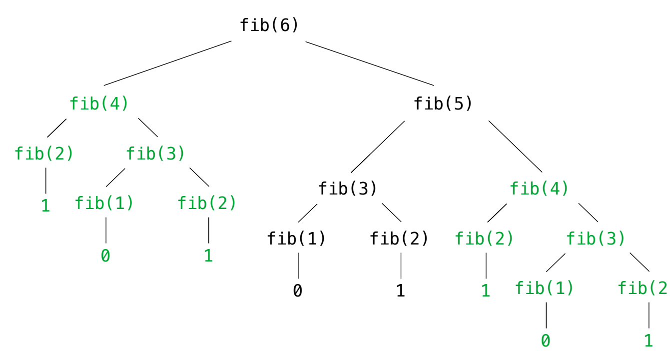
The computational process of fib evolves into a tree structure



Demo

Repetition in Tree-Recursive Computation

This process is highly repetitive; fib is called on the same argument multiple times



Memoization

Idea: Remember the results that have been computed before

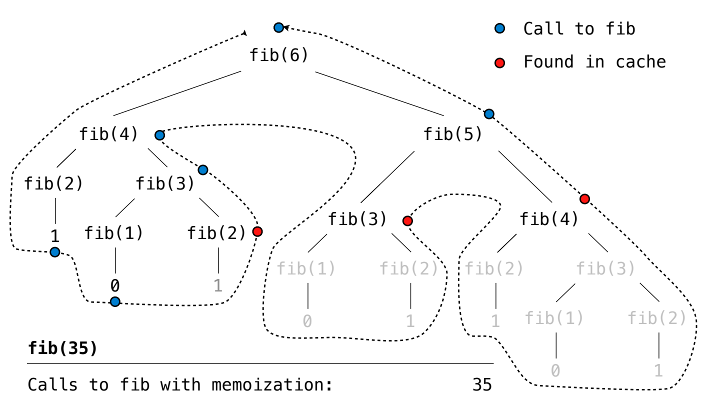
```
def memo(f):
    cache = {}
    def memoized(n):
        if n not in cache:
            cache[n] = f(n)
        return cache[n]
    return memoized
```

Keys are arguments that map to return values

Same behavior as f, if f is a pure function

Demo

Memoized Tree Recursion



fib(35)

Calls to fib with memoization:	35
Calls to fib without memoization:	18,454,929

Iteration vs Memoized Tree Recursion

Iterative and memoized implementations are not the same.

The first Fibonacci number

```
def fib_iter(n):
    prev, curr = 1, 0
    for _ in range(n-1):
        prev, curr = curr, prev + curr
    return curr
```

```
@memo
def fib(n):
    if n == 1:
        return 0
    if n == 2:
        return 1
    return fib(n-2) + fib(n-1)
```

Time	Space
n steps	3 names
n steps	n entries

3 names: Independent of problem size

n entries: Scales with problem size

Counting Change

$1 = $0.50 + $0.25 + $0.10 + $0.10 + $0.05

$1 = 1 half dollar, 1 quarter, 2 dimes, 1 nickel

$1 = 2 quarters, 2 dimes, 30 pennies

$1 = 100 pennies

How many ways are there to change a dollar?

How many ways to change $0.11 with nickels & pennies?

$0.11 can be changed with nickels & pennies by

A. Not using any more nickels; $0.11 with just pennies

B. Using at least one nickel; $0.06 with nickels & pennies

Counting Change Recursively

How many ways are there to change a dollar?

The number of ways to change an amount **a** using **n** kinds =

- The number of ways to change **a** using all but the first kind

+

- The number of ways to change **(a - d)** using all **n** kinds, where **d** is the denomination of the first kind of coin.

```
def count_change(a, kinds=(50, 25, 10, 5, 1)):

    <base cases>

    d = kinds[0]
    return count_change(a, kinds[1:]) + count_change(a-d, kinds)
```

Demo