

# 61A Lecture 14

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Friday, September 28

# Testing for Identity

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Demo

# Implementing Dice

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## Implementing Dice

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Random numbers are useful for experimentation

## Implementing Dice

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Random numbers are useful for experimentation

They also appear in lots of algorithms, e.g.,

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- Primality tests

## Implementing Dice

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Random numbers are useful for experimentation

They also appear in lots of algorithms, e.g.,

- Primality tests
- Machine learning techniques

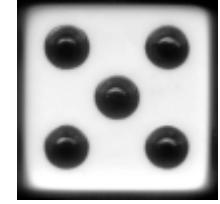
## Implementing Dice

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- Primality tests
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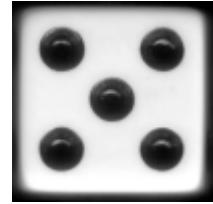
# Implementing Dice

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Random numbers are useful for experimentation

They also appear in lots of algorithms, e.g.,

- Primality tests
- Machine learning techniques



```
def make_dice(sides=6):
    seed = 1
    multiplier = pow(7, 5)
    big_prime = pow(2, 31) - 1
    def dice():
        nonlocal seed
        seed = (multiplier * seed) % big_prime
        return (sides*seed) // big_prime + 1
    return dice
```

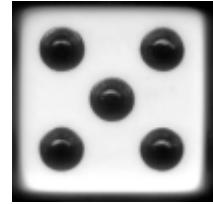
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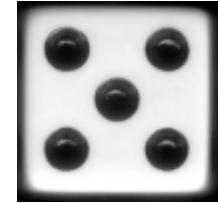
```
def make_dice(sides=6):    16807
    seed = 1
    multiplier = pow(7, 5)
    big_prime = pow(2, 31) - 1
    def dice():
        nonlocal seed
        seed = (multiplier * seed) % big_prime
        return (sides*seed) // big_prime + 1
    return dice
```

# Implementing Dice

Random numbers are useful for experimentation

They also appear in lots of algorithms, e.g.,

- Primality tests
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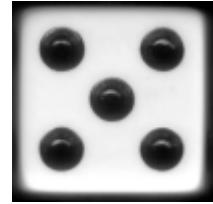
```
def make_dice(sides=6):    16807
    seed = 1
    multiplier = pow(7, 5)
    big_prime = pow(2, 31) - 1 2147483647
    def dice():
        nonlocal seed
        seed = (multiplier * seed) % big_prime
        return (sides*seed) // big_prime + 1
    return dice
```

# Implementing Dice

Random numbers are useful for experimentation

They also appear in lots of algorithms, e.g.,

- Primality tests
- Machine learning techniques

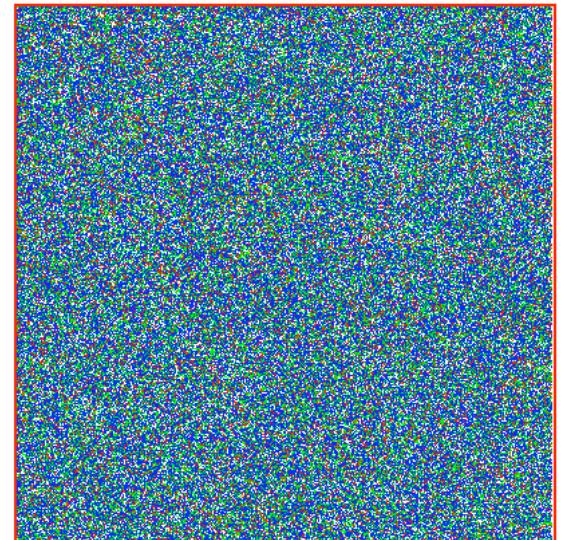


```
def make_dice(sides=6):
    seed = 1
    multiplier = pow(7, 5)
    big_prime = pow(2, 31) - 1
    def dice():
        nonlocal seed
        seed = (multiplier * seed) % big_prime
        return (sides*seed) // big_prime + 1
    return dice
```

16807

2147483647

P1 = 16807, P2 = 0, N = 2147483647



100000 dots drawn, seed = 1

<http://www.math.utah.edu/~pa/Random/Random.html>

S.K. Park and K.W. Miller, " Random Number Generators: Good Ones Are Hard To Find", Communications of the ACM, October 1988, pp. 1192-1201.

