61A Lecture 20

Friday, October 12

What Are Programs?

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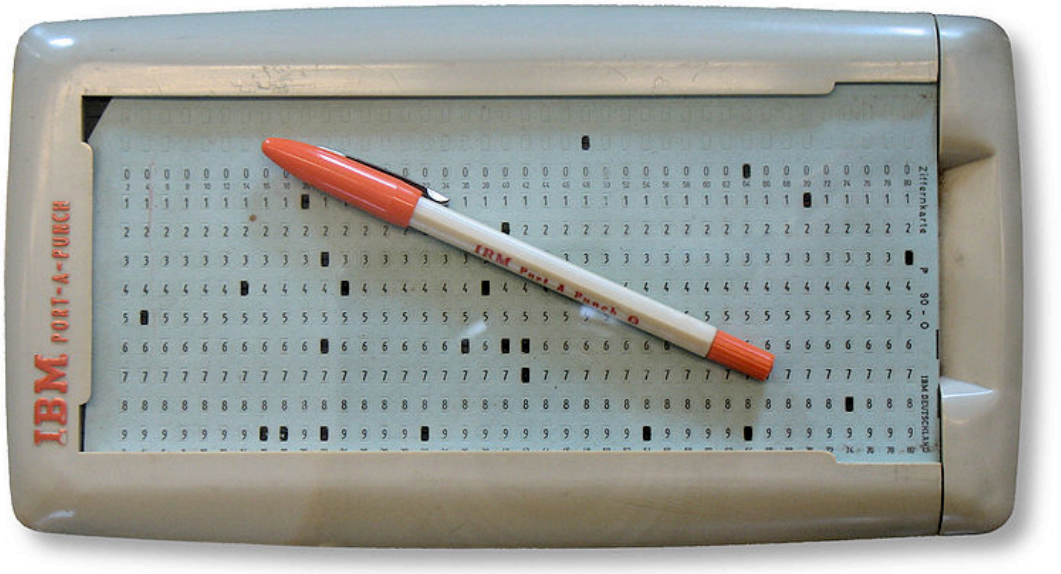

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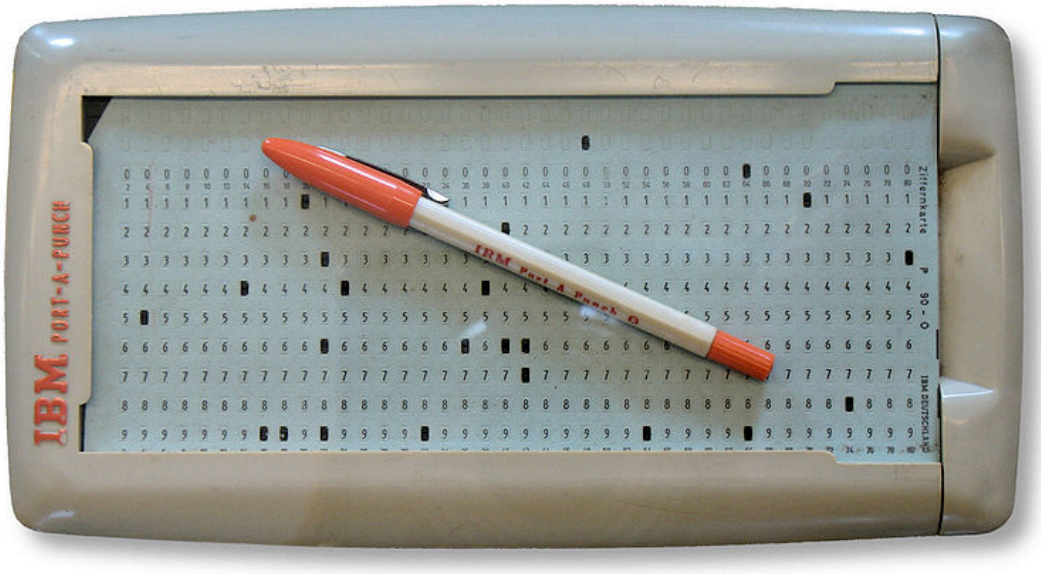

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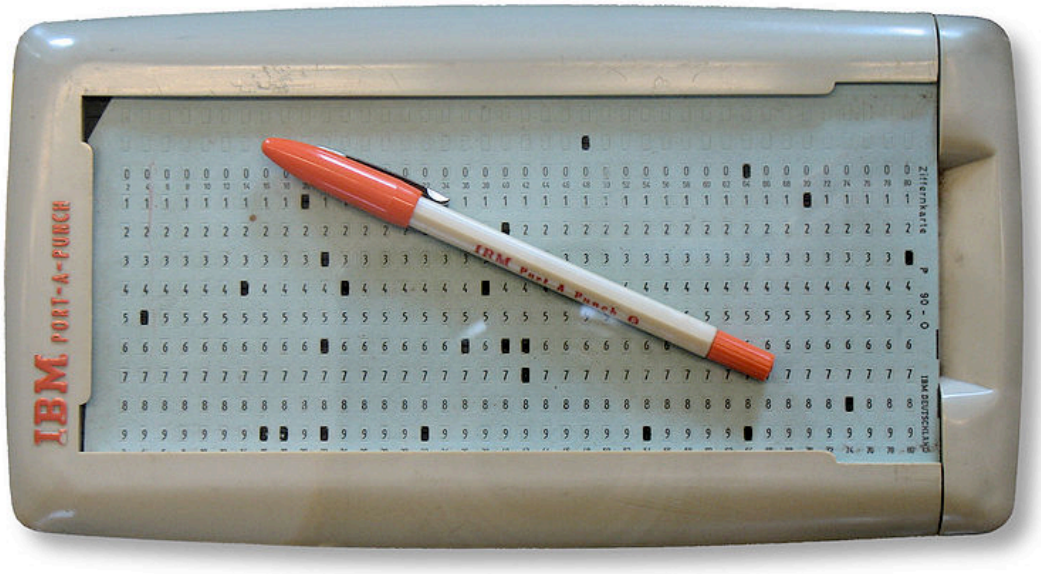

Now, we type programs as text files using editors like Emacs

Programs are just text (or cards) until we interpret them

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How Are Evaluation Procedures Applied?

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Evaluation rule for call expressions:

1. Evaluate the operator and operand subexpressions.
2. Apply the function that is the value of the operator subexpression to the arguments that are the values of the operand subexpressions.

Applying user-defined functions:

1. Create a new local frame that extends the environment with which the function is associated.
2. Bind the arguments to the function's formal parameter names in that frame.
3. Execute the body of the function in the environment beginning at that frame.

