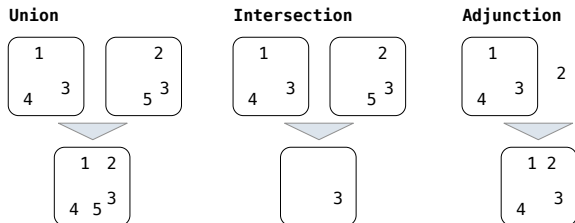


The interface for sets:

- Membership testing: Is a value an element of a set?
- Adjunction: Return a set with all elements in s and a value v.
- Union: Return a set with all elements in set1 or set2.
- Intersection: Return a set with any elements in set1 and set2.

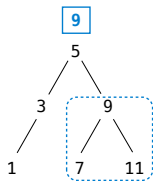


Proposal 1: A set is represented by a recursive list that contains no duplicate items.

Proposal 2: A set is represented by a recursive list with unique elements ordered from least to greatest.

Proposal 3: A set is represented as a Tree. Each entry is:

- Larger than all entries in its left branch and
- Smaller than all entries in its right branch



	Proposal 1	2	3
Membership	$\Theta(n)$	$\Theta(n)$	$\Theta(\log n)$
Adjunction	$\Theta(n)$	$\Theta(n)$	$\Theta(\log n)$
Union	$\Theta(n^2)$	$\Theta(n)$	$\Theta(n)$
Intersection	$\Theta(n^2)$	$\Theta(n)$	$\Theta(n)$

If 9 is in the set, it is somewhere in this branch

Exceptions are raised with a raise statement.

```
raise <expression>
```

<expression> must evaluate to an exception instance or class.

Exceptions are constructed like any other object; they are just instances of classes that inherit from BaseException.

```
try:
    <try suite>
except <exception class> as <name>:
    <except suite>
...
```

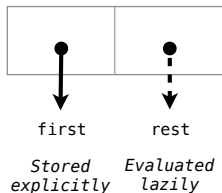
The <try suite> is executed first;

If, during the course of executing the <try suite>, an exception is raised that is not handled otherwise, and

If the class of the exception inherits from <exception class>, then

The <except suite> is executed, with <name> bound to the exception

Streams are lazily computed recursive lists



```
class Stream(object):
    """A lazily computed recursive list."""
    class empty(object):
        def __repr__(self):
            return 'Stream.empty'
        empty = empty()

    def __init__(self, first, compute_rest=lambda: Stream.empty):
        assert callable(compute_rest), 'compute_rest must be callable.'
        self.first = first
        self._compute_rest = compute_rest
        self._rest = None

    @property
    def rest(self):
        """Please don't reference directly!"""
        """Return the rest of the stream, computing it if necessary."""
        if self._compute_rest is not None:
            self._rest = self._compute_rest()
            self._compute_rest = None
        return self._rest

    def integer_stream(first=1):
        def compute_rest():
            return integer_stream(first+1)
        return Stream(first, compute_rest)

    def filter_stream(fn, s):
        if s is Stream.empty:
            return s
        def compute_rest():
            return filter_stream(fn, s.rest)
        if fn(s.first):
            return Stream(s.first, compute_rest)
        else:
            return compute_rest()

    def primes(pos_stream):
        def not_divisible(x):
            return x % pos_stream.first != 0
        def compute_rest():
            return primes(filter_stream(not_divisible, pos_stream.rest))
        return Stream(pos_stream.first, compute_rest)

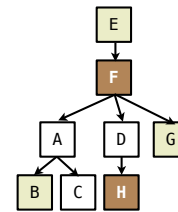
    def map_stream(fn, s):
        if s is Stream.empty:
            return s
        def compute_rest():
            return map_stream(fn, s.rest)
        return Stream(fn(s.first), compute_rest)
```

A simple fact expression in the Logic language declares a relation to be true.