# Implementing a Mutable Container Object

---

Demo

## Dispatch Functions

---

A technique for packing multiple behaviors into one function

## Dispatch Functions

---

A technique for packing multiple behaviors into one function

```
def pair(x, y):
    """Return a function that behaves like a pair."""
    def dispatch(m):
        if m == 0:
            return x
        elif m == 1:
            return y
    return dispatch
```

# Dispatch Functions

---

A technique for packing multiple behaviors into one function

```
def pair(x, y):
    """Return a function that behaves like a pair."""
    def dispatch(m):
        if m == 0:
            return x
        elif m == 1:
            return y
    return dispatch
```

Message argument can be anything, but strings are most common

# Dispatch Functions

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A technique for packing multiple behaviors into one function

```
def pair(x, y):
    """Return a function that behaves like a pair."""
    def dispatch(m):
        if m == 0:
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            return y
    return dispatch
```

Message argument can be anything, but strings are most common

The body of a dispatch function is always the same:

# Dispatch Functions

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A technique for packing multiple behaviors into one function

```
def pair(x, y):
    """Return a function that behaves like a pair."""
    def dispatch(m):
        if m == 0:
            return x
        elif m == 1:
            return y
    return dispatch
```

Message argument can be anything, but strings are most common

The body of a dispatch function is always the same:

- One conditional statement with several clauses

# Dispatch Functions

---

A technique for packing multiple behaviors into one function

```
def pair(x, y):
    """Return a function that behaves like a pair."""
    def dispatch(m):
        if m == 0:
            return x
        elif m == 1:
            return y
    return dispatch
```

Message argument can be anything, but strings are most common

The body of a dispatch function is always the same:

- One conditional statement with several clauses
- Headers perform equality tests on the message

# Message Passing

---



# Message Passing

---

An approach to organizing the relationship among different pieces of a program



# Message Passing

---

An approach to organizing the relationship among different pieces of a program

Different objects pass messages to each other



# Message Passing

---

An approach to organizing the relationship among different pieces of a program

Different objects pass messages to each other

- What is your fourth element?



# Message Passing

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An approach to organizing the relationship among different pieces of a program

Different objects pass messages to each other

- What is your fourth element?
- Change your third element to this new value. (please?)



# Message Passing

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An approach to organizing the relationship among different pieces of a program

Different objects pass messages to each other

- What is your fourth element?
- Change your third element to this new value. (please?)

Encapsulates the behavior of all operations on a piece of data



# Message Passing

---

An approach to organizing the relationship among different pieces of a program

Different objects pass messages to each other

- What is your fourth element?
- Change your third element to this new value. (please?)

Encapsulates the behavior of all operations on a piece of data

Important historical role:  
The message passing approach strongly influenced object-oriented programming  
(next lecture)



# A Mutable Container That Uses Message Passing

---

## A Mutable Container That Uses Message Passing

---

```
def container_dispatch(contents):
```

## A Mutable Container That Uses Message Passing

---

```
def container_dispatch(contents):  
  
    def dispatch(message, value=None):
```

## A Mutable Container That Uses Message Passing

---

```
def container_dispatch(contents):  
  
    def dispatch(message, value=None):  
  
        nonlocal contents
```

## A Mutable Container That Uses Message Passing

---

```
def container_dispatch(contents):  
  
    def dispatch(message, value=None):  
  
        nonlocal contents  
  
        if message == 'get':
```

## A Mutable Container That Uses Message Passing

---

```
def container_dispatch(contents):

    def dispatch(message, value=None):

        nonlocal contents

        if message == 'get':

            return contents
```

## A Mutable Container That Uses Message Passing

---

```
def container_dispatch(contents):

    def dispatch(message, value=None):

        nonlocal contents

        if message == 'get':

            return contents

        if message == 'put':
```

## A Mutable Container That Uses Message Passing

---

```
def container_dispatch(contents):

    def dispatch(message, value=None):

        nonlocal contents

        if message == 'get':

            return contents

        if message == 'put':

            contents = value
```

## A Mutable Container That Uses Message Passing

---

```
def container_dispatch(contents):

    def dispatch(message, value=None):

        nonlocal contents

        if message == 'get':

            return contents

        if message == 'put':

            contents = value

    return dispatch
```

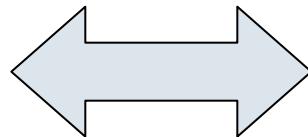
# A Mutable Container That Uses Message Passing

