61A Lecture 4

Friday, August 31

The Fibonacci Sequence

Example: http://goo.gl/dcaf0

The Fibonacci Sequence

0, 1, 1, 2, 3, 5, 8, 13, ...

Example: http://goo.gl/dcaf0

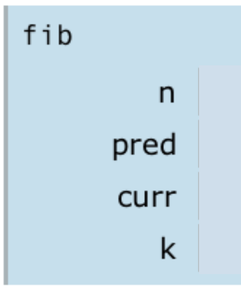
The Fibonacci Sequence

0, 1, 1, 2, 3, 5, 8, 13, ...

```
def fib(n):
    """Compute the nth Fibonacci number, for n >= 2."""
    pred, curr = 0, 1   # First two Fibonacci numbers
    k = 2               # Tracks which Fib number is curr
    while k < n:
        pred, curr = curr, pred + curr
        k = k + 1
    return curr
```

Example: http://goo.gl/dcaf0

The Fibonacci Sequence

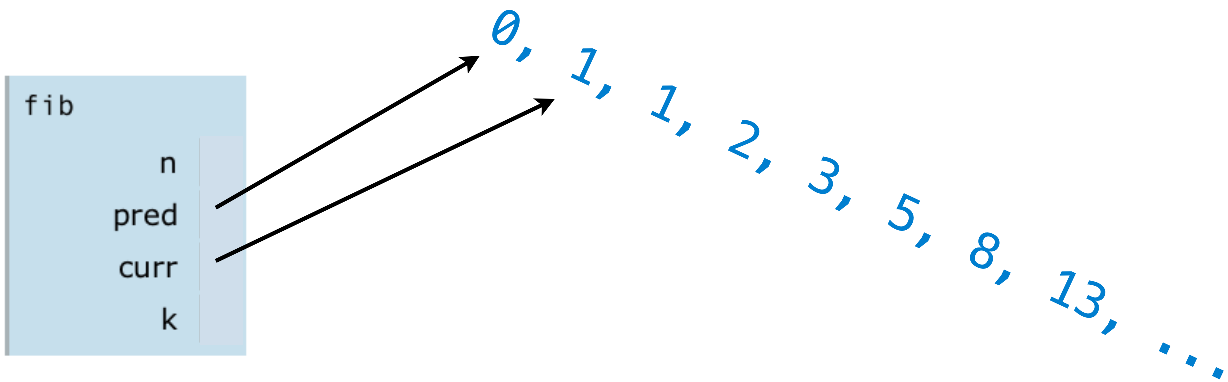


0, 1, 1, 2, 3, 5, 8, 13, ...

```
def fib(n):
    """Compute the nth Fibonacci number, for n >= 2."""
    pred, curr = 0, 1   # First two Fibonacci numbers
    k = 2               # Tracks which Fib number is curr
    while k < n:
        pred, curr = curr, pred + curr
        k = k + 1
    return curr
```

Example: http://goo.gl/dcaf0

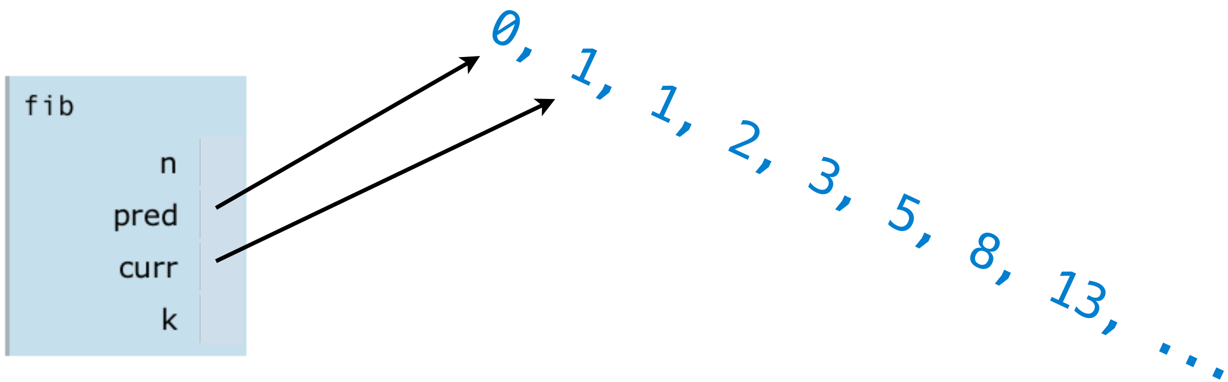
The Fibonacci Sequence




```
def fib(n):
    """Compute the nth Fibonacci number, for n >= 2."""
    pred, curr = 0, 1   # First two Fibonacci numbers
    k = 2               # Tracks which Fib number is curr
    while k < n:
        pred, curr = curr, pred + curr
        k = k + 1
    return curr
```

Example: http://goo.gl/dcaf0

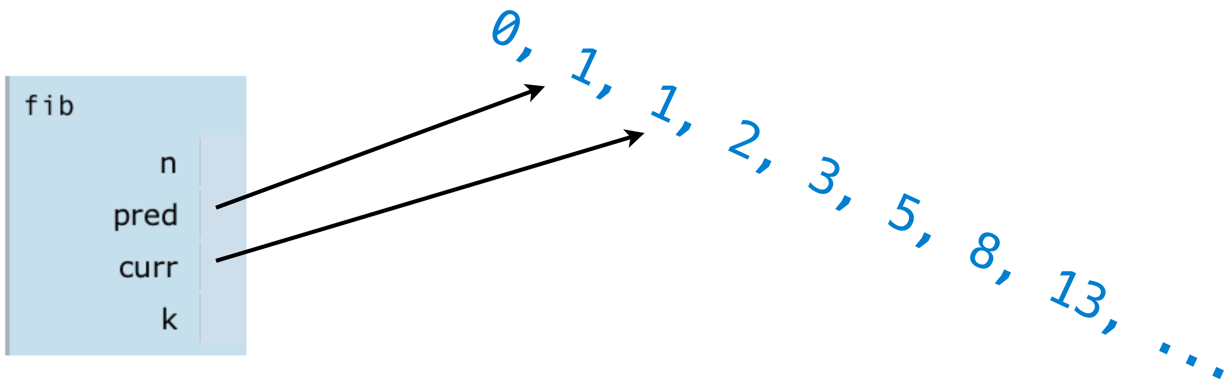
The Fibonacci Sequence




```
def fib(n):
    """Compute the nth Fibonacci number, for n >= 2."""
    pred, curr = 0, 1   # First two Fibonacci numbers
    k = 2               # Tracks which Fib number is curr
    while k < n:
        pred, curr = curr, pred + curr
        k = k + 1
    return curr
```

Example: http://goo.gl/dcaf0

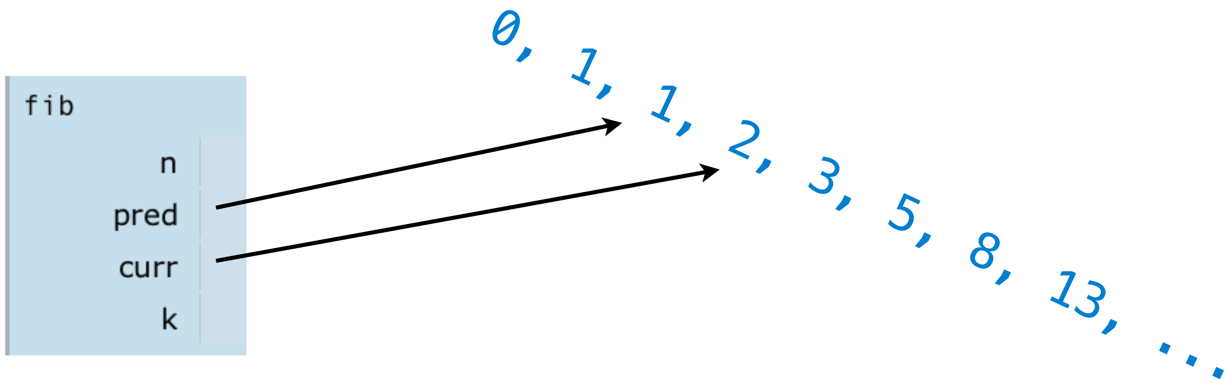
The Fibonacci Sequence




```python
def fib(n):
    """Compute the nth Fibonacci number, for n >= 2."""
    pred, curr = 0, 1   # First two Fibonacci numbers
    k = 2               # Tracks which Fib number is curr
    while k < n:
        pred, curr = curr, pred + curr
        k = k + 1
    return curr
```

