

CS 61A Lecture 9

Friday, September 14

The Sequence Abstraction

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red, orange, yellow, green, blue, indigo, violet.

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The sequence abstraction is shared among several types.

Tuples are Sequences

(Demo)

Box-and-Pointer Notation

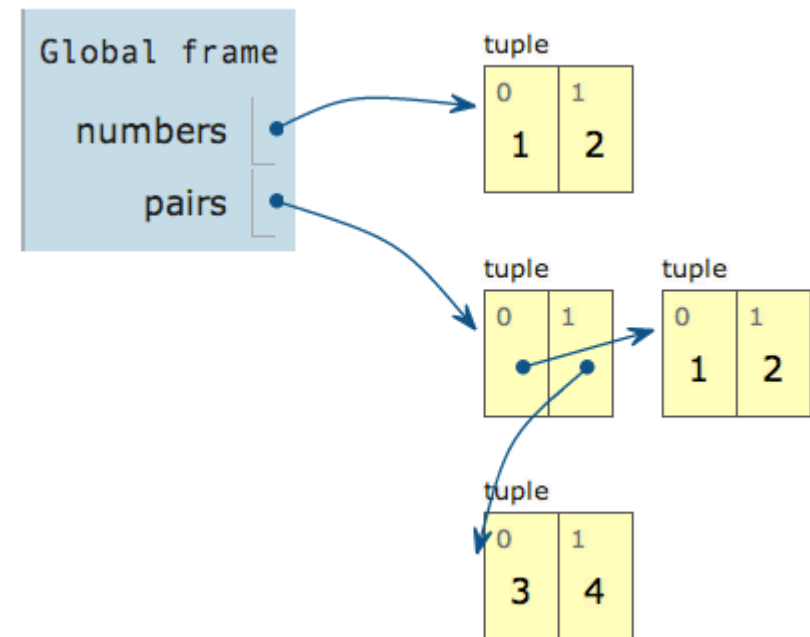
```
1 numbers = (1, 2)  
→ 2 pairs = ((1, 2), (3, 4))
```

[Edit code](#)

< Back

Program has terminated

Forward >



The Closure Property of Data Types

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Tuples can contain tuples as elements

Recursive Lists

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Behavior condition(s):

If a recursive list s is constructed from a first element f and a recursive list r , then

- `first(s)` returns f , and
- `rest(s)` returns r , which is a recursive list.