Language Syntax:

- A relation is a Scheme list.
- A fact expression is a Scheme list of relations.

```
logic> (fact (parent delano herbert))
logic> (fact (parent abraham barack))
logic> (fact (parent abraham clinton))
logic> (fact (parent fillmore abraham))
logic> (fact (parent fillmore delano))
logic> (fact (parent fillmore grover))
logic> (fact (parent eisenhower fillmore))
```



Relations can contain relations in addition to atoms.

```
logic> (fact (dog (name abraham) (color white)))
logic> (fact (dog (name barack) (color tan)))
logic> (fact (dog (name clinton) (color white)))
logic> (fact (dog (name delano) (color white)))
logic> (fact (dog (name eisenhower) (color tan)))
logic> (fact (dog (name fillmore) (color brown)))
logic> (fact (dog (name grover) (color tan)))
logic> (fact (dog (name herbert) (color brown)))
```



Variables can refer to atoms or relations in queries.

```
logic> (query (parent abraham ?child))
Success!
child: barack
child: clinton

logic> (query (dog (name clinton) (color ?color)))
Success!
color: white

logic> (query (dog (name clinton) ?info))
Success!
info: (color white)
```

A fact can include multiple relations and variables as well:

```
(fact <conclusion> <hypothesis> <hypothesis1> ... <hypothesisN>)
```

Means <conclusion> is true if all <hypothesisk> are true.

```
logic> (fact (child ?c ?p) (parent ?p ?c))
logic> (query (child herbert delano))
Success!

logic> (query (child eisenhower clinton))
Failure.

logic> (query (child ?child fillmore))
Success!
child: abraham
child: delano
child: grover
```

A fact is recursive if the same relation is mentioned in a hypothesis and the conclusion.

```
logic> (fact (ancestor ?a ?y) (parent ?a ?y))
logic> (fact (ancestor ?a ?y) (parent ?a ?z) (ancestor ?z ?y))

logic> (query (ancestor ?a herbert))
Success!
a: delano
a: fillmore
a: eisenhower
```

The Logic interpreter performs a search in the space of relations for each query to find a satisfying assignment.

- (parent delano herbert) ; (1), a simple fact
- (ancestor delano herbert) ; (2), from (1) and the 1st ancestor fact
- (parent fillmore delano) ; (3), a simple fact
- (ancestor fillmore herbert) ; (4), from (2), (3), & the 2nd ancestor fact

Two lists append to form a third list if:

- The first list is empty and the second and third are the same
- The rest of 1 and 2 append to form the rest of 3

```
logic> (fact (append-to-form () ?x ?x))
logic> (fact (append-to-form (?a . ?r) ?y (?a . ?z))
        (append-to-form ?r ?y ?z))
```

```
class Letters(object):
    """An iterator over letters."""
    def __init__(self):
        self.current = 'a'
    def __next__(self):
        if self.current > 'd':
            raise StopIteration
        result = self.current
        self.current = chr(ord(result)+1)
        return result
    def __iter__(self):
        return self

def letters_generator():
    """A generator function."""
    current = 'a'
    while current <= 'd':
        yield current
        current = chr(ord(current)+1)

class LetterIterable(object):
    """An iterable over letters."""
    def __iter__(self):
        current = 'a'
        while current <= 'd':
            yield current
            current = chr(ord(current)+1)

>>> letters = Letters()
>>> letters.__next__()
'a'
>>> letters.__next__()
'b'
>>> letters.__next__()
'c'
>>> letters.__next__()
'd'
>>> letters.__next__()
Traceback ...
StopIteration

>>> for x in Letters():
    print(x)
a
b
c
d

>>> for x in letters_generator():
    print(x)
a
b
c
d

>>> for x in LetterIterable():
    print(x)
a
b
c
d
```

- A generator is an iterator backed by a generator function.
- When a generator function is called, it returns a generator.

Scheme programs consist of expressions, which can be:

- Primitive expressions: 2, 3.3, true, +, quotient, ...
- Combinations: (quotient 10 2), (not true), ...

Numbers are self-evaluating; symbols are bound to values. Call expressions have an operator and 0 or more operands.

A combination that is not a call expression is a *special form*:

- If expression: (if <predicate> <consequent> <alternative>)
- Binding names: (define <name> <expression>)
- New procedures: (define (<name> <formal parameters>) <body>)

```
> (define pi 3.14)      > (define (abs x)
> (* pi 2)             (if (< x 0)
6.28                   (- x)
                       x))
> (abs -3)
3
```

Lambda expressions evaluate to anonymous functions.

```
(lambda (<formal-parameters>) <body>)
```

Two equivalent expressions:

```
(define (plus4 x) (+ x 4))
(define plus4 (lambda (x) (+ x 4)))
```

An operator can be a call expression too:

```
((lambda (x y z) (+ x y (square z))) 1 2 3)
```



In the late 1950s, computer scientists used confusing names.