Example: http://goo.gl/dcaf0

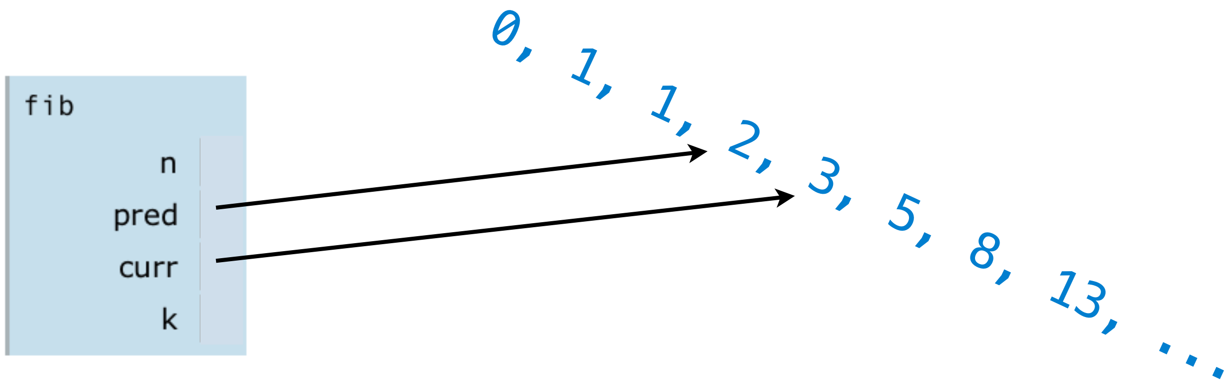
The Fibonacci Sequence




```
def fib(n):
    """Compute the nth Fibonacci number, for n >= 2."""
    pred, curr = 0, 1   # First two Fibonacci numbers
    k = 2               # Tracks which Fib number is curr
    while k < n:
        pred, curr = curr, pred + curr
        k = k + 1
    return curr
```

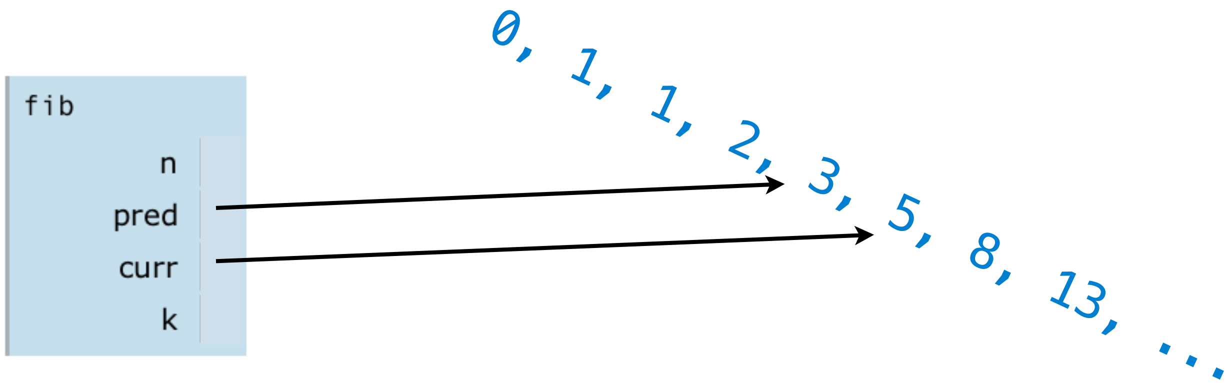
Example: http://goo.gl/dcaf0

The Fibonacci Sequence




```
def fib(n):
    """Compute the nth Fibonacci number, for n >= 2."""
    pred, curr = 0, 1   # First two Fibonacci numbers
    k = 2               # Tracks which Fib number is curr
    while k < n:
        pred, curr = curr, pred + curr
        k = k + 1
    return curr
```

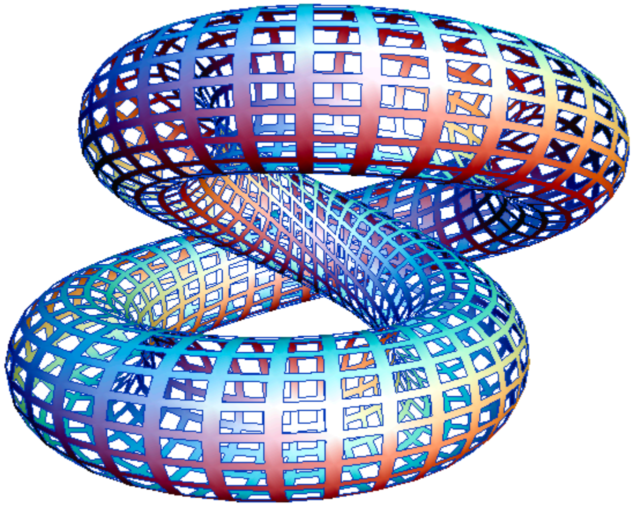
The Fibonacci Sequence




```python
def fib(n):
    """Compute the nth Fibonacci number, for n >= 2."""
    pred, curr = 0, 1   # First two Fibonacci numbers
    k = 2               # Tracks which Fib number is curr
    while k < n:
        pred, curr = curr, pred + curr
        k = k + 1
    return curr
```

Practical Guidance: the Art of the Function

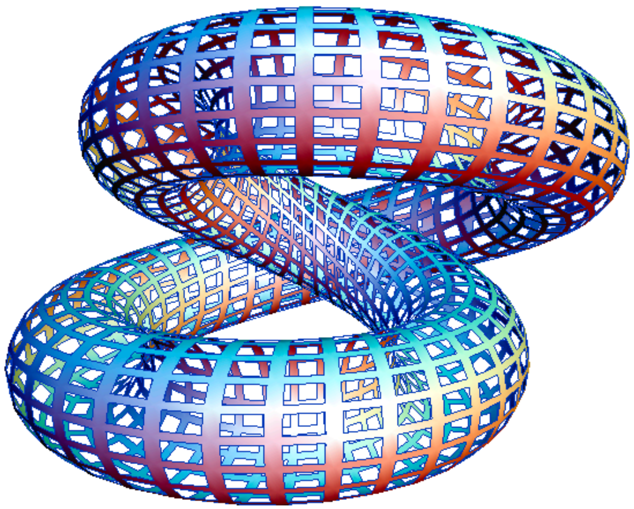
Practical Guidance: the Art of the Function



$$\begin{bmatrix} r_0 \\ \theta \\ z \end{bmatrix} = \begin{bmatrix} 3+\sin t+\cos u \\ 2t \\ \sin u+2\cos t \end{bmatrix}, t=0\ldots2\pi, u=0\ldots2\pi$$

Practical Guidance: the Art of the Function

Give each function exactly one job.