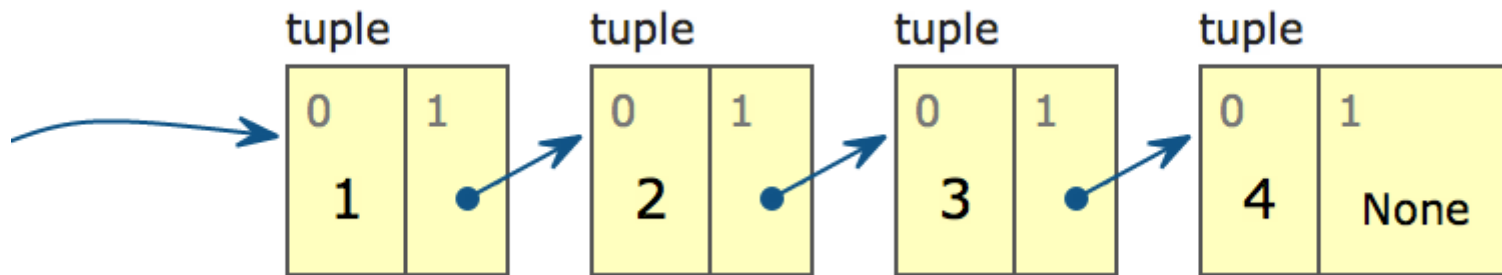
Implementing Recursive Lists with Pairs

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1 , 2 , 3 , 4

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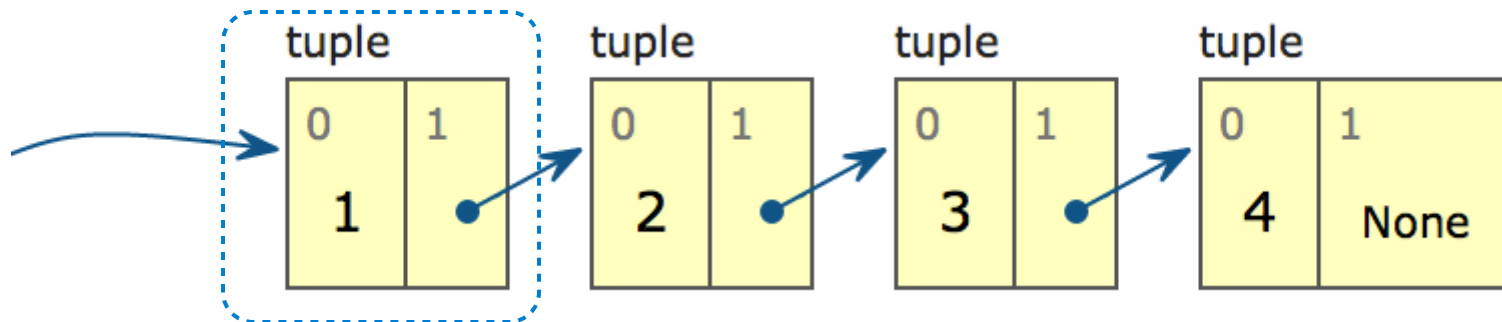
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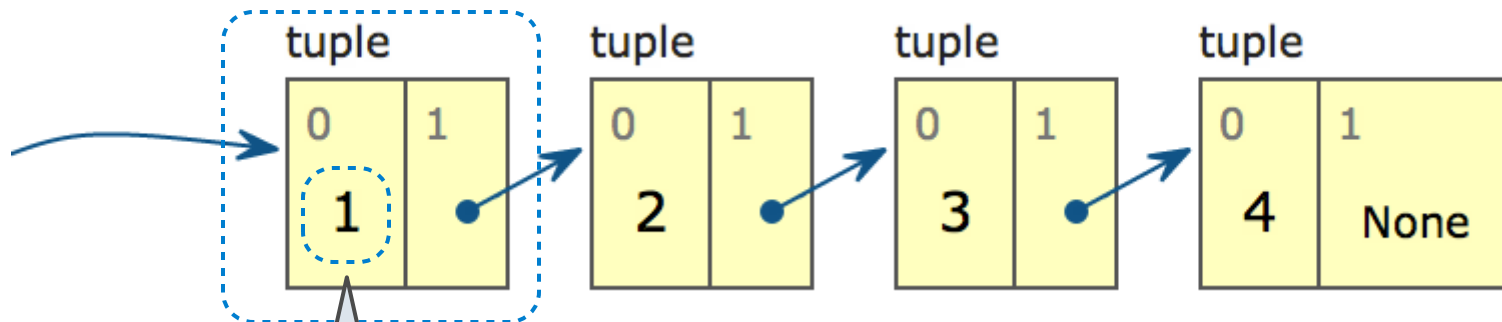
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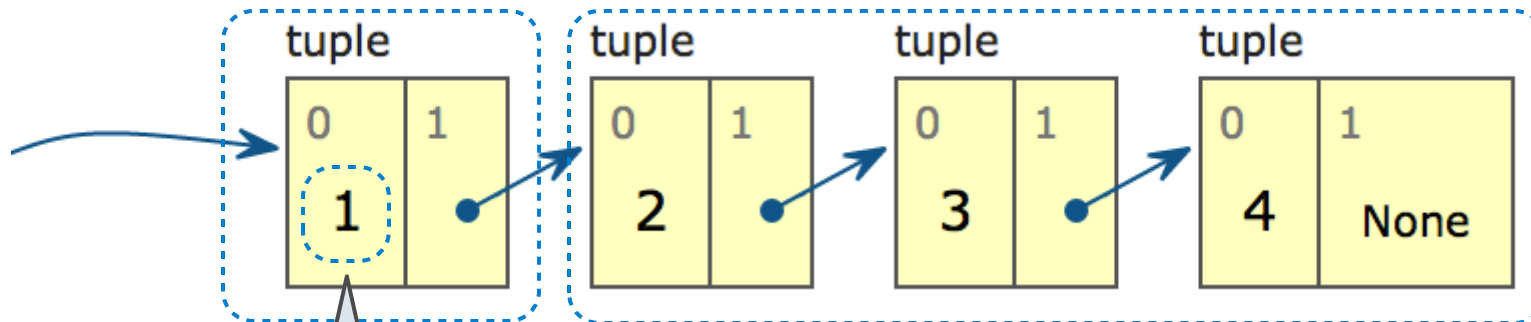