Execution rule for def statements:

1. Create a new function value with the specified name, formal parameters, and function body.
2. Associate that function with the current environment.
3. Bind the name of the function to the function value in the first frame of the current environment.

Execution rule for assignment statements:

1. Evaluate the expression(s) on the right of the equal sign.
2. Simultaneously bind the names on the left to those values in the first frame of the current environment.

Execution rule for conditional statements:

Each clause is considered in order.

1. Evaluate the header's expression.
2. If it is a true value, execute the suite, then skip the remaining clauses in the statement.

Evaluation rule for or expressions:

1. Evaluate the subexpression <left>.
2. If the result is a true value v, then the expression evaluates to v.
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Evaluation rule for and expressions:

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Evaluation rule for not expressions:

1. Evaluate <exp>; The value is True if the result is a false value, and False otherwise.

Execution rule for while statements:

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The most fundamental idea in computer science:

An *interpreter*, which determines the meaning of expressions in a programming language, is just another program.

Recursive Functions

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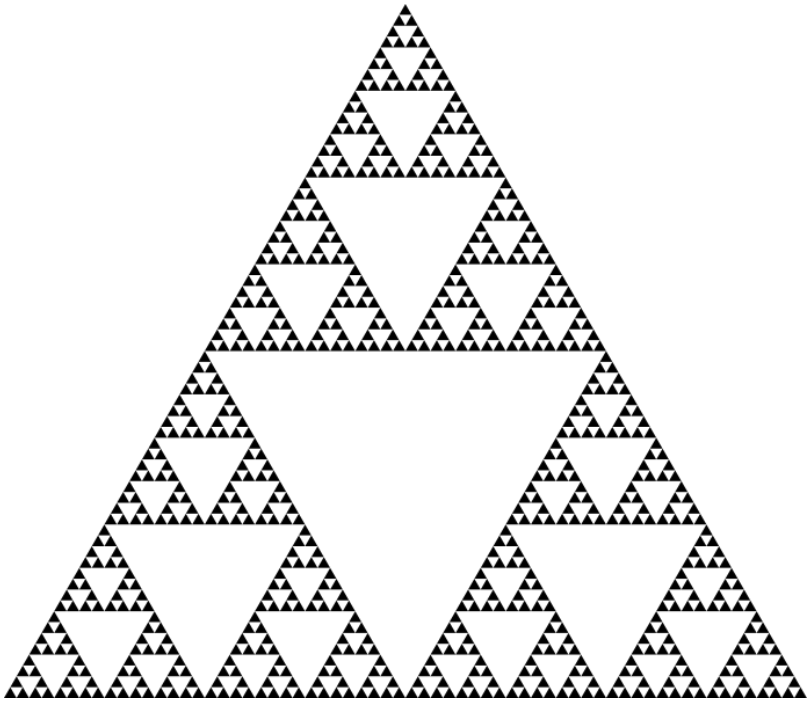

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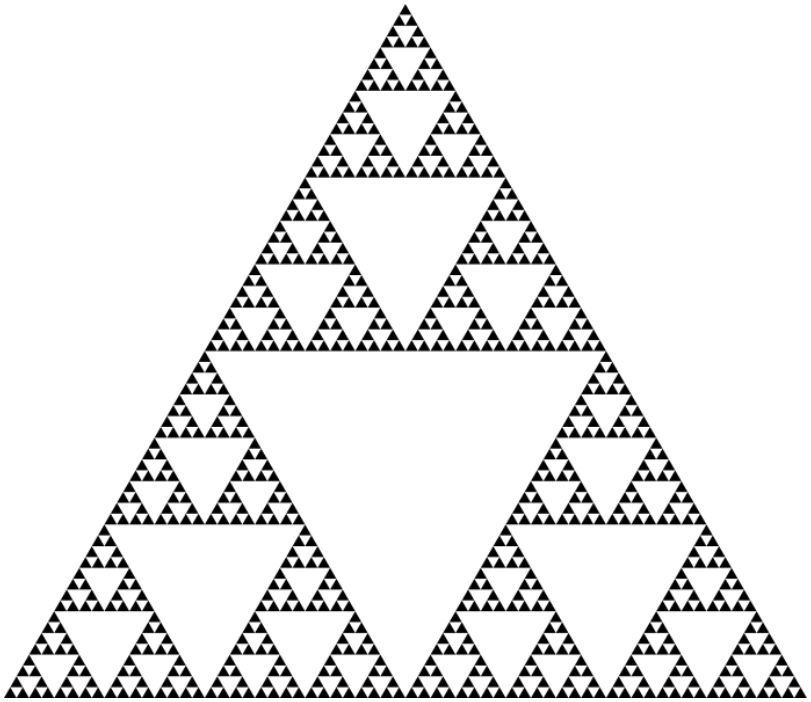

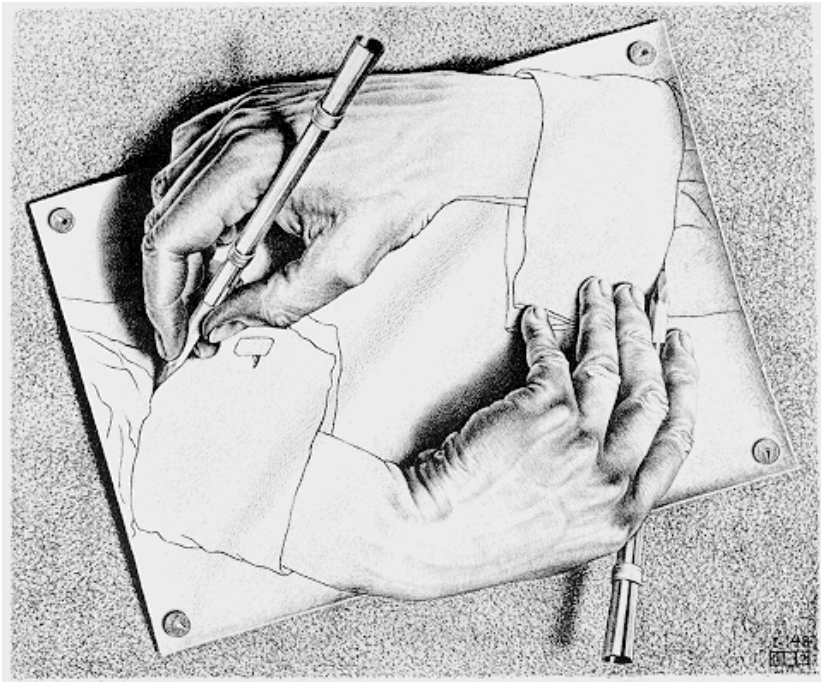

Drawing Hands, by M. C. Escher (lithograph, 1948)