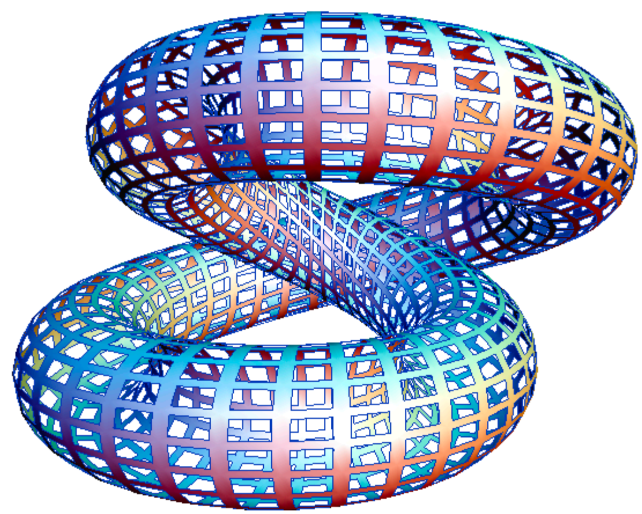


$$\begin{bmatrix} r_0 \\ \theta \\ z \end{bmatrix} = \begin{bmatrix} 3+\sin t+\cos u \\ 2t \\ \sin u+2\cos t \end{bmatrix}, t=0\ldots 2\pi, u=0\ldots 2\pi$$

Practical Guidance: the Art of the Function

```
Give each function exactly one job.
```

vs



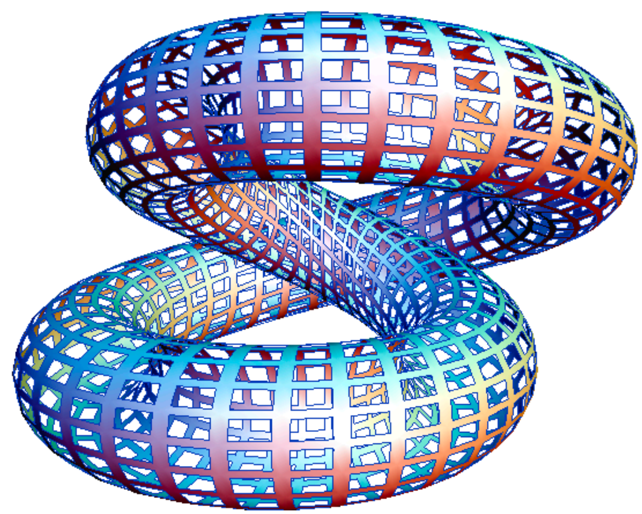
$$\begin{bmatrix} r_0 \\ \theta \\ z \end{bmatrix} = \begin{bmatrix} 3+\sin t+\cos u \\ 2t \\ \sin u+2\cos t \end{bmatrix}, t=0\ldots2\pi, u=0\ldots2\pi$$

Practical Guidance: the Art of the Function

Give each function exactly one job.

vs

Don't repeat yourself (DRY). Implement a computational process just once, but execute it many times.



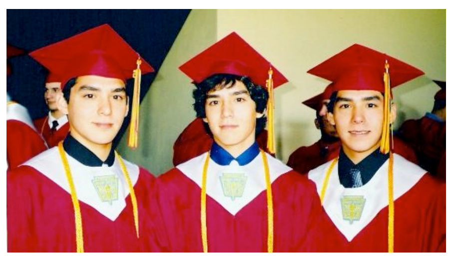
$$\begin{bmatrix} r_0 \\ \theta \\ z \end{bmatrix} = \begin{bmatrix} 3+\sin t+\cos u \\ 2t \\ \sin u+2\cos t \end{bmatrix}, t=0\ldots2\pi, u=0\ldots2\pi$$

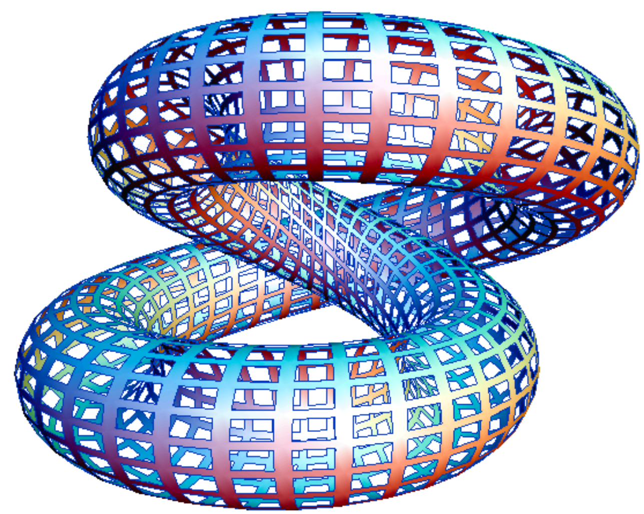
Practical Guidance: the Art of the Function

Give each function exactly one job.

vs

Don't repeat yourself (DRY). Implement a computational process just once, but execute it many times.





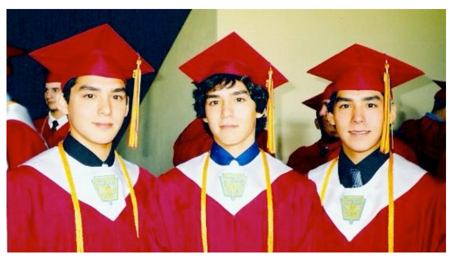
$$\begin{bmatrix} r_0 \\ \theta \\ z \end{bmatrix} = \begin{bmatrix} 3+\sin t+\cos u \\ 2t \\ \sin u+2\cos t \end{bmatrix}, t=0\ldots 2\pi, u=0\ldots 2\pi$$

Practical Guidance: the Art of the Function

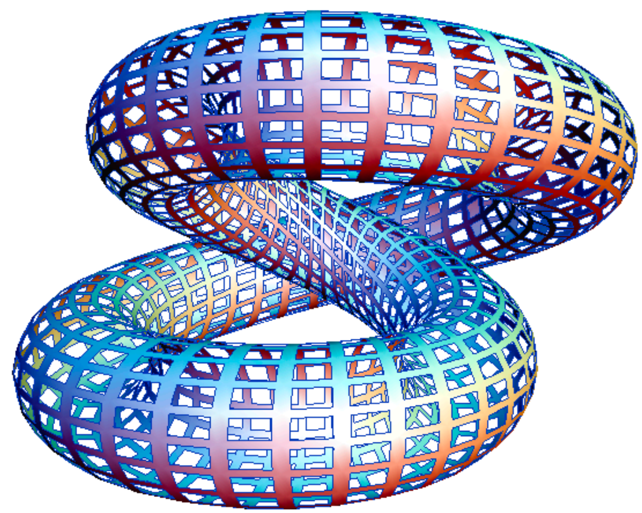
Give each function exactly one job.

vs

Don't repeat yourself (DRY). Implement a computational process just once, but execute it many times.



Define functions generally.