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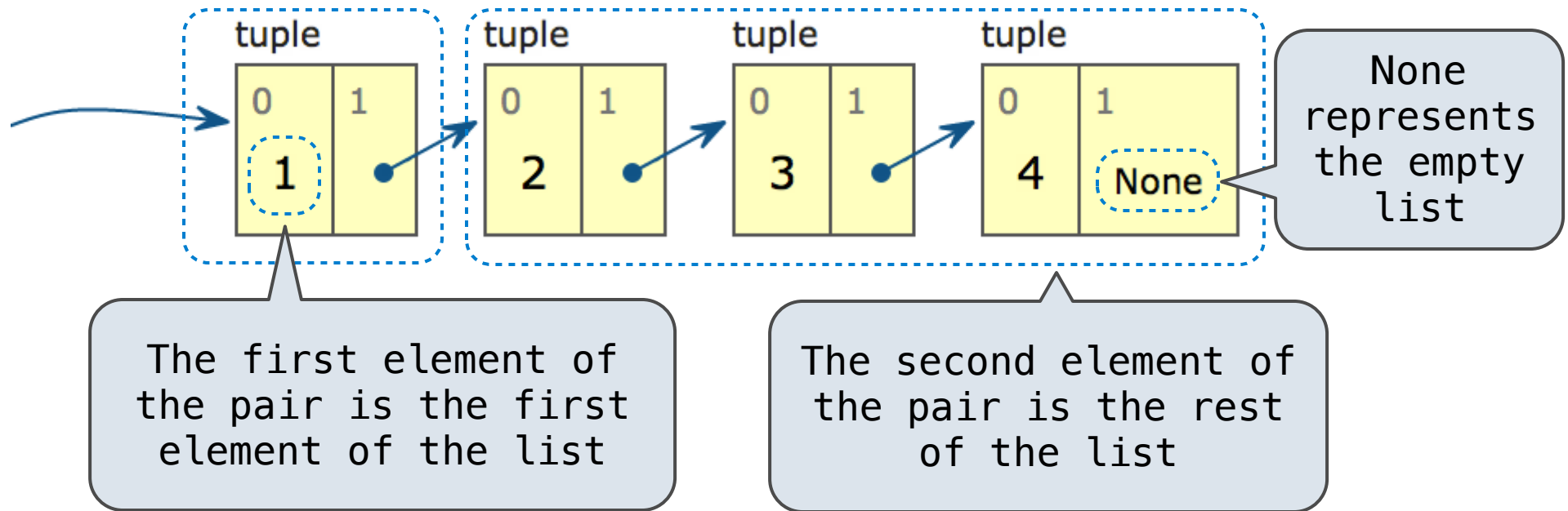
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