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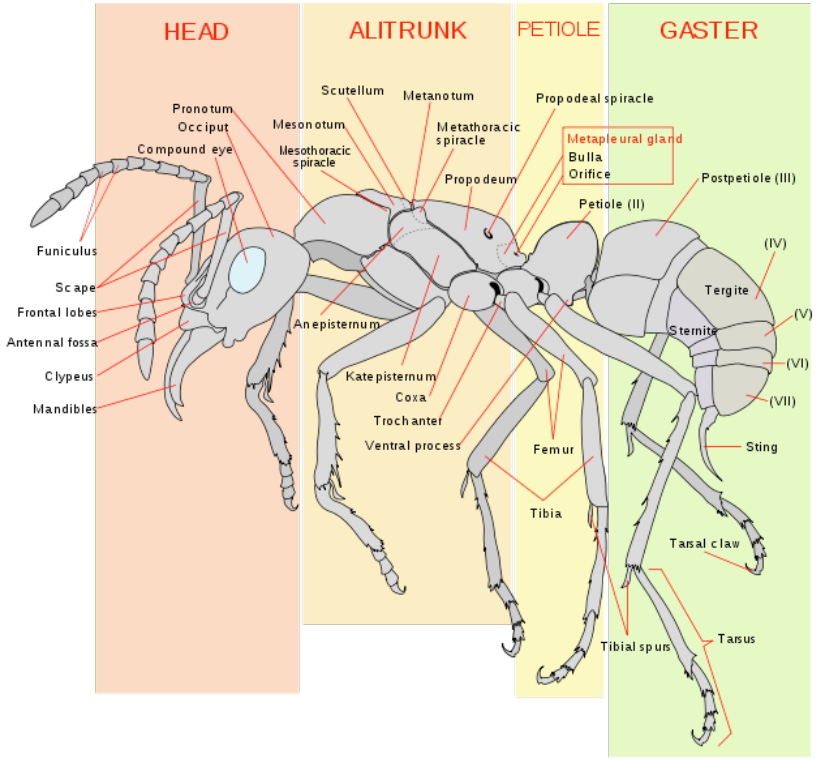

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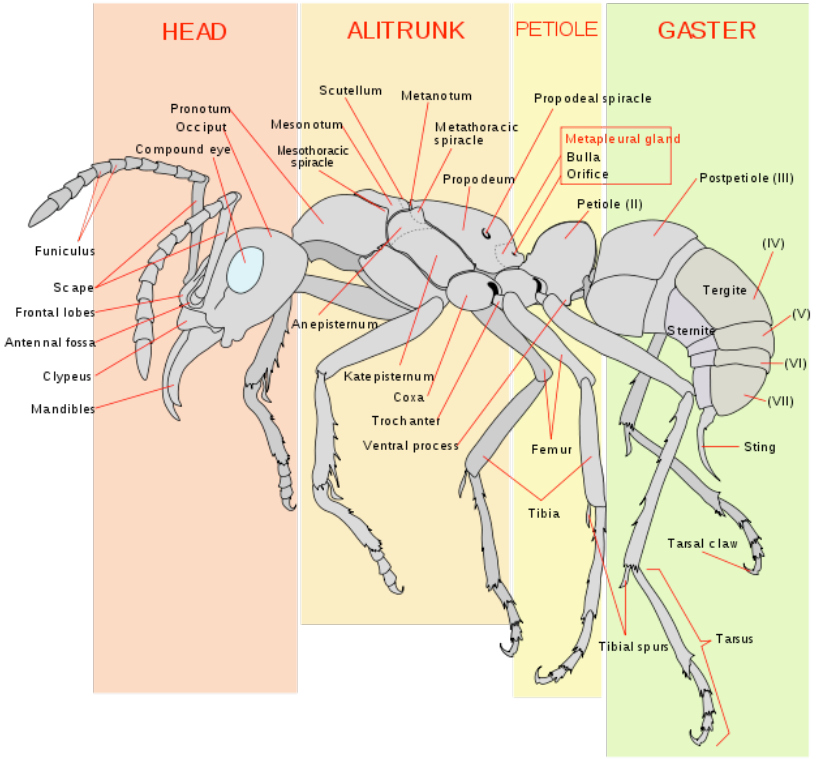


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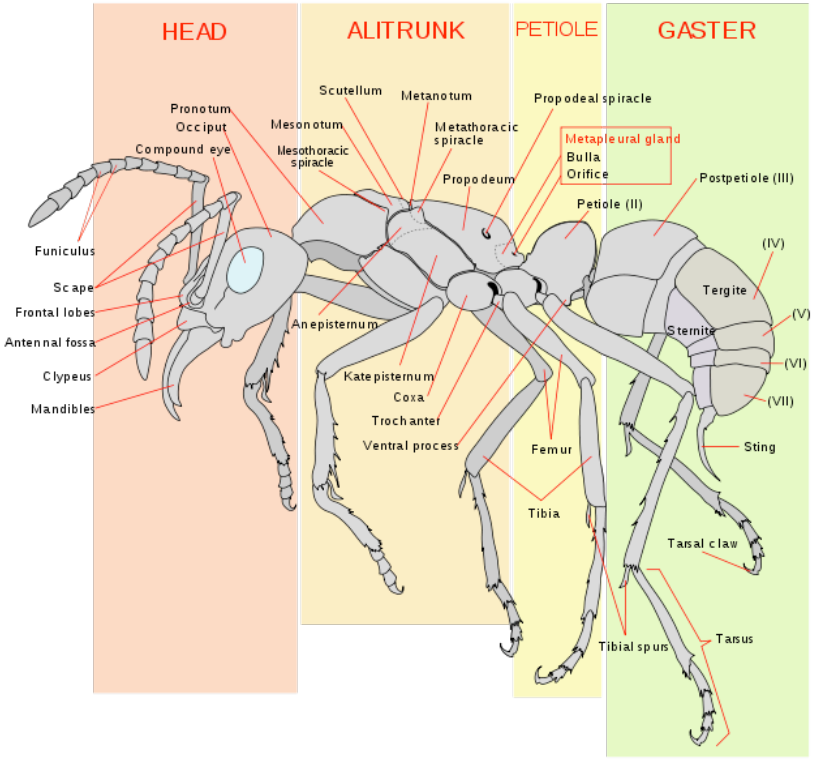