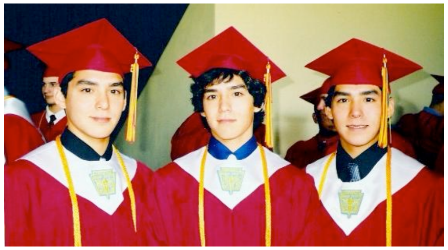
$$\begin{bmatrix} r_0 \\ \theta \\ z \end{bmatrix} = \begin{bmatrix} 3+\sin t+\cos u \\ 2t \\ \sin u+2\cos t \end{bmatrix}, t=0\ldots2\pi, u=0\ldots2\pi$$

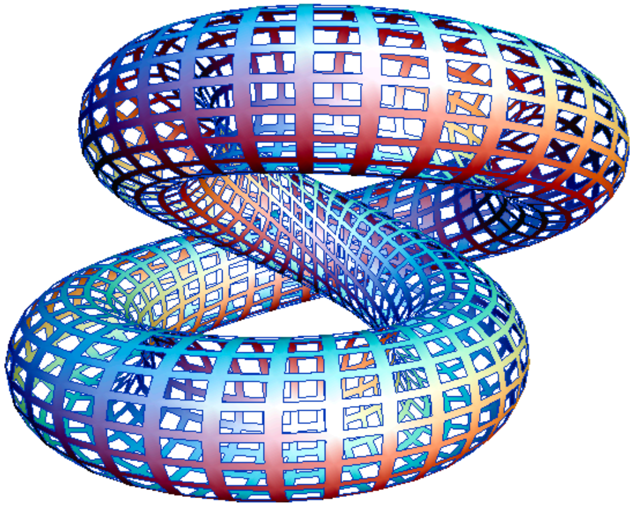
Practical Guidance: the Art of the Function

Give each function exactly one job.

vs

Don't repeat yourself (DRY). Implement a computational process just once, but execute it many times.





Define functions generally.

$$\begin{bmatrix} r_0 \\ \theta \\ z \end{bmatrix} = \begin{bmatrix} 3+\sin t+\cos u \\ 2t \\ \sin u+2\cos t \end{bmatrix}, t=0\ldots 2\pi, u=0\ldots 2\pi$$

Generalizing Patterns with Arguments

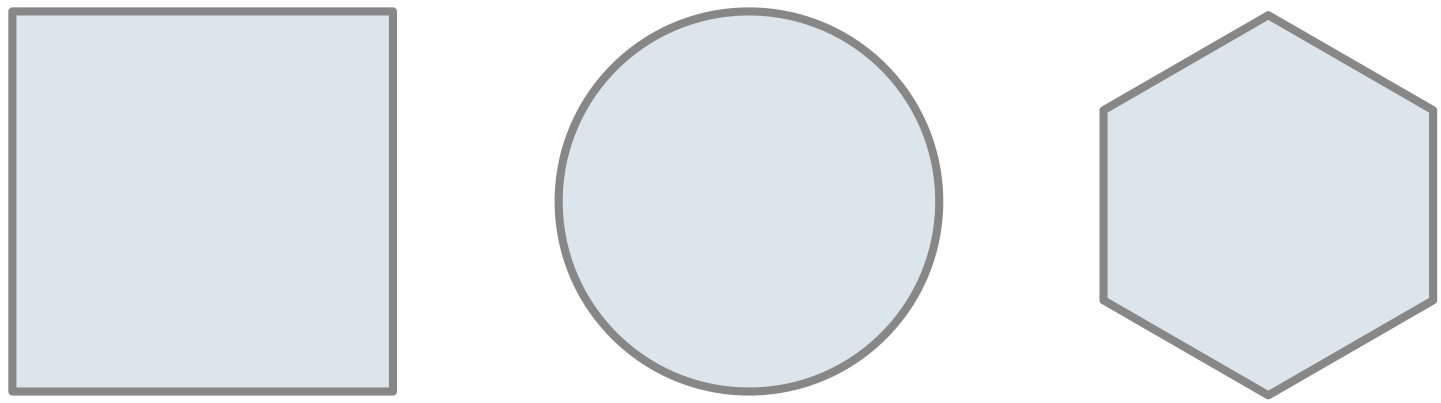
Generalizing Patterns with Arguments

Regular geometric shapes relate length and area.

Generalizing Patterns with Arguments

Regular geometric shapes relate length and area.

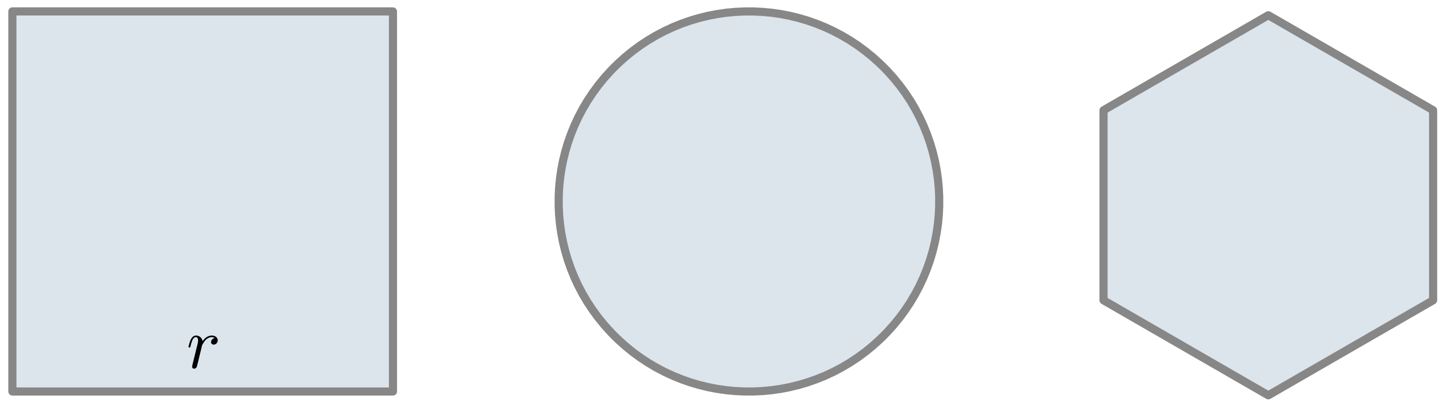
Shape:



Generalizing Patterns with Arguments

Regular geometric shapes relate length and area.

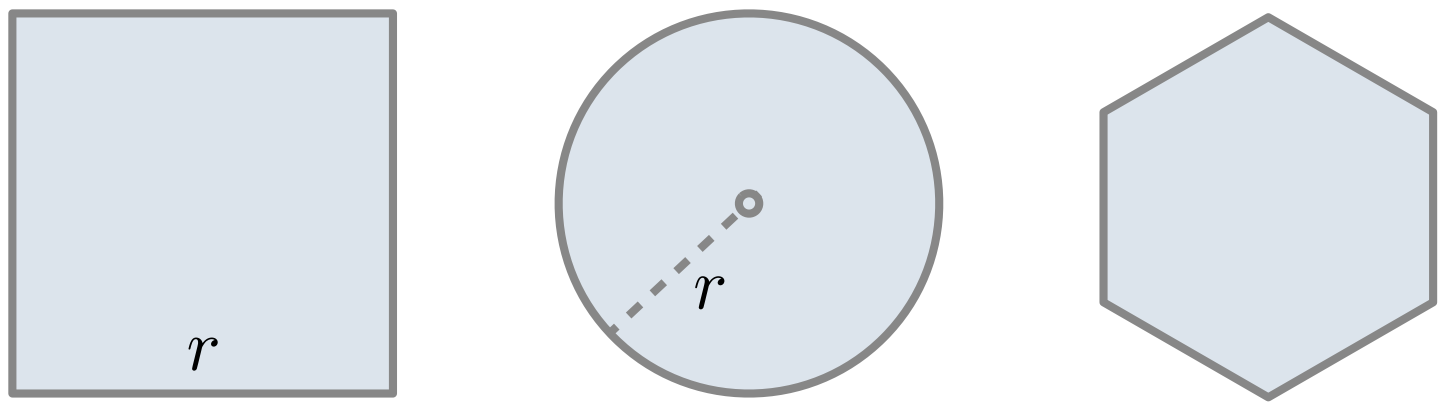
Shape:



Generalizing Patterns with Arguments

Regular geometric shapes relate length and area.