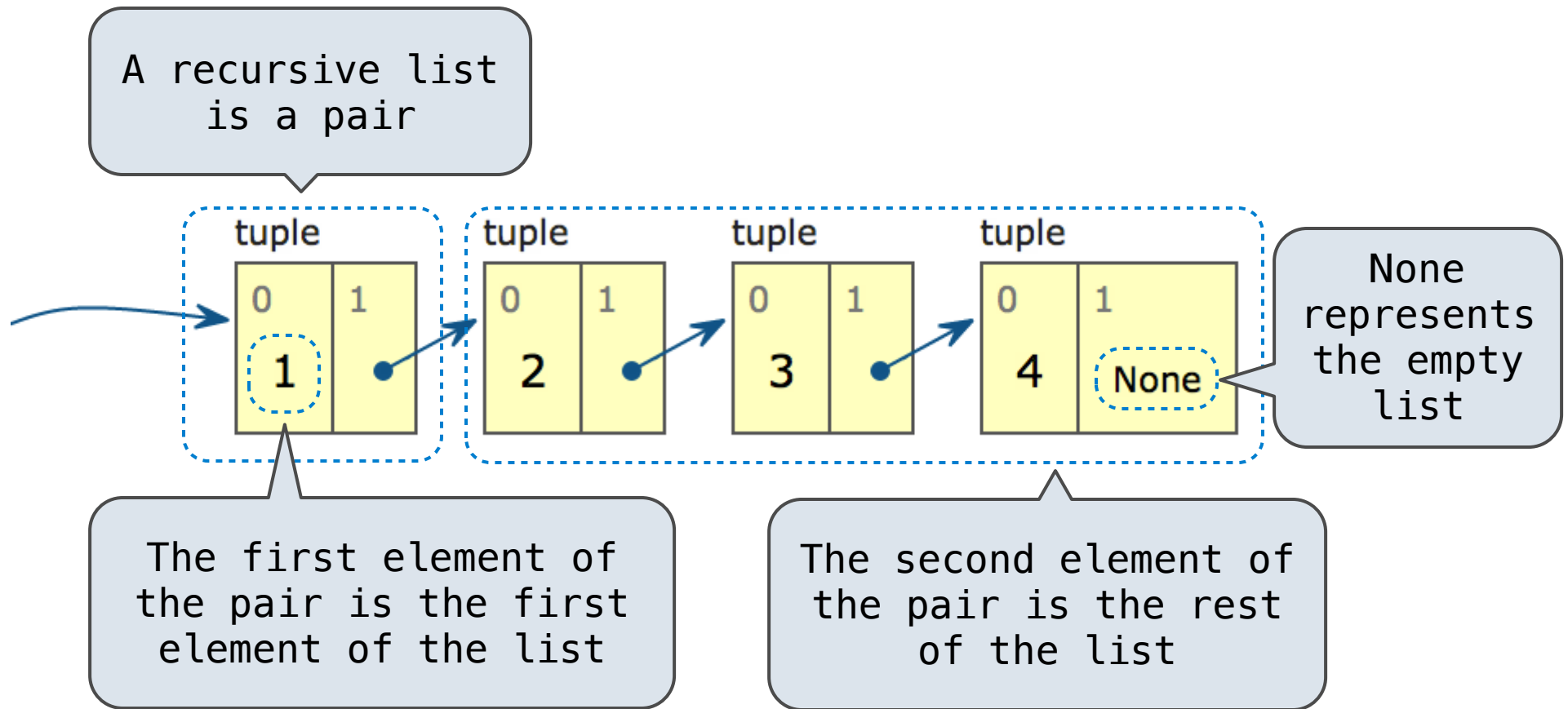
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Implementing Recursive Lists with Pairs