---

```
def container_dispatch(contents):           def container(contents):  
  
    def dispatch(message, value=None):  
  
        nonlocal contents  
  
        if message == 'get':  
  
            return contents  
  
        if message == 'put':  
  
            contents = value  
  
    return dispatch  
  
def container(contents):  
  
    def get():  
  
        return contents  
  
    def put(value):  
  
        nonlocal contents  
  
        contents = value  
  
    return get, put
```

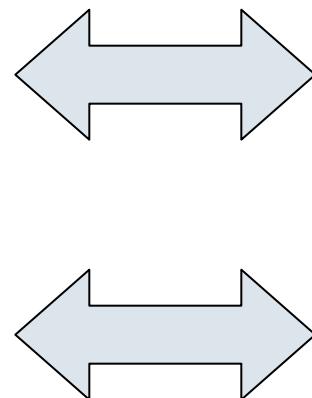
# A Mutable Container That Uses Message Passing

```
def container_dispatch(contents):           def container(contents):  
  
    def dispatch(message, value=None):  
  
        nonlocal contents  
  
        if message == 'get':  
            return contents  
  
        if message == 'put':  
            contents = value  
  
    return dispatch  
  
def container(contents):  
  
    def get():  
        return contents  
  
    def put(value):  
        nonlocal contents  
        contents = value  
  
    return get, put
```



# A Mutable Container That Uses Message Passing

```
def container_dispatch(contents):  
  
    def dispatch(message, value=None):  
  
        nonlocal contents  
  
        if message == 'get':  
  
            return contents  
  
        if message == 'put':  
  
            contents = value  
  
    return dispatch
```



```
def container(contents):  
  
    def get():  
  
        return contents  
  
    def put(value):  
  
        nonlocal contents  
  
        contents = value  
  
    return get, put
```

# A Mutable Container That Uses Message Passing

```
def container_dispatch(contents):  
  
    def dispatch(message, value=None):  
  
        nonlocal contents  
  
        if message == 'get':  
  
            return contents  
  
        if message == 'put':  
  
            contents = value  
  
    return dispatch
```

```
def container(contents):  
  
    def get():  
  
        return contents  
  
    def put(value):  
  
        nonlocal contents  
  
        contents = value  
  
    return get, put
```

Demo

# Implementing Mutable Recursive Lists

---

# Implementing Mutable Recursive Lists

---

Recursive List  
Refresher Demo

# Implementing Mutable Recursive Lists

---

```
def mutable_rlist():
```

Recursive List  
Refresher Demo

# Implementing Mutable Recursive Lists

---

```
def mutable_rlist():
    contents = empty_rlist
```

Recursive List  
Refresher Demo

# Implementing Mutable Recursive Lists

---

```
def mutable_rlist():
    contents = empty_rlist
    def dispatch(message, value=None):
```

Recursive List  
Refresher Demo

# Implementing Mutable Recursive Lists

---

```
def mutable_rlist():
    contents = empty_rlist
    def dispatch(message, value=None):
        nonlocal contents
```

Recursive List  
Refresher Demo

# Implementing Mutable Recursive Lists

---

```
def mutable_rlist():
    contents = empty_rlist
    def dispatch(message, value=None):
        nonlocal contents
        if message == 'len':
```

Recursive List  
Refresher Demo

# Implementing Mutable Recursive Lists

---

```
def mutable_rlist():
    contents = empty_rlist
    def dispatch(message, value=None):
        nonlocal contents
        if message == 'len':
            return len_rlist(contents)
```

Recursive List  
Refresher Demo

# Implementing Mutable Recursive Lists

---

```
def mutable_rlist():
    contents = empty_rlist
    def dispatch(message, value=None):
        nonlocal contents
        if message == 'len':
            return len_rlist(contents)
        elif message == 'getitem':
```

Recursive List  
Refresher Demo

# Implementing Mutable Recursive Lists

---

```
def mutable_rlist():
    contents = empty_rlist
    def dispatch(message, value=None):
        nonlocal contents
        if message == 'len':
            return len_rlist(contents)
        elif message == 'getitem':
            return getitem_rlist(contents, value)
```

Recursive List  
Refresher Demo

# Implementing Mutable Recursive Lists

---

```
def mutable_rlist():
    contents = empty_rlist
    def dispatch(message, value=None):
        nonlocal contents
        if message == 'len':
            return len_rlist(contents)
        elif message == 'getitem':
            return getitem_rlist(contents, value)
        elif message == 'push_first':
```