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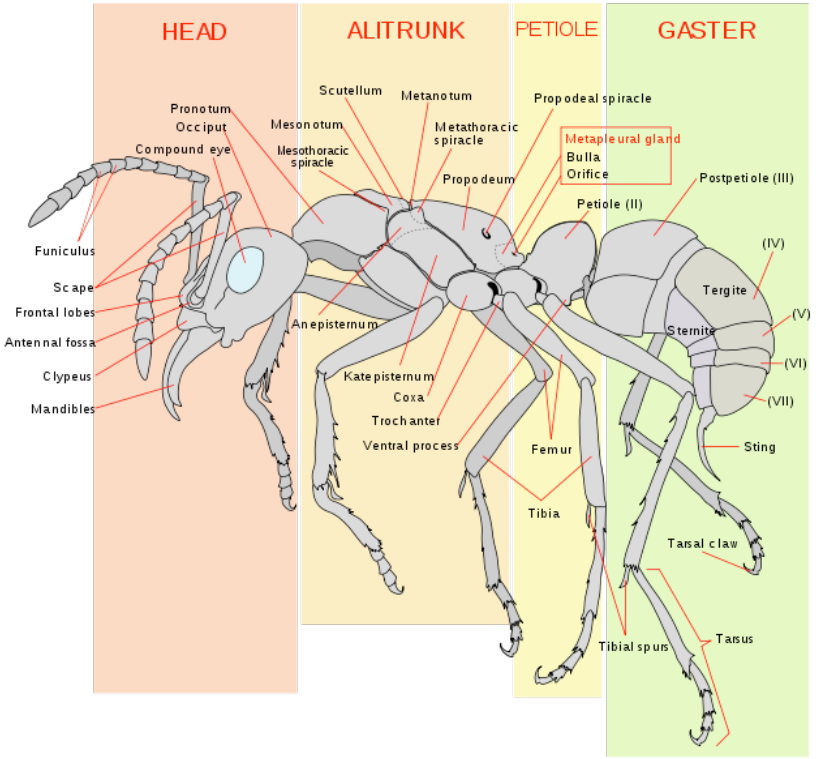


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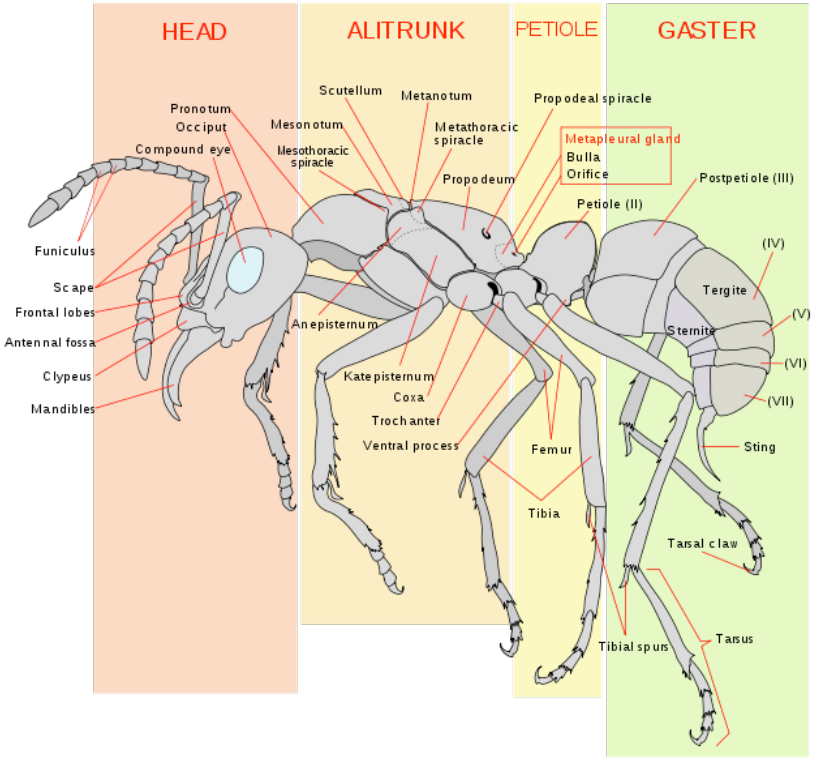


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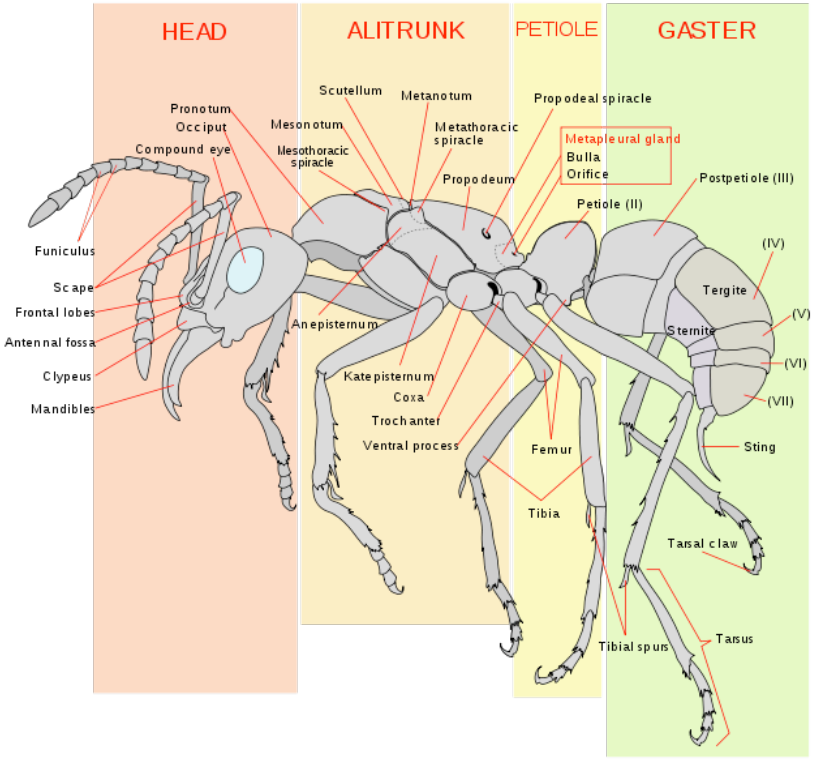