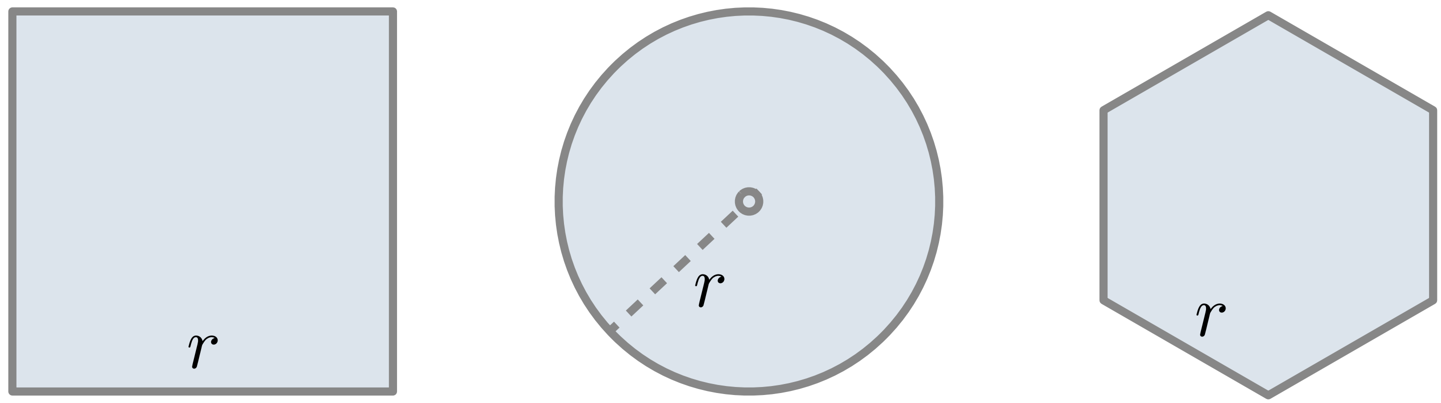
Shape:



Generalizing Patterns with Arguments

Regular geometric shapes relate length and area.

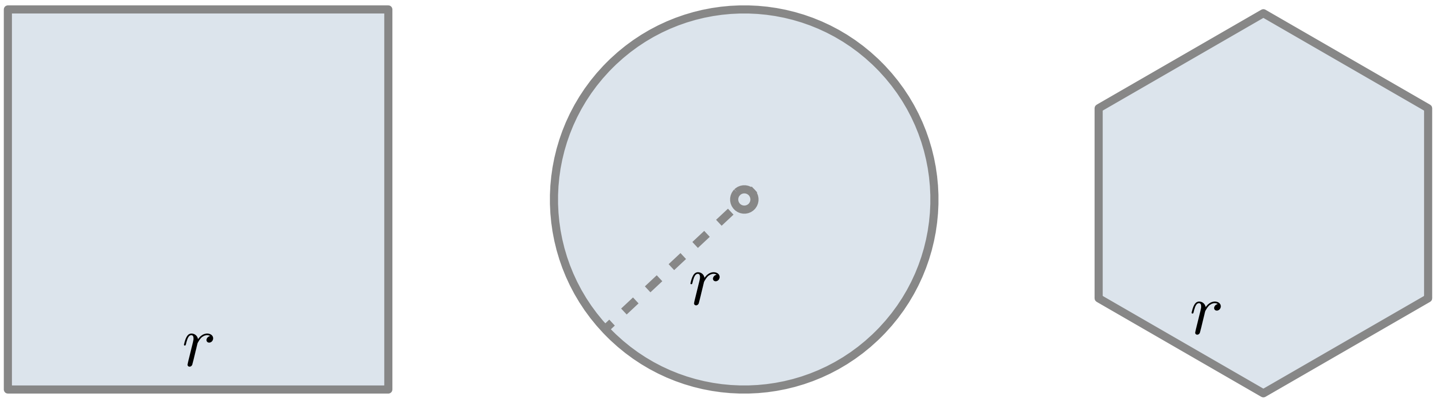
Shape:



Generalizing Patterns with Arguments

Regular geometric shapes relate length and area.

Shape:

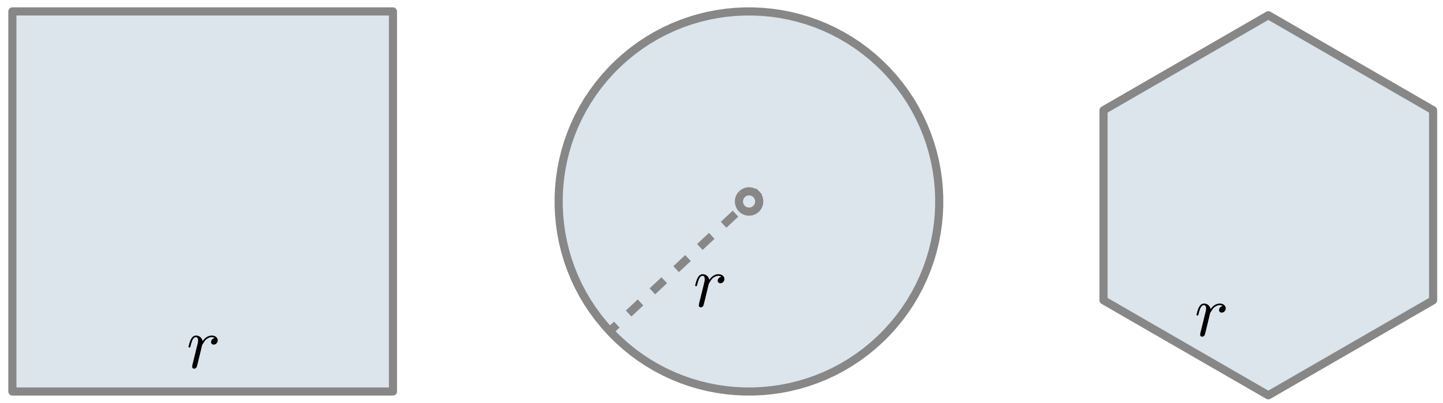


Area:

Generalizing Patterns with Arguments

Regular geometric shapes relate length and area.

Shape:

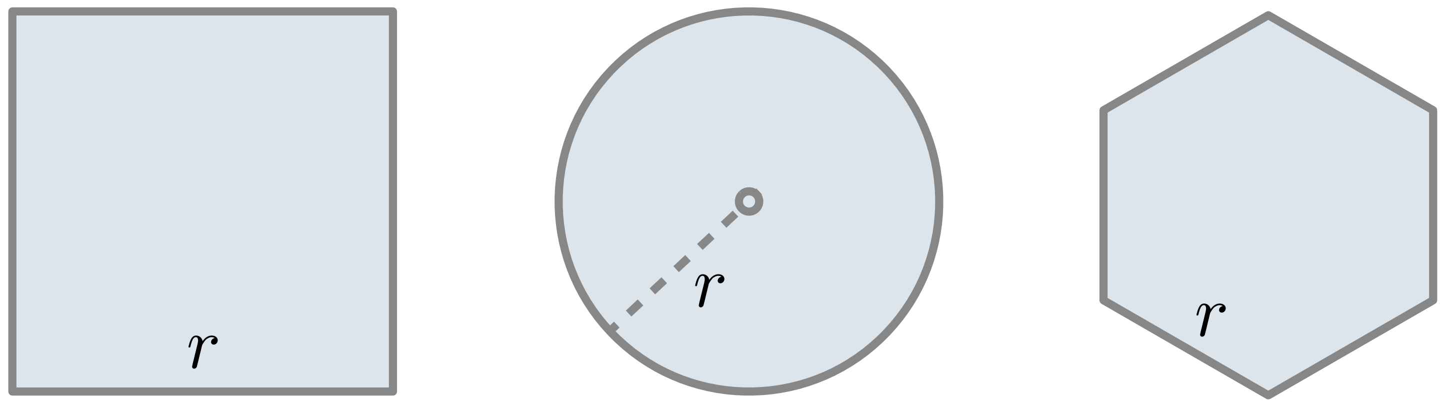


Area: r^2

Generalizing Patterns with Arguments

Regular geometric shapes relate length and area.

