61A Lecture 19

Wednesday, October 10

Generic Functions, Continued

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What's different? Today's generic functions apply to multiple arguments that don't share a common interface

Rational Numbers

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`Rational numbers represented as a numerator and denominator`

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    def __init__(self, numer, denom):
        g = gcd(numer, denom)
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Greatest common divisor

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    def __repr__(self):
        return 'Rational({0}, {1})'.format(self.numer, self.denom)
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def add_rational(x, y):
    nx, dx = x.numer, x.denom
    ny, dy = y.numer, y.denom
    return Rational(nx * dy + ny * dx, dx * dy)
```

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def add_rational(x, y):
    nx, dx = x.numer, x.denom
    ny, dy = y.numer, y.denom
    return Rational(nx * dy + ny * dx, dx * dy)

def mul_rational(x, y):
    return Rational(x.numer * y.numer, x.denom * y.denom)
```

Greatest common divisor

Complex Numbers: the Rectangular Representation

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```
class ComplexRI(object):

    def __init__(self, real, imag):
        self.real = real
        self.imag = imag

    @property
    def magnitude(self):
        return (self.real ** 2 + self.imag ** 2) ** 0.5

    @property
    def angle(self):
        return atan2(self.imag, self.real)

    def __repr__(self):
        return 'ComplexRI({0}, {1})'.format(self.real,
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```
def add_complex(z1, z2):
     return ComplexRI(z1.real + z2.real,
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                                            self.imag)
```

Might be either ComplexMA
or ComplexRI instances

```
def add_complex(z1, z2):
    return ComplexRI(z1.real + z2.real,
                     z1.imag + z2.imag)
```

Special Methods

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Adding instances of user-defined classes with `__add__`.

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ComplexRI(3.0, 2.0)
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>>> ComplexRI(0, 1) * ComplexRI(0, 1)
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http://getpython3.com/diveintopython3/special-method-names.html

http://docs.python.org/py3k/reference/datamodel.html#special-method-names

The Independence of Data Types

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Data abstraction and class definitions keep types separate

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`add_rational  mul_rational`

Rational numbers as numerators & denominators

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add_complex mul_complex

add_rational mul_rational

Complex numbers as two-dimensional vectors

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How do we add a complex number and a rational number together?

`add_rational` `mul_rational`

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How do we add a complex number and a rational number together?

add_rational mul_rational

Rational numbers as numerators & denominators

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Complex numbers as two-dimensional vectors

There are many different techniques for doing this!

Type Dispatching

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Define a different function for each possible combination of types for which an operation (e.g., addition) is valid

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def iscomplex(z):
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def add_complex_and_rational(z, r):
    return ComplexRI(z.real + r.numer/r.denom, z.imag)
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Tag-Based Type Dispatching

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def type_tag(x):
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type_tag.tags = {ComplexRI: 'com',
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def add(z1, z2):
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def add(z1, z2):
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add.implementations = {}
add.implementations[('com', 'com')] = add_complex
add.implementations[('rat', 'rat')] = add_rational
add.implementations[('com', 'rat')] = add_complex_and_rational
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```
lambda r, z: add_complex_and_rational(z, r)
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add, subtract, multiply, divide

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Arg 1	**Arg 2**	**Add**	**Multiply**
Complex	Complex		
Rational	Rational		
Complex	Rational		
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Type Dispatching

Message Passing

Data-Directed Programming

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```
def apply(operator_name, x, y):
    tags = (type_tag(x), type_tag(y))
    key = (operator_name, tags)
    return apply.implementations[key](x, y)
```

Data-Directed Programming

There's nothing addition-specific about add_by_type

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>>> def rational_to_complex(x):
        return ComplexRI(x.numer/x.denom, 0)
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>>> coercions = {('rat', 'com'): rational_to_complex}
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Question: Have we been repeating ourselves with data-directed programming?

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    if tx != ty:
        if (tx, ty) in coercions:
            tx, x = ty, coercions[(tx, ty)](x)
```

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1. Attempt to coerce arguments into values of the same type

2. Apply type-specific (not cross-type) operations

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Demo

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