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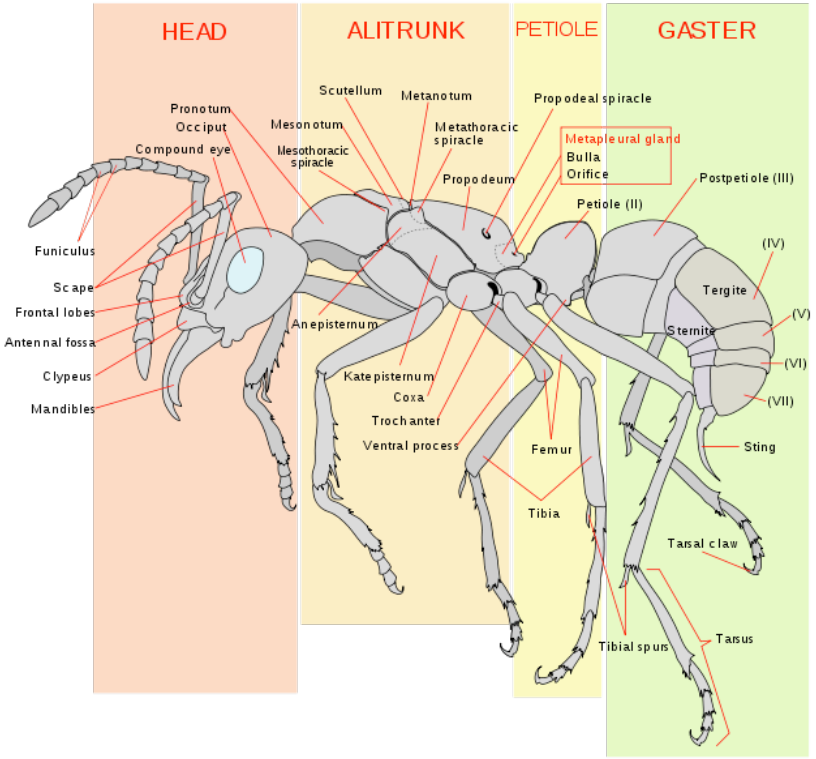


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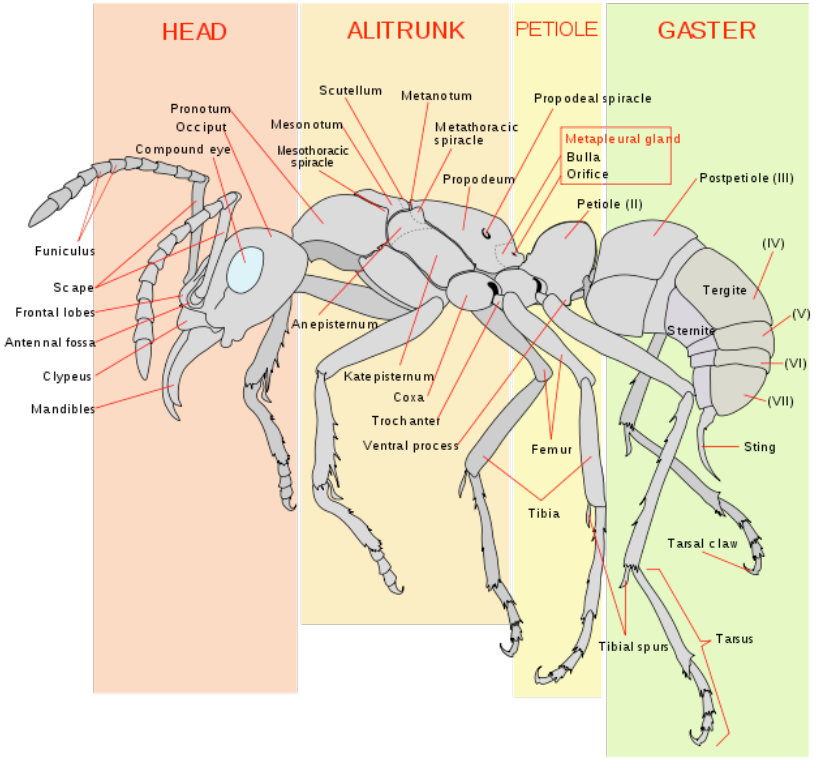


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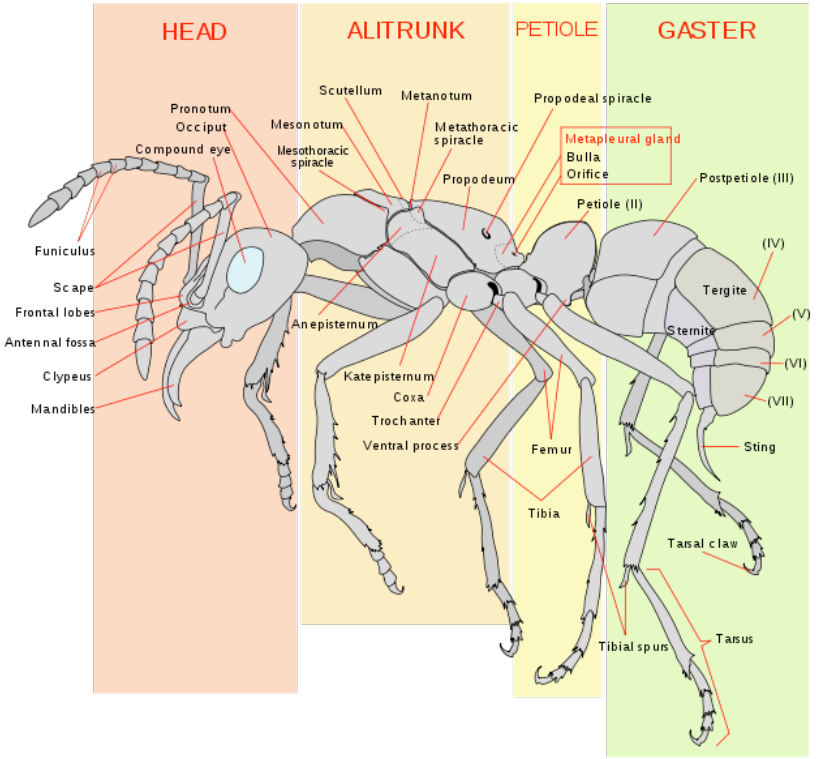


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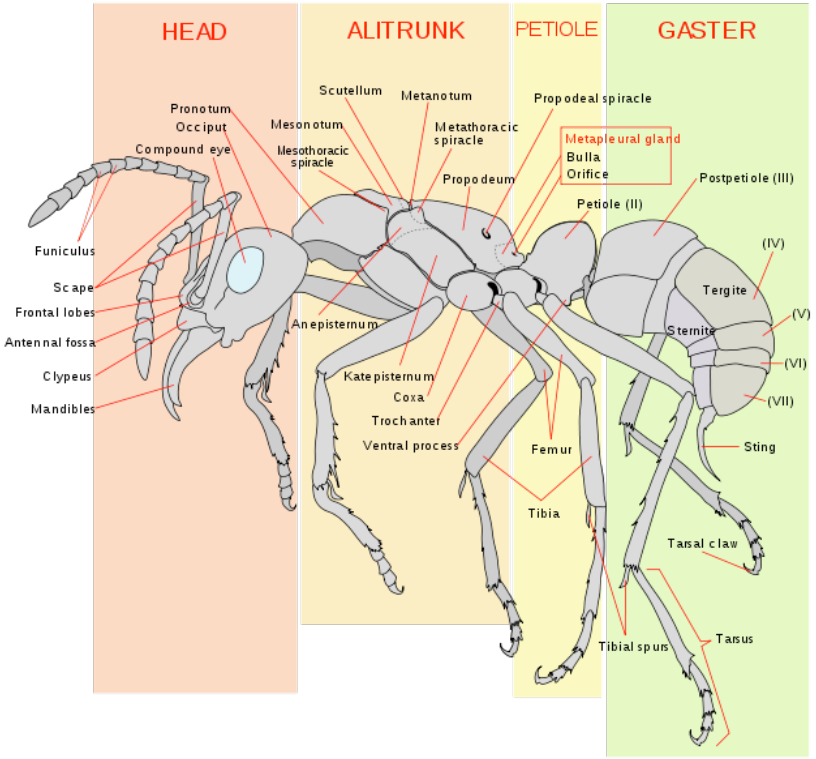