Shape:

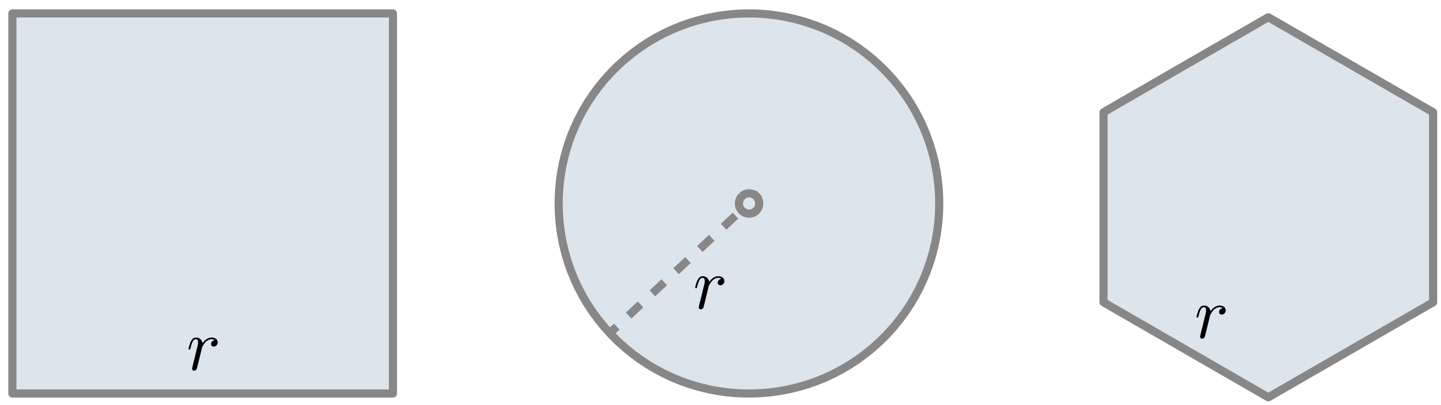


Area: r^2 $\pi \cdot r^2$

Generalizing Patterns with Arguments

Regular geometric shapes relate length and area.

Shape:



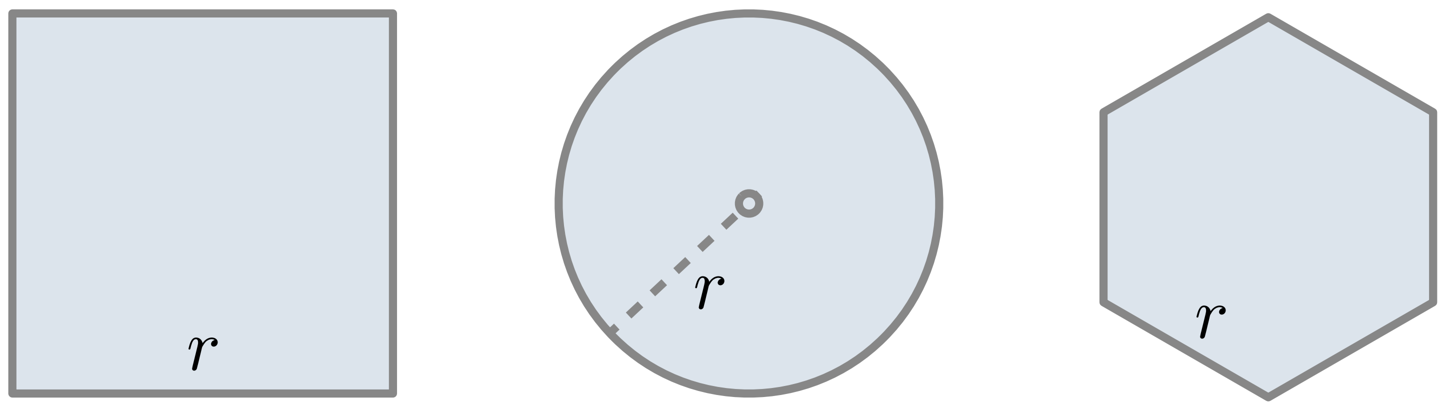
Area:

$$r^2 \qquad \pi \cdot r^2 \qquad \frac{3\sqrt{3}}{2} \cdot r^2$$

Generalizing Patterns with Arguments

Regular geometric shapes relate length and area.

Shape:

