61A Lecture 8

Wednesday, September 12

Data Abstraction

- Compound objects combine primitive objects together
- A date: a year, a month, and a day
- A geographic position: latitude and longitude
- An *abstract data type* lets us manipulate compound objects as units
- Isolate two parts of any program that uses data:
  - How data are represented (as parts)
  - How data are manipulated (as units)
- Data abstraction: A methodology by which functions enforce an abstraction barrier between ***representation*** and ***use***

All Programmers

Great Programmers

Rational Numbers

$$\frac{\texttt{numerator}}{\texttt{denominator}}$$

Exact representation of fractions

A pair of integers

As soon as division occurs, the exact representation is lost!

Assume we can compose and decompose rational numbers:

Constructor
- `rational(n, d)` *returns a rational number* x

Selectors
- `numer(x)` *returns the numerator of* x
- `denom(x)` *returns the denominator of* x

Rational Number Arithmetic

Example:

$$\frac{3}{2} * \frac{3}{5} = \frac{9}{10}$$

$$\frac{3}{2} + \frac{3}{5} = \frac{21}{10}$$

General Form:

$$\frac{nx}{dx} * \frac{ny}{dy} = \frac{nx*ny}{dx*dy}$$

$$\frac{nx}{dx} + \frac{ny}{dy} = \frac{nx*dy + ny*dx}{dx*dy}$$

Rational Number Arithmetic Implementation

```
def mul_rational(x, y):
    return rational(numer(x) * numer(y), denom(x) * denom(y))
```

Constructor

Selectors

```
def add_rational(x, y):
    nx, dx = numer(x), denom(x)
    ny, dy = numer(y), denom(y)
    return rational(nx * dy + ny * dx, dx * dy)

def eq_rational(x, y):
    return numer(x) * denom(y) == numer(y) * denom(x)
```

Wishful thinking

- `rational(n, d)` *returns a rational number* `x`
- `numer(x)` *returns the numerator of* `x`
- `denom(x)` *returns the denominator of* `x`

Tuples

```
>>> pair = (1, 2)
>>> pair
(1, 2)
```

A tuple literal:
Comma-separated expression

```
>>> x, y = pair
>>> x
1
>>> y
2
```

"Unpacking" a tuple

```
>>> pair[0]
1
>>> pair[1]
2
>>> from operator import getitem
>>> getitem(pair, 0)
1
>>> getitem(pair, 1)
2
```

Element selection

More tuples next lecture

Representing Rational Numbers

```
def rational(n, d):
    """Construct a rational number x that represents n/d."""
    return (n, d)
```

Construct a tuple

```
from operator import getitem

def numer(x):
    """Return the numerator of rational number x."""
    return getitem(x, 0)

def denom(x):
    """Return the denominator of rational number x."""
    return getitem(x, 1)
```

Select from a tuple

Reducing to Lowest Terms

Example:

$$\frac{3}{2} * \frac{5}{3} = \boxed{\frac{5}{2}} \qquad \frac{2}{5} + \frac{1}{10} = \boxed{\frac{1}{2}}$$

$$\frac{15}{6} * \frac{1/3}{1/3} = \frac{5}{2} \qquad \frac{25}{50} * \frac{1/25}{1/25} = \frac{1}{2}$$

```
from fractions import gcd
```

Greatest common divisor

```
def rational(n, d):
    """Construct a rational number x that represents n/d."""
    g = gcd(n, d)
    return (n//g, d//g)
```

Abstraction Barriers

Rational numbers as whole data values

`add_rationals  mul_rationals  eq_rationals`

Rational numbers as numerators & denominators

`rational  numer  denom`

Rational numbers as tuples

`tuple  getitem`

However tuples are implemented in Python

Violating Abstraction Barriers

Does not use constructors

Twice!

```
add_rational( (1, 2), (1, 4) )
```

```
def divide_rational(x, y):
    return (x[0] * y[1], x[1] * y[0])
```

No selectors!

And no constructor!

What is Data?

- We need to guarantee that constructor and selector functions together specify the right behavior.
- **Behavior condition:** If we construct rational number x from numerator n and denominator d, then numer(x)/denom(x) must equal n/d.
- An abstract data type is some collection of selectors and constructors, together with some behavior condition(s).
- If behavior conditions are met, the representation is valid.

You can recognize data types by behavior, not by bits

Behavior Conditions of a Pair

To implement our rational number abstract data type,
we used a two-element tuple (also known as a pair).

What is a pair?

Constructors, selectors, and behavior conditions:

If a pair p was constructed from elements x and y, then

- getitem_pair(p, 0) returns x, and
- getitem_pair(p, 1) returns y.

Together, selectors are the inverse of the constructor

Generally true of *container types*.

Not true for rational numbers because of GCD

Functional Pair Implementation

```
def pair(x, y):
    """Return a functional pair."""
    def dispatch(m):
        if m == 0:
            return x
        elif m == 1:
            return y
    return dispatch
```

This function represents a pair

Constructor is a higher-order function

```
def getitem_pair(p, i):
    """Return the element at index i of pair p."""
    return p(i)
```

Selector defers to the object itself

Using a Functionally Implemented Pair

```
>>> p = pair(1, 2)

>>> getitem_pair(p, 0)
1

>>> getitem_pair(p, 1)
2
```

As long as we do not violate the abstraction barrier, we don't need to know that pairs are just functions

If a pair p was constructed from elements x and y, then

- getitem_pair(p, 0) returns x, and
- getitem_pair(p, 1) returns y.

This pair representation is valid!