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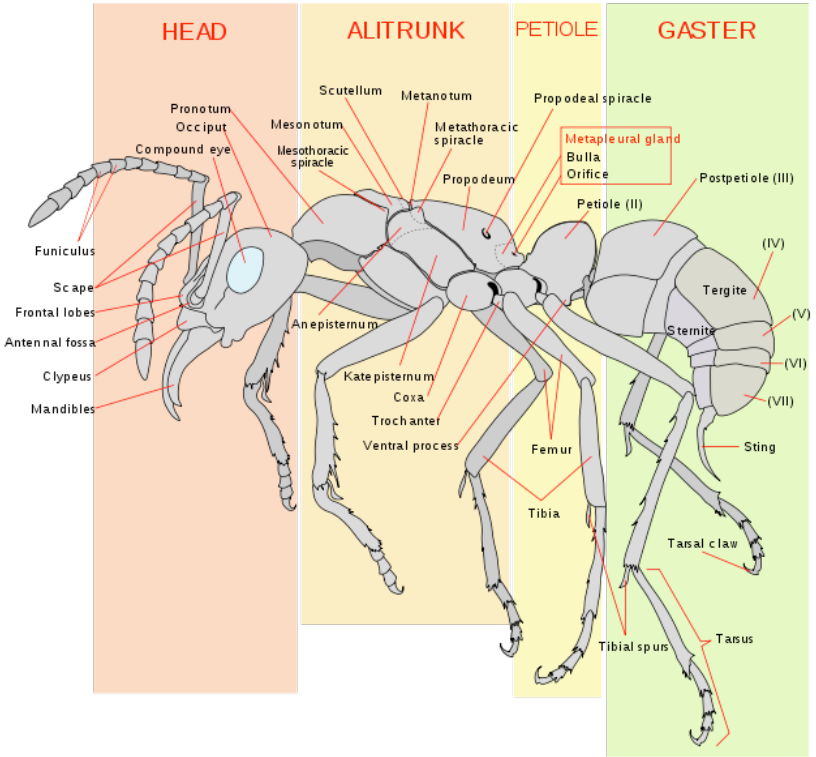


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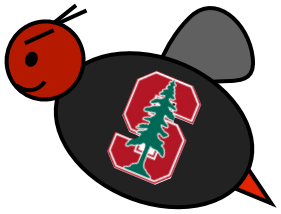

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The Recursive Leap of Faith

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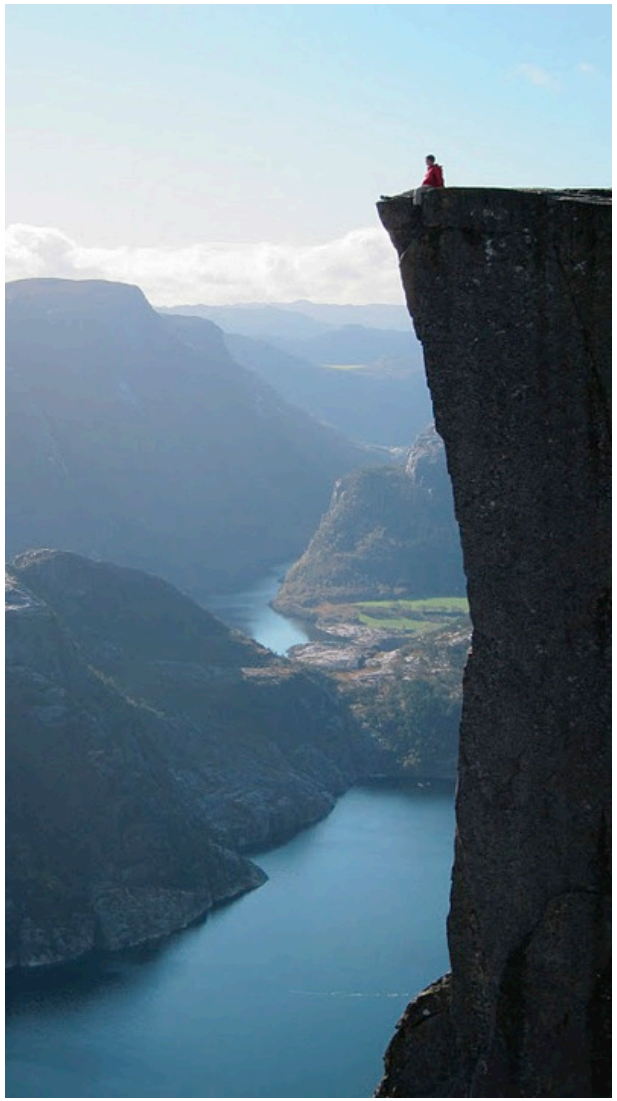