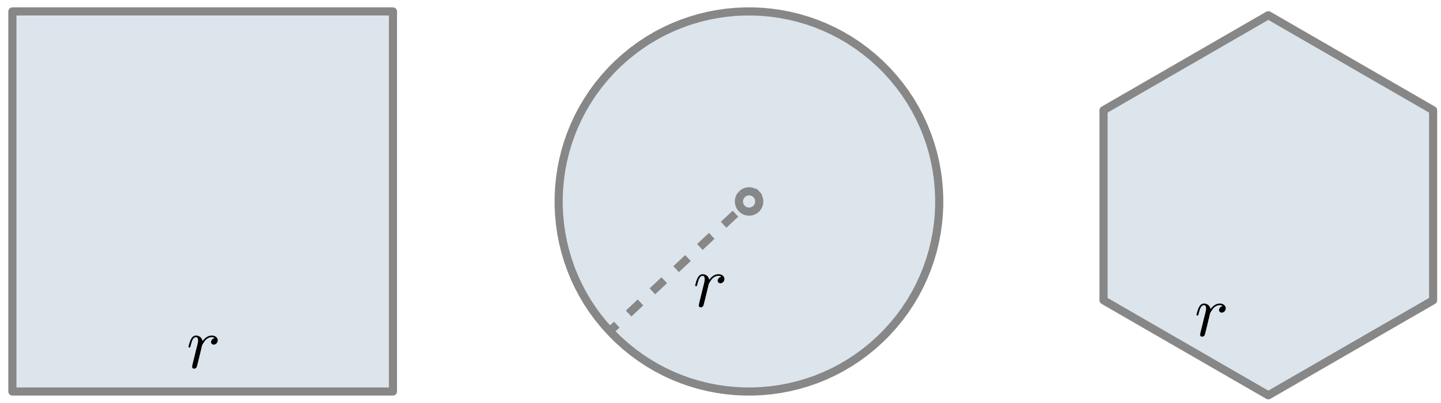


Area: $1 \cdot r^2$ $\pi \cdot r^2$ $\frac{3\sqrt{3}}{2} \cdot r^2$

Generalizing Patterns with Arguments

`Regular geometric shapes relate length and area.`

`Shape:`



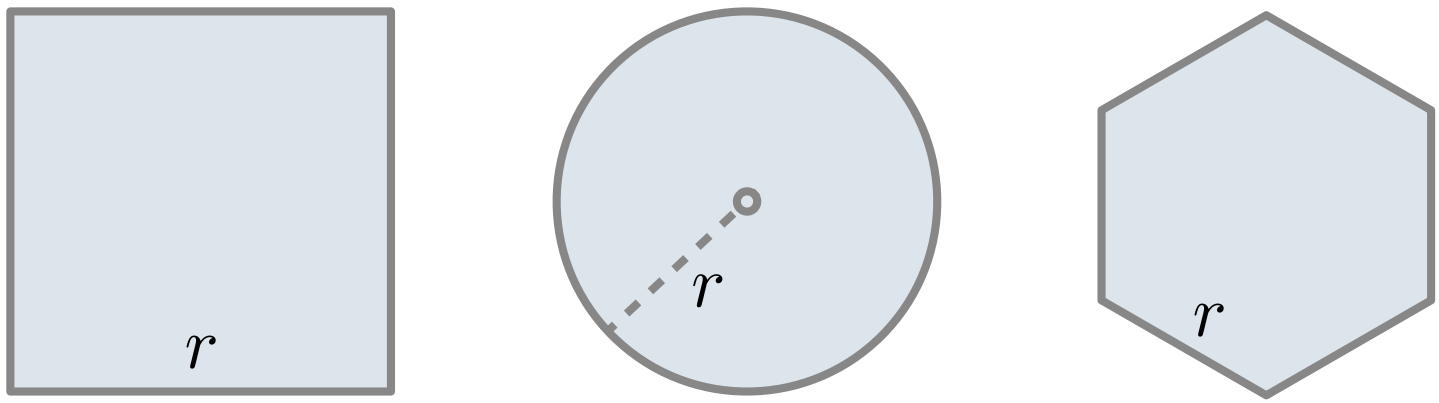
`Area:`

$$1 \cdot r^2 \qquad \pi \cdot r^2 \qquad \frac{3\sqrt{3}}{2} \cdot r^2$$

Generalizing Patterns with Arguments

Regular geometric shapes relate length and area.

Shape:



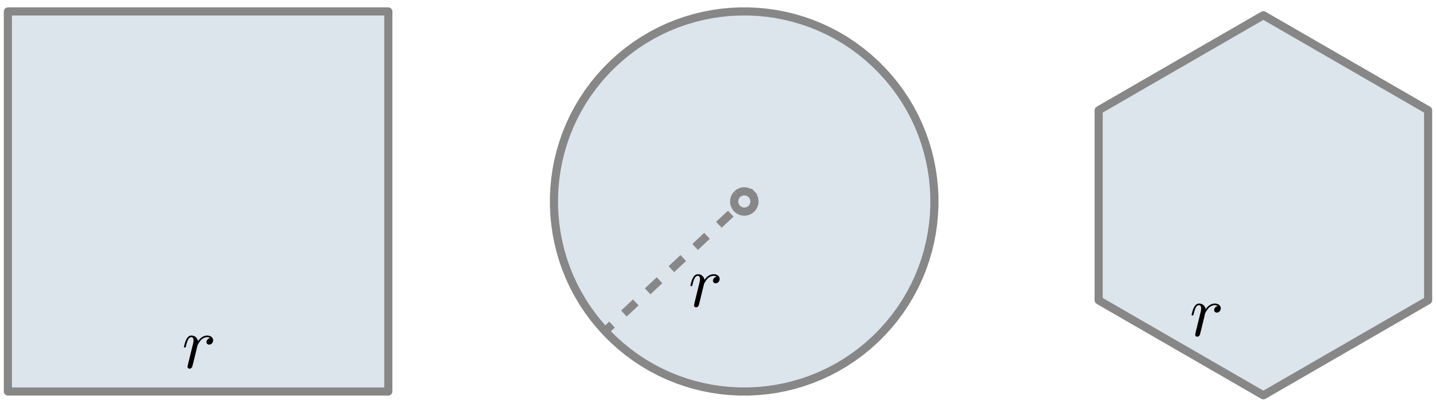
Area:

$1 \cdot r^2$ $\qquad$ $\pi \cdot r^2$ $\qquad$ $\frac{3\sqrt{3}}{2} \cdot r^2$

Generalizing Patterns with Arguments

Regular geometric shapes relate length and area.

Shape:

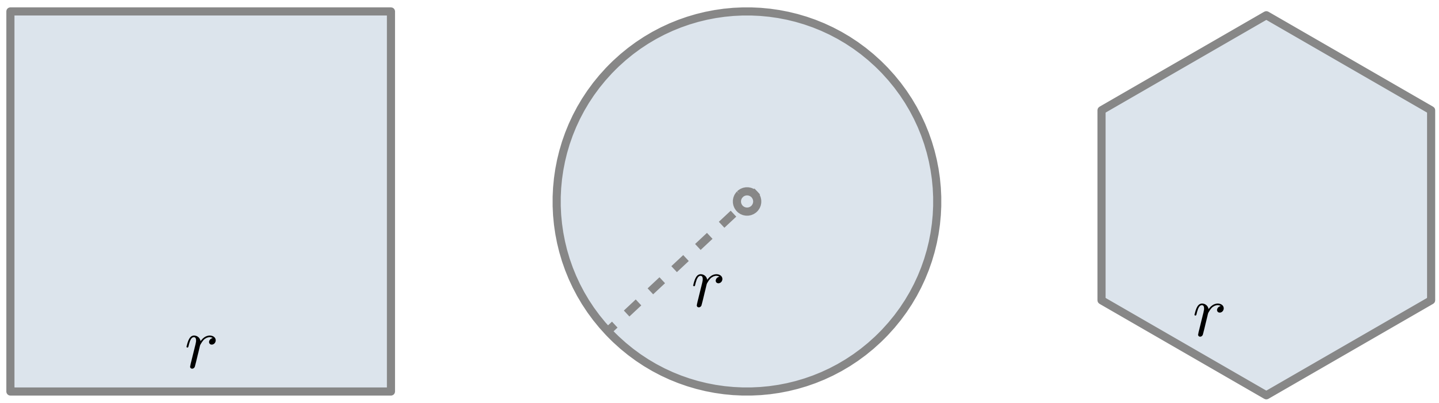


Area: $1 \cdot r^2$ $\pi \cdot r^2$ $\frac{3\sqrt{3}}{2} \cdot r^2$

Generalizing Patterns with Arguments

Regular geometric shapes relate length and area.

Shape:



Area: $\boxed{1} \cdot r^2$ $\boxed{\pi} \cdot r^2$ $\boxed{\frac{3\sqrt{3}}{2}} \cdot r^2$

Finding common structure allows for shared implementation

Generalizing Over Computational Processes

Generalizing Over Computational Processes

The common structure among functions may itself be a computational process, rather than a number.

Generalizing Over Computational Processes

The common structure among functions may itself be a computational process, rather than a number.

$$\sum_{k=1}^{5} k = 1 + 2 + 3 + 4 + 5 \qquad = 15$$

$$\sum_{k=1}^{5} k^3 = 1^3 + 2^3 + 3^3 + 4^3 + 5^3 \qquad = 225$$

$$\sum_{k=1}^{5} \frac{8}{(4k-3)\cdot(4k-1)} = \frac{8}{3} + \frac{8}{35} + \frac{8}{99} + \frac{8}{195} + \frac{8}{323} \qquad = 3.04$$

Generalizing Over Computational Processes

The common structure among functions may itself be a computational process, rather than a number.

$$\sum_{k=1}^{5} k = 1 + 2 + 3 + 4 + 5 \qquad = 15$$

$$\sum_{k=1}^{5} k^3 = 1^3 + 2^3 + 3^3 + 4^3 + 5^3 \qquad = 225$$

$$\sum_{k=1}^{5} \frac{8}{(4k-3)\cdot(4k-1)} = \frac{8}{3} + \frac{8}{35} + \frac{8}{99} + \frac{8}{195} + \frac{8}{323} \qquad = 3.04$$

Generalizing Over Computational Processes

The common structure among functions may itself be a computational process, rather than a number.

$$\sum_{k=1}^{5} \boxed{k} = 1 + 2 + 3 + 4 + 5 \qquad = 15$$

$$\sum_{k=1}^{5} \boxed{k^3} = 1^3 + 2^3 + 3^3 + 4^3 + 5^3 \qquad = 225$$

$$\sum_{k=1}^{5} \frac{8}{(4k-3)\cdot(4k-1)} = \frac{8}{3} + \frac{8}{35} + \frac{8}{99} + \frac{8}{195} + \frac{8}{323} \qquad = 3.04$$