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(Demo)

Implementing the Sequence Abstraction

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Length. A sequence has a finite length.

Element selection. A sequence has an element corresponding to any non-negative integer index less than its length, starting at 0 for the first element.

Implementing the Sequence Abstraction

```
def len_rlist(s):  
    """Return the length of recursive list s."""  
    length = 0  
    while s != empty_rlist:  
        s, length = rest(s), length + 1  
    return length
```

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def getitem_rlist(s, i):
    """Return the element at index i of recursive list s."""
    while i > 0:
        s, i = rest(s), i - 1
    return first(s)
```

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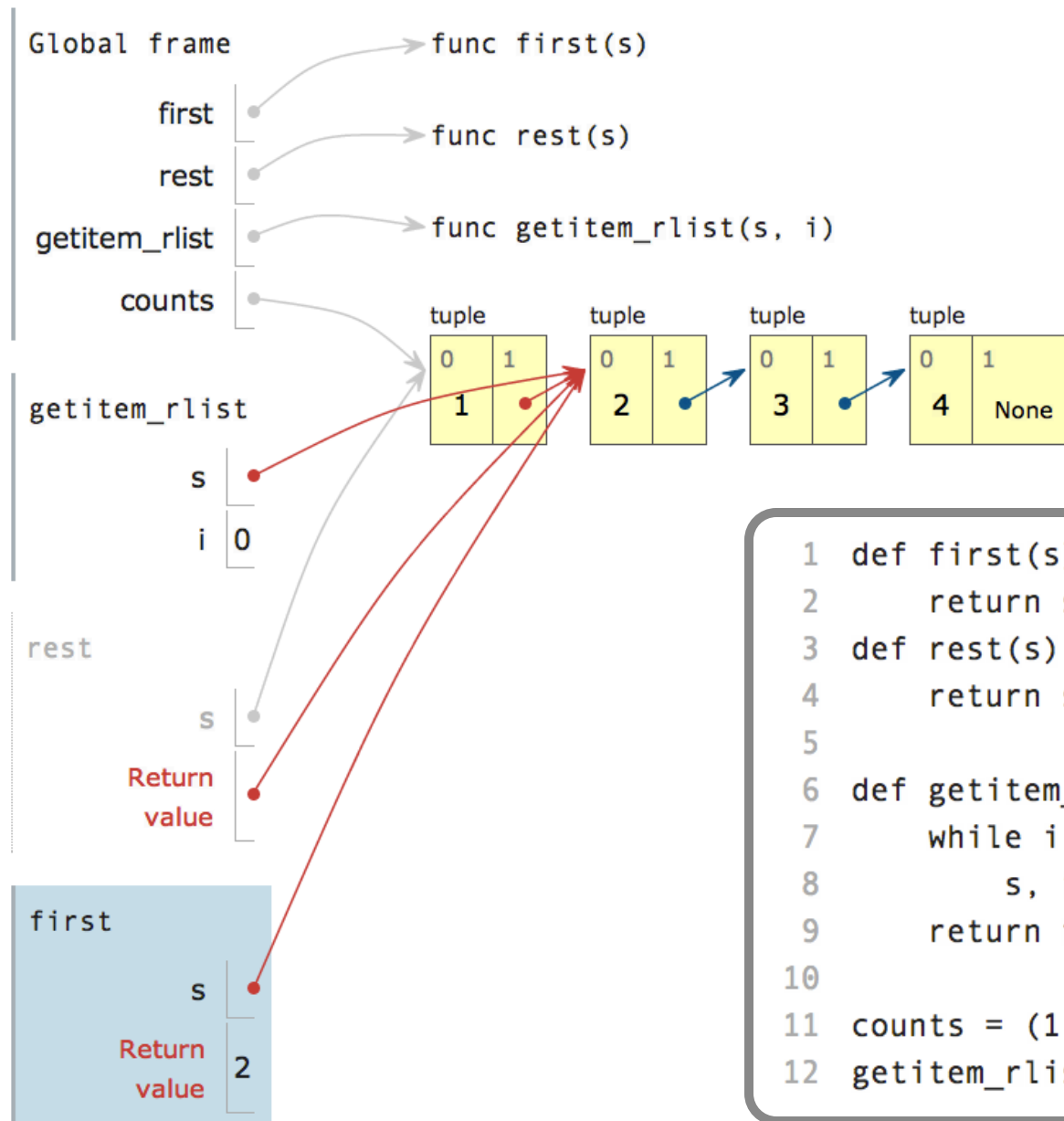
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Environment Diagram for getitem_rlist



```
1 def first(s):
2     return s[0]
3 def rest(s):
4     return s[1]
5
6 def getitem_rlist(s, i):
7     while i > 0:
8         s, i = rest(s), i - 1
9     return first(s)
10
11 counts = (1, (2, (3, (4, None))))
12 getitem_rlist(counts, 1)
```

Sequence Iteration

**Not on
Midterm 1**

(Demo)

Sequence Iteration

(Demo)

```
def count(s, value):  
    total = 0  
    for elem in s:
```

Name bound in the first frame
of the current environment