Photo by Kevin Lee, Preikestolen, Norway

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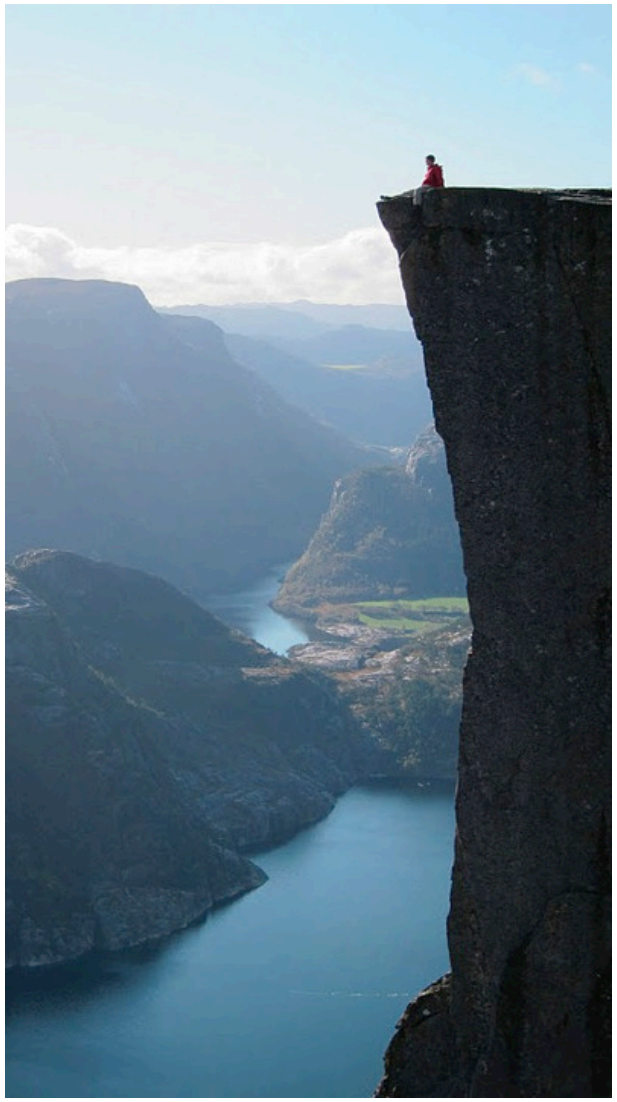

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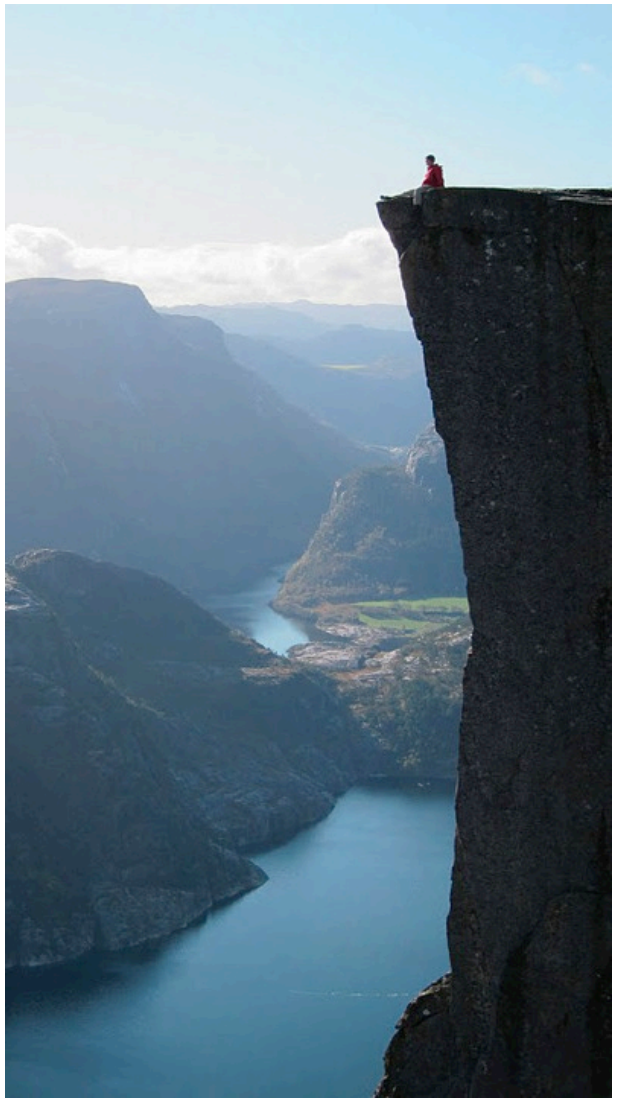

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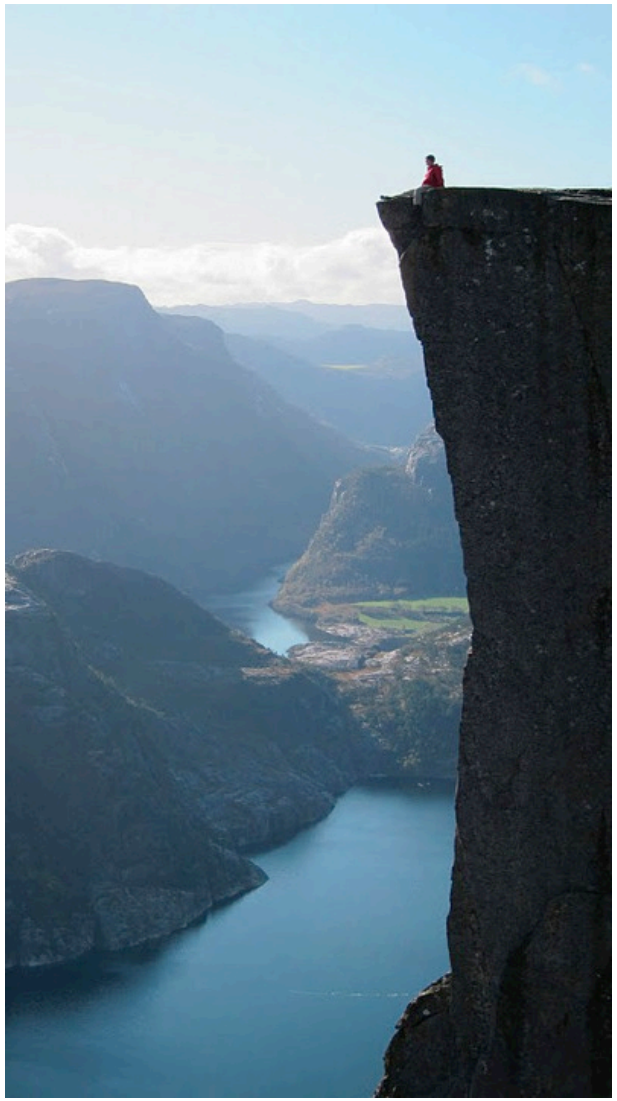