Recursive List  
Refresher Demo

# Implementing Mutable Recursive Lists

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```
def mutable_rlist():
    contents = empty_rlist
    def dispatch(message, value=None):
        nonlocal contents
        if message == 'len':
            return len_rlist(contents)
        elif message == 'getitem':
            return getitem_rlist(contents, value)
        elif message == 'push_first':
            contents = make_rlist(value, contents)
```

Recursive List  
Refresher Demo

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def mutable_rlist():
    contents = empty_rlist
    def dispatch(message, value=None):
        nonlocal contents
        if message == 'len':
            return len_rlist(contents)
        elif message == 'getitem':
            return getitem_rlist(contents, value)
        elif message == 'push_first':
            contents = make_rlist(value, contents)
        elif message == 'pop_first':
```

Recursive List  
Refresher Demo

# Implementing Mutable Recursive Lists

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```
def mutable_rlist():
    contents = empty_rlist
    def dispatch(message, value=None):
        nonlocal contents
        if message == 'len':
            return len_rlist(contents)
        elif message == 'getitem':
            return getitem_rlist(contents, value)
        elif message == 'push_first':
            contents = make_rlist(value, contents)
        elif message == 'pop_first':
            f = first(contents)
            contents = rest(contents)
            return f
        else:
            raise ValueError("Unknown message type")
    return dispatch
```

Recursive List  
Refresher Demo

# Implementing Mutable Recursive Lists

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def mutable_rlist():
    contents = empty_rlist
    def dispatch(message, value=None):
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        if message == 'len':
            return len_rlist(contents)
        elif message == 'getitem':
            return getitem_rlist(contents, value)
        elif message == 'push_first':
            contents = make_rlist(value, contents)
        elif message == 'pop_first':
            f = first(contents)
            contents = rest(contents)
```

Recursive List  
Refresher Demo

# Implementing Mutable Recursive Lists

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            return getitem_rlist(contents, value)
        elif message == 'push_first':
            contents = make_rlist(value, contents)
        elif message == 'pop_first':
            f = first(contents)
            contents = rest(contents)
            return f
    return dispatch
```

Recursive List  
Refresher Demo

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def mutable_rlist():
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    def dispatch(message, value=None):
        nonlocal contents
        if message == 'len':
            return len_rlist(contents)
        elif message == 'getitem':
            return getitem_rlist(contents, value)
        elif message == 'push_first':
            contents = make_rlist(value, contents)
        elif message == 'pop_first':
            f = first(contents)
            contents = rest(contents)
            return f
        elif message == 'str':
```

Recursive List  
Refresher Demo

# Implementing Mutable Recursive Lists

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def mutable_rlist():
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    def dispatch(message, value=None):
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        if message == 'len':
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        elif message == 'getitem':
            return getitem_rlist(contents, value)
        elif message == 'push_first':
            contents = make_rlist(value, contents)
        elif message == 'pop_first':
            f = first(contents)
            contents = rest(contents)
            return f
        elif message == 'str':
            return str(contents)
```

Recursive List  
Refresher Demo

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    def dispatch(message, value=None):
        nonlocal contents
        if message == 'len':
            return len_rlist(contents)
        elif message == 'getitem':
            return getitem_rlist(contents, value)
        elif message == 'push_first':
            contents = make_rlist(value, contents)
        elif message == 'pop_first':
            f = first(contents)
            contents = rest(contents)
            return f
        elif message == 'str':
            return str(contents)
    return dispatch
```

Recursive List  
Refresher Demo

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            return getitem_rlist(contents, value)
        elif message == 'push_first':
            contents = make_rlist(value, contents)
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            f = first(contents)
            contents = rest(contents)
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    return dispatch
```

Recursive List  
Refresher Demo

Demo

# Implementing Dictionaries

---

# Implementing Dictionaries

---

```
def dictionary():
    """Return a functional implementation of a dictionary."""
```

# Implementing Dictionaries

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```
def dictionary():
    """Return a functional implementation of a dictionary."""
    records = []
```

# Implementing Dictionaries

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```
def dictionary():
    """Return a functional implementation of a dictionary."""
    records = []

    def getitem(key):
        for k, v in records:
            if k == key:
                return v
```

# Implementing Dictionaries

---

```
def dictionary():
    """Return a functional implementation of a dictionary."""
    records = []

    def getitem(key):
        for k, v in records:
            if k == key:
                return v

    def setitem(key, value):
        records.append((key, value))
```

# Implementing Dictionaries

---