```
        if elem == value:  
            total = total + 1  
    return total
```

For Statement Execution Procedure

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for <name> in <expression>:  
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1. Evaluate the header <expression>, which must yield an iterable value.
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 - A. Bind <name> to that element in the local environment.
 - B. Execute the <suite>.

Sequence Unpacking in For Statements

**Not on
Midterm 1**

Sequence Unpacking in For Statements

```
>>> pairs = ((1, 2), (2, 2), (2, 3), (4, 4))
```

```
>>> same_count = 0
```

Sequence Unpacking in For Statements

A sequence of
fixed-length sequences

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```

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A name for each element in
a fixed-length sequence

Each name is bound to a value,
as in multiple assignment

```
>>> for x, y in pairs:
    if x == y:
        same_count = same_count + 1
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The Range Type

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$\dots, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, \dots$

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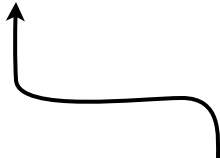
`range(-2, 2)`

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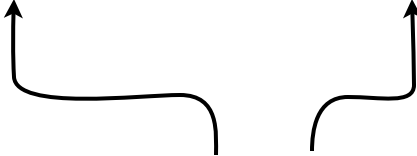
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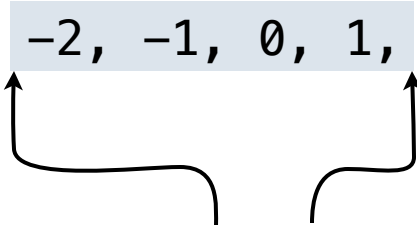
The diagram illustrates the range of integers from -2 to 2. Two curved arrows originate from the text 'range(-2, 2)' below. The left arrow starts at the first '-2' and points up to the '-2' in the sequence. The right arrow starts at the '2' and points up to the '2' in the sequence. The numbers -2 and 2 in the code are highlighted in green.

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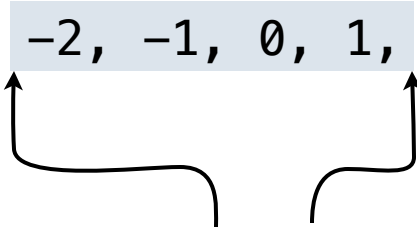
The diagram illustrates the range function in Python. It shows a sequence of integers from -5 to 5. The integers -2, -1, 0, and 1 are highlighted with a light blue background. Two curved arrows originate from the text 'range(-2, 2)' below. One arrow points to the -2 in the sequence, and the other points to the 1 in the sequence, indicating that the range function generates the sequence of integers from -2 up to, but not including, 2.

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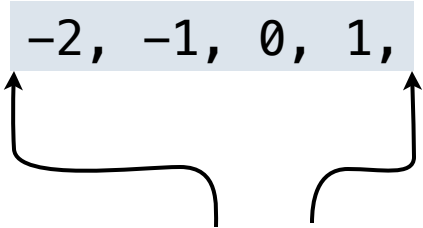
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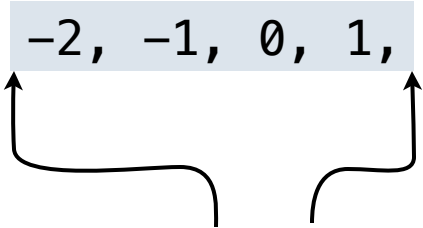
Element selection: starting value + index

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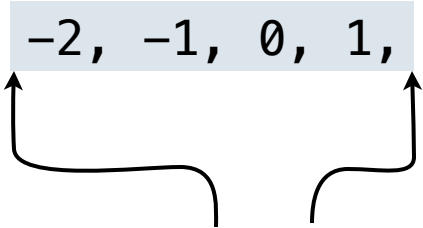
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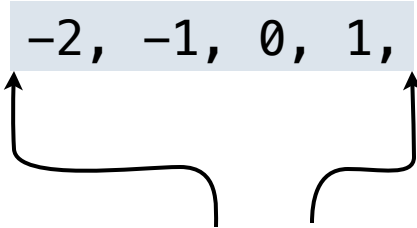
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>>> 2 in digits
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```

Slicing.

```
>>> digits[0:2]
(1, 8)
>>> digits[1:]
(8, 2, 8)
```