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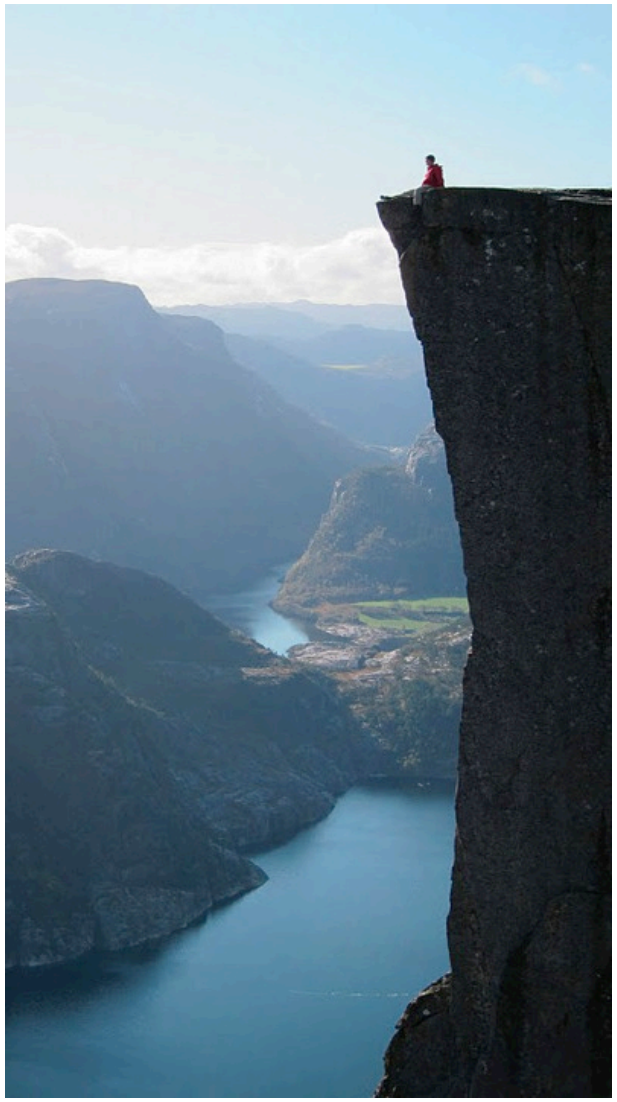

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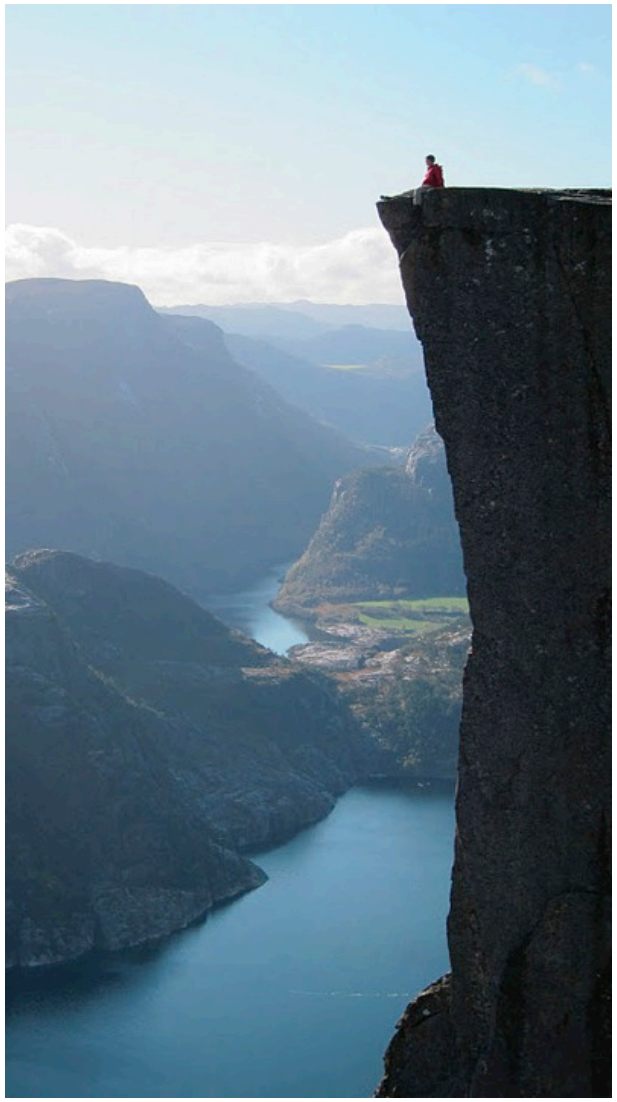

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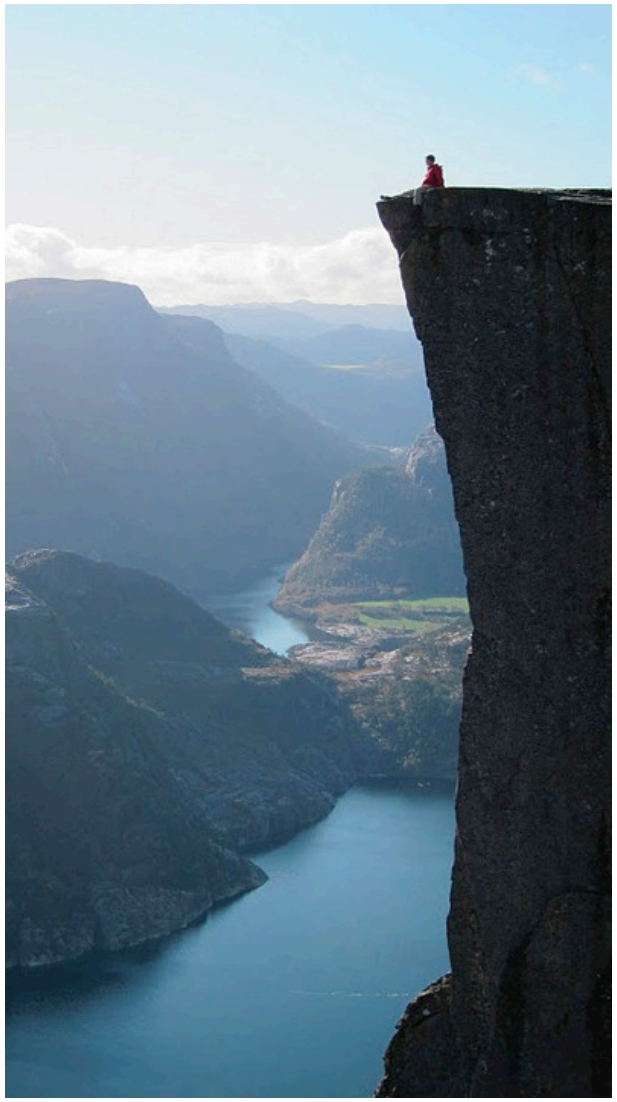
4. Verify that fact(n) is correct, assuming that fact(n-1) correct.



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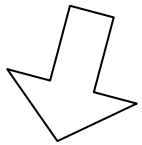

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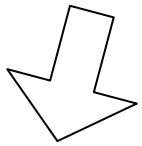

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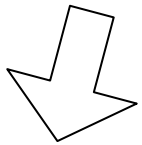

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```
reverse(s[1:]) + s[0]
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Base Case: The reverse of an empty string is itself.

Converting Recursion to Iteration

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def reverse_iter(s):
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Arguments to a recursive call