```
def dictionary():
    """Return a functional implementation of a dictionary."""
    records = []

    def getitem(key):
        for k, v in records:
            if k == key:
                return v

    def setitem(key, value):
        for item in records:
            if item[0] == key:
                item[1] = value
        return
```

# Implementing Dictionaries

---

```
def dictionary():
    """Return a functional implementation of a dictionary."""
    records = []

    def getitem(key):
        for k, v in records:
            if k == key:
                return v

    def setitem(key, value):
        for item in records:
            if item[0] == key:
                item[1] = value
                return
        records.append([key, value])
```

# Implementing Dictionaries

---

```
def dictionary():
    """Return a functional implementation of a dictionary."""
    records = []

    def getitem(key):
        for k, v in records:
            if k == key:
                return v

    def setitem(key, value):
        for item in records:
            if item[0] == key:
                item[1] = value
                return
        records.append([key, value])

    def dispatch(message, key=None, value=None):
        if message == "getitem":
            return getitem(key)
        elif message == "setitem":
            return setitem(key, value)
        elif message == "dispatch":
```

# Implementing Dictionaries

---

```
def dictionary():
    """Return a functional implementation of a dictionary."""
    records = []

    def getitem(key):
        for k, v in records:
            if k == key:
                return v

    def setitem(key, value):
        for item in records:
            if item[0] == key:
                item[1] = value
                return
        records.append([key, value])

    def dispatch(message, key=None, value=None):
        if message == 'getitem':
            return getitem(key)
```

# Implementing Dictionaries

---

```
def dictionary():
    """Return a functional implementation of a dictionary."""
    records = []

    def getitem(key):
        for k, v in records:
            if k == key:
                return v

    def setitem(key, value):
        for item in records:
            if item[0] == key:
                item[1] = value
                return
        records.append([key, value])

    def dispatch(message, key=None, value=None):
        if message == 'getitem':
            return getitem(key)
        elif message == 'setitem':
            setitem(key, value)
```

# Implementing Dictionaries

---

```
def dictionary():
    """Return a functional implementation of a dictionary."""
    records = []

    def getitem(key):
        for k, v in records:
            if k == key:
                return v

    def setitem(key, value):
        for item in records:
            if item[0] == key:
                item[1] = value
                return
        records.append([key, value])

    def dispatch(message, key=None, value=None):
        if message == 'getitem':
            return getitem(key)
        elif message == 'setitem':
            setitem(key, value)
        elif message == 'keys':
            return tuple(k for k, _ in records)

    return dispatch
```

# Implementing Dictionaries

---

```
def dictionary():
    """Return a functional implementation of a dictionary."""
    records = []

    def getitem(key):
        for k, v in records:
            if k == key:
                return v

    def setitem(key, value):
        for item in records:
            if item[0] == key:
                item[1] = value
                return
        records.append([key, value])

    def dispatch(message, key=None, value=None):
        if message == 'getitem':
            return getitem(key)
        elif message == 'setitem':
            setitem(key, value)
        elif message == 'keys':
            return tuple(k for k, _ in records)
        elif message == 'values':
            return tuple(v for _, v in records)

    return dispatch
```

# Implementing Dictionaries

---

```
def dictionary():
    """Return a functional implementation of a dictionary."""
    records = []

    def getitem(key):
        for k, v in records:
            if k == key:
                return v

    def setitem(key, value):
        for item in records:
            if item[0] == key:
                item[1] = value
                return
        records.append([key, value])

    def dispatch(message, key=None, value=None):
        if message == 'getitem':
            return getitem(key)
        elif message == 'setitem':
            setitem(key, value)
        elif message == 'keys':
            return tuple(k for k, _ in records)
        elif message == 'values':
            return tuple(v for _, v in records)

    return dispatch
```

# Implementing Dictionaries

```
def dictionary():
    """Return a functional implementation of a dictionary."""
    records = []

    def getitem(key):
        for k, v in records:
            if k == key:
                return v

    def setitem(key, value):
        for item in records:
            if item[0] == key:
                item[1] = value
                return
        records.append([key, value])

    def dispatch(message, key=None, value=None):
        if message == 'getitem':
            return getitem(key)
        elif message == 'setitem':
            setitem(key, value)
        elif message == 'keys':
            return tuple(k for k, _ in records)
        elif message == 'values':
            return tuple(v for _, v in records)

    return dispatch
```

Question: Do we need a nonlocal statement here?

# Implementing Dictionaries