Generalizing Over Computational Processes

```
The common structure among functions may itself be a
computational process, rather than a number.
```

$$\sum_{k=1}^{5} k = 1 + 2 + 3 + 4 + 5 \qquad = 15$$

$$\sum_{k=1}^{5} k^3 = 1^3 + 2^3 + 3^3 + 4^3 + 5^3 \qquad = 225$$

$$\sum_{k=1}^{5} \frac{8}{(4k-3)\cdot(4k-1)} = \frac{8}{3} + \frac{8}{35} + \frac{8}{99} + \frac{8}{195} + \frac{8}{323} \qquad = 3.04$$

Summation Example

```
def cube(k):
    return pow(k, 3)

def summation(n, term):
    """Sum the first n terms of a sequence.

    >>> summation(5, cube)
    225
    """
    total, k = 0, 1
    while k <= n:
        total, k = total + term(k), k + 1
    return total
```

Summation Example

```
def cube(k):
    return pow(k, 3)
```

Function of a single argument (not called term)

```
def summation(n, term):
    """Sum the first n terms of a sequence.

    >>> summation(5, cube)
    225
    """
    total, k = 0, 1
    while k <= n:
        total, k = total + term(k), k + 1
    return total
```

Summation Example

```
def cube(k):
    return pow(k, 3)

def summation(n, term):
    """Sum the first n terms of a sequence.

    >>> summation(5, cube)
    225
    """
    total, k = 0, 1
    while k <= n:
        total, k = total + term(k), k + 1
    return total
```

Function of a single argument (not called term)

A formal parameter that will be bound to a function

Summation Example

```
def cube(k):
    return pow(k, 3)

def summation(n, term):
    """Sum the first n terms of a sequence.

    >>> summation(5, cube)
    225
    """
    total, k = 0, 1
    while k <= n:
        total, k = total + term(k), k + 1
    return total
```

Function of a single argument (not called term)

A formal parameter that will be bound to a function

The function bound to term gets called here

Summation Example

```
def cube(k):
    return pow(k, 3)

def summation(n, term):
    """Sum the first n terms of a sequence.

    >>> summation(5, cube)
    225
    """
    total, k = 0, 1
    while k <= n:
        total, k = total + term(k), k + 1
    return total
```

Function of a single argument (not called term)

A formal parameter that will be bound to a function

The cube function is passed as an argument value

The function bound to term gets called here

Summation Example

```
def cube(k):
    return pow(k, 3)

def summation(n, term):
    """Sum the first n terms of a sequence.

    >>> summation(5, cube)
    225
    """
    total, k = 0, 1
    while k <= n:
        total, k = total + term(k), k + 1
    return total
```

Function of a single argument (not called term)

A formal parameter that will be bound to a function

The cube function is passed as an argument value

The function bound to term gets called here

$0 + 1^3 + 2^3 + 3^3 + 4^3 + 5^5$

Locally Defined Functions

Locally Defined Functions

Functions defined within other function bodies
are bound to names in the local frame

Locally Defined Functions

Functions defined within other function bodies
are bound to names in the local frame

```
def make_adder(n):
    """Return a function that takes one argument k and returns k + n.

    >>> add_three = make_adder(3)
    >>> add_three(4)
    7
    """
    def adder(k):
        return k + n
    return adder
```

Locally Defined Functions

Functions defined within other function bodies
are bound to names in the local frame

A function that
returns a function

```
def make_adder(n):
    """Return a function that takes one argument k and returns k + n.

    >>> add_three = make_adder(3)
    >>> add_three(4)
    7
    """
    def adder(k):
        return k + n
    return adder
```

Locally Defined Functions

Functions defined within other function bodies
are bound to names in the local frame

A function that
returns a function

```
def make_adder(n):
    """Return a function that takes one argument k and returns k + n.

    >>> add_three = make_adder(3)
    >>> add_three(4)
    7
    """
    def adder(k):
        return k + n
    return adder
```

The name add_three is
bound to a function

Locally Defined Functions

Functions defined within other function bodies
are bound to names in the local frame

A function that
returns a function

```
def make_adder(n):
    """Return a function that takes one argument k and returns k + n.

    >>> add_three = make_adder(3)
    >>> add_three(4)
    7
    """
    def adder(k):
        return k + n
    return adder
```

The name add_three is
bound to a function

A local
def statement

Locally Defined Functions

Functions defined within other function bodies
are bound to names in the local frame

A function that
returns a function

```
def make_adder(n):
    """Return a function that takes one argument k and returns k + n.

    >>> add_three = make_adder(3)
    >>> add_three(4)
    7
    """
    def adder(k):
        return k + n
    return adder
```

The name add_three is
bound to a function

A local
def statement

Can refer to names in
the enclosing function

Call Expressions as Operator Expressions

```
make_adder(1)(2)
```

```
def make_adder(n):
    def adder(k):
        return k + n
    return adder
make_adder(1)(2)
```

Call Expressions as Operator Expressions

`make_adder(1)(2)`

`make_adder(1)      (        2        )`

```
def make_adder(n):
    def adder(k):
        return k + n
    return adder
make_adder(1)(2)
```

Call Expressions as Operator Expressions

```
make_adder(1)(2)
```

$$\underbrace{\texttt{make\_adder(1)}}_{\textit{Operator}} \quad (\quad 2 \quad)$$

```
def make_adder(n):
    def adder(k):
        return k + n
    return adder
make_adder(1)(2)
```

Call Expressions as Operator Expressions

```
make_adder(1)(2)
```

```
make_adder(1)     (       2          )
```

Operator — *Operand 0*

```
def make_adder(n):
    def adder(k):
        return k + n
    return adder
make_adder(1)(2)
```

Call Expressions as Operator Expressions

`make_adder(1)(2)`

`make_adder(1)` `(` `2` `)`

Operator *Operand 0*

An expression that evaluates to a function

```
def make_adder(n):
    def adder(k):
        return k + n
    return adder
make_adder(1)(2)
```

Call Expressions as Operator Expressions

`make_adder(1)(2)`

`make_adder(1)` (`2`)

Operator — An expression that evaluates to a function

Operand 0 — An expression that evaluates to any value

```
def make_adder(n):
    def adder(k):
        return k + n
    return adder
make_adder(1)(2)
```

The Purpose of Higher-Order Functions

The Purpose of Higher-Order Functions

Functions are first-class: Functions can be manipulated as values in our programming language.

The Purpose of Higher-Order Functions

Functions are first-class: Functions can be manipulated as values in our programming language.

Higher-order function: A function that takes a function as an argument value or returns a function as a return value

The Purpose of Higher-Order Functions

Functions are first-class: Functions can be manipulated as values in our programming language.

Higher-order function: A function that takes a function as an argument value or returns a function as a return value

Higher-order functions:

The Purpose of Higher-Order Functions

Functions are first-class: Functions can be manipulated as values in our programming language.

Higher-order function: A function that takes a function as an argument value or returns a function as a return value

Higher-order functions:

- Express general methods of computation

The Purpose of Higher-Order Functions

Functions are first-class: Functions can be manipulated as values in our programming language.

Higher-order function: A function that takes a function as an argument value or returns a function as a return value

Higher-order functions:

- Express general methods of computation
- Remove repetition from programs

The Purpose of Higher-Order Functions

Functions are first-class: Functions can be manipulated as values in our programming language.

Higher-order function: A function that takes a function as an argument value or returns a function as a return value

Higher-order functions:

- Express general methods of computation
- Remove repetition from programs
- Separate concerns among functions

Pig Introduction

(Demo)