```
def dictionary():
    """Return a functional implementation of a dictionary."""
    records = []

    def getitem(key):
        for k, v in records:
            if k == key:
                return v

    def setitem(key, value):
        for item in records:
            if item[0] == key:
                item[1] = value
                return
        records.append([key, value])

    def dispatch(message, key=None, value=None):
        if message == 'getitem':
            return getitem(key)
        elif message == 'setitem':
            setitem(key, value)
        elif message == 'keys':
            return tuple(k for k, _ in records)
        elif message == 'values':
            return tuple(v for _, v in records)

    return dispatch
```

Question: Do we need a nonlocal statement here?

Demo

# Dispatch Dictionaries

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Combination idea: All intermediate quantities have values too.

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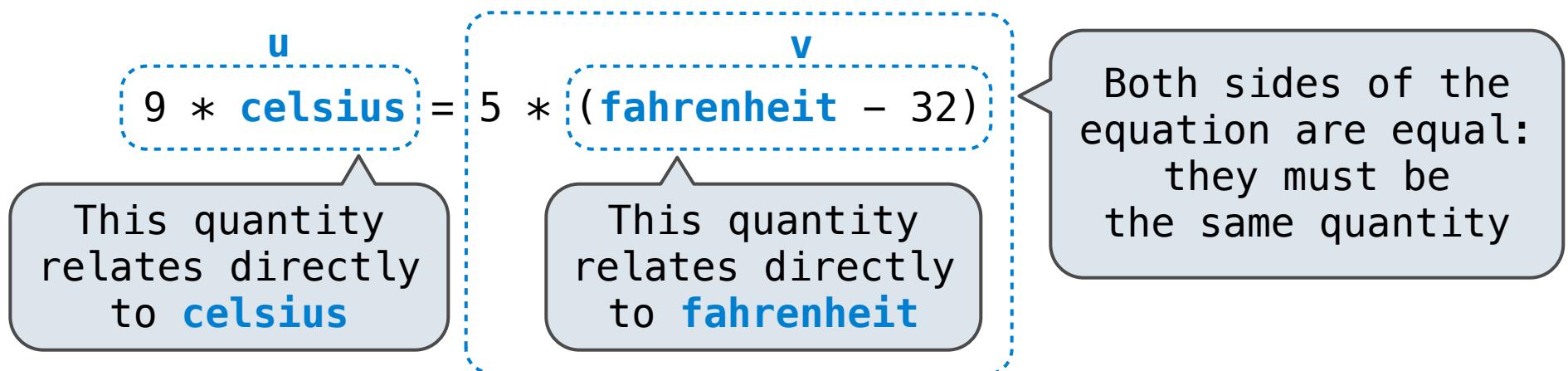
This quantity relates directly to **celsius**

This quantity relates directly to **fahrenheit**

The diagram illustrates the temperature conversion formula  $9 * \text{celsius} = 5 * (\text{fahrenheit} - 32)$ . Two terms in the equation are highlighted with dashed circles:  $9 * \text{celsius}$  and  $5 * (\text{fahrenheit} - 32)$ . Below each highlighted term is a callout box containing the text "This quantity relates directly to [term]". The first callout box is associated with the term  $9 * \text{celsius}$  and the second is associated with  $5 * (\text{fahrenheit} - 32)$ .

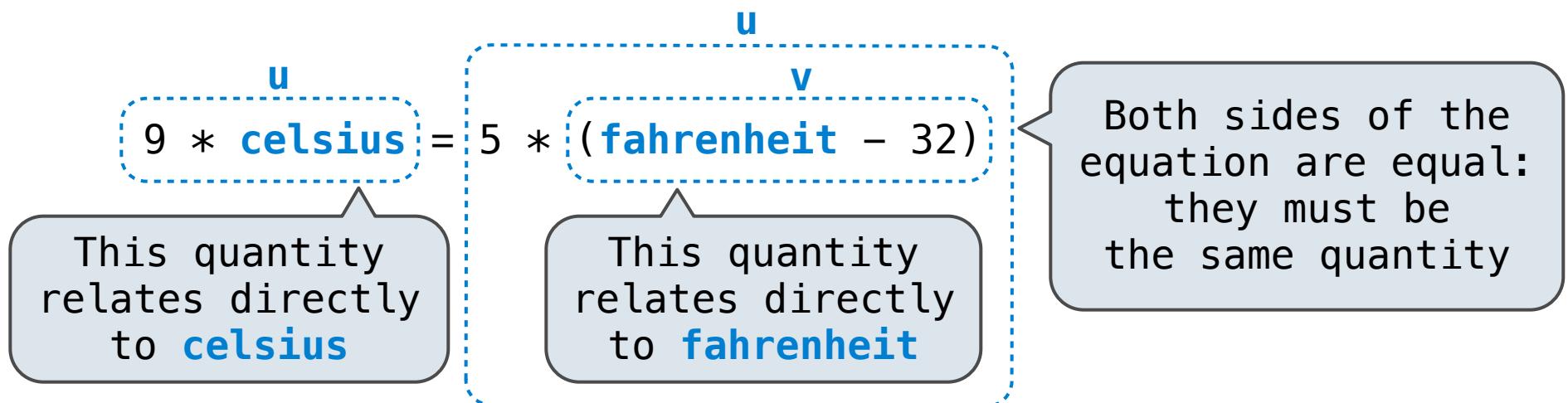
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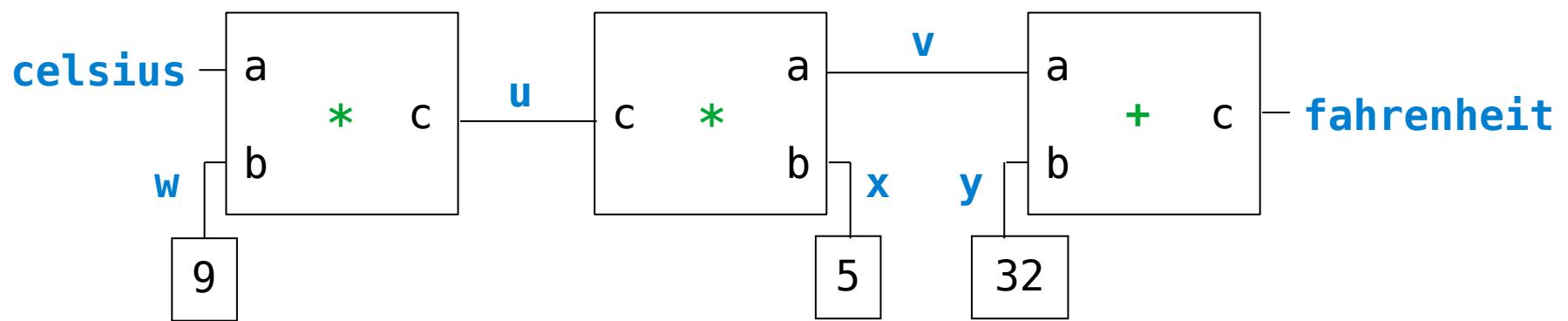
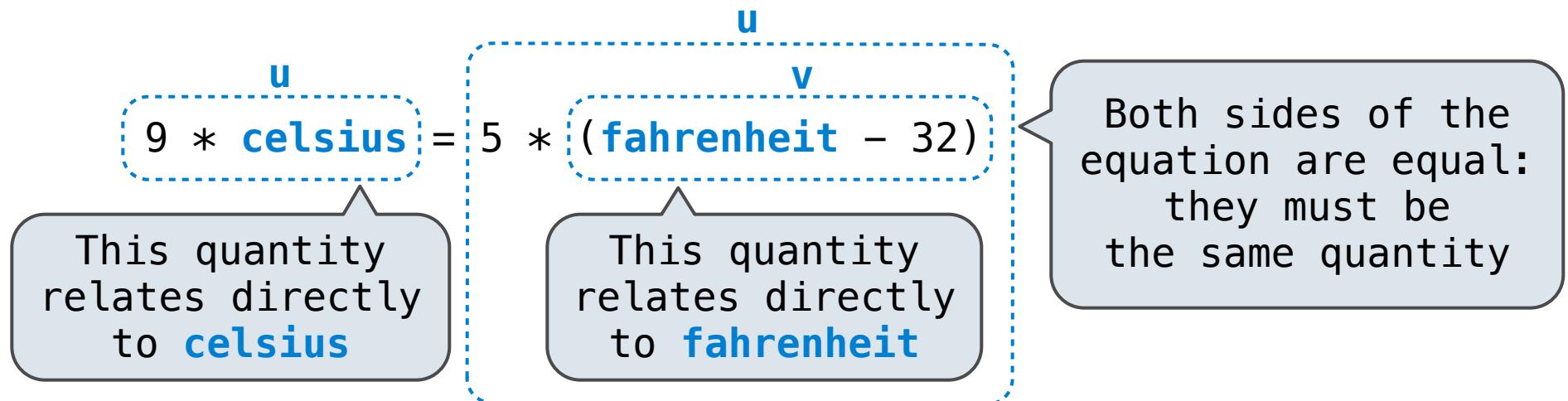
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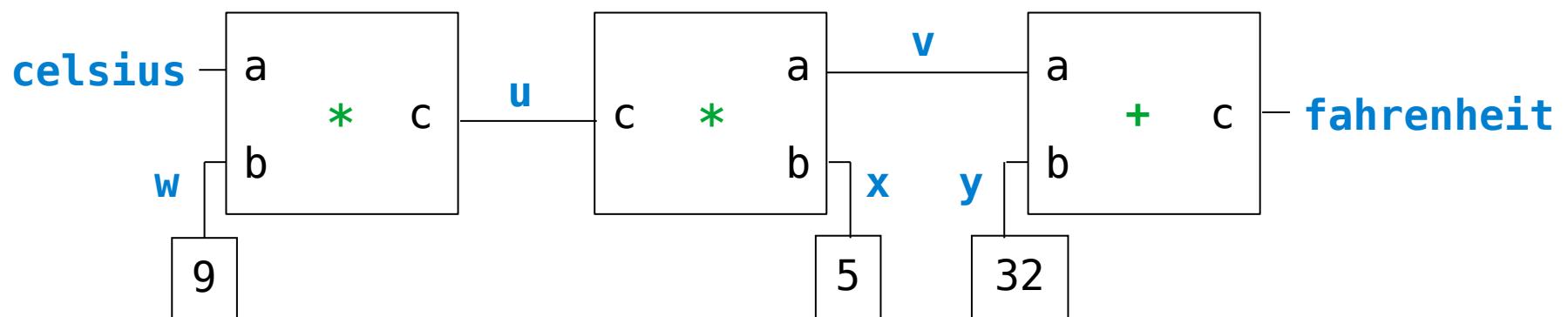
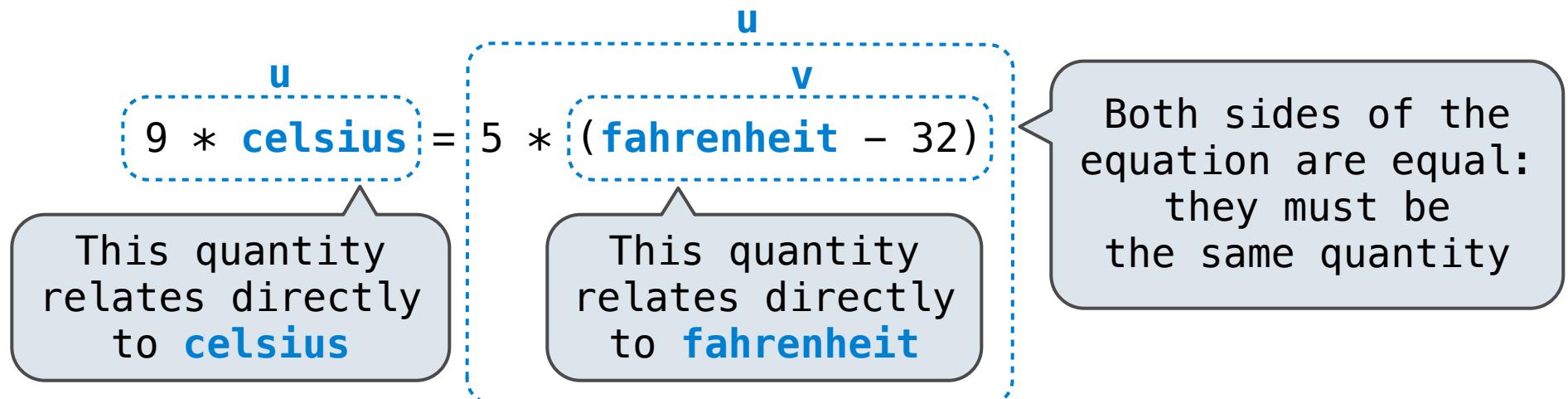
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Demo