- **cons**: Two-argument procedure that **creates a pair**
- **car**: Procedure that returns the **first element** of a pair
- **cdr**: Procedure that returns the **second element** of a pair
- **nil**: The empty list

They also used a non-obvious notation for recursive lists.

- A (recursive) Scheme list is a pair in which the second element is nil or a Scheme list.
- Scheme lists are written as space-separated combinations.
- A dotted list has an arbitrary value for the second element of the last pair. Dotted lists may not be well-formed lists.

```
> (define x (cons 1 2))
> x
(1 . 2)
> (car x)
1
> (cdr x)
2
> (cons 1 (cons 2 (cons 3 (cons 4 nil))))
(1 2 3 4)
```

Not a well-formed list!

Symbols normally refer to values; how do we refer to symbols?

```
> (define a 1)
> (define b 2)
> (list a b)
(1 2)
```

No sign of "a" and "b" in the resulting value

Quotation is used to refer to symbols directly in Lisp.

```
> (list 'a 'b)
(a b)
> (list 'a b)
(a 2)
```

Symbols are now values

Quotation can also be applied to combinations to form lists.

```
> (car '(a b c))
a
> (cdr '(a b c))
(b c)
```

Dots can be used in a quoted list to specify the second element of the final pair.

```
> (cdr (cdr '(1 2 . 3)))
3
```

However, dots appear in the output only of ill-formed lists.

```
> '(1 2 . 3)      [1] → [2] → [3]
(1 2 . 3)
> '(1 2 . (3 4)) [1] → [2] → [3] → [4] → nil
(1 2 3 4)
> '(1 2 3 . nil) [1] → [2] → [3] → nil
(1 2 3)
> (cdr '((1 2) . (3 4 . (5))))
(3 4 5)
```

The way in which names are looked up in Scheme and Python is called *lexical scope* (or *static scope*).

Lexical scope: The parent of a frame is the environment in which a procedure was *defined*. (lambda ...)

Dynamic scope: The parent of a frame is the environment in which a procedure was *called*. (mu ...)

```
> (define f (mu (x) (+ x y)))
> (define g (lambda (x y) (f (+ x y))))
> (g 3 7)
13
```

```
for <name> in <expression>:
  <suite>
```

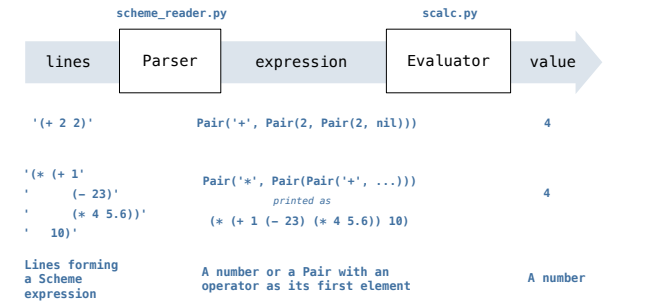
1. Evaluate the header <expression>, which yields an iterable object.
2. For each element in that sequence, in order:
 - A. Bind <name> to that element in the first frame of the current environment.
 - B. Execute the <suite>.

An iterable object has a method `__iter__` that returns an iterator.

```
>>> counts = [1, 2, 3]
>>> for item in counts:
>>>     print(item)
1
2
3
```

```
>>> items = counts.__iter__()
>>> try:
>>>     while True:
>>>         item = items.__next__()
>>>         print(item)
>>>     except StopIteration:
>>>         pass
```

A basic interpreter has two parts: a *parser* and an *evaluator*.



A Scheme list is written as elements in parentheses:

```
(<element> <element> ... <element>)
```

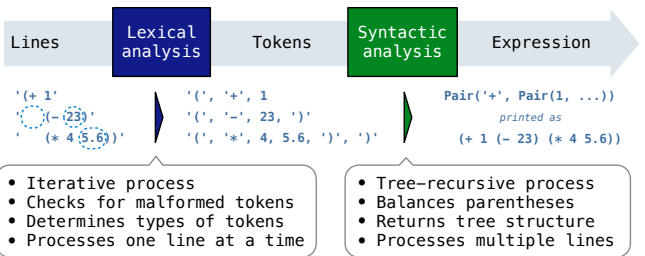
A recursive Scheme list

Each <element> can be a combination or atom (primitive).

```
(+ (* 3 (+ (* 2 4) (+ 3 5))) (+ (- 10 7) 6))
```

The task of *parsing* a language involves coercing a string representation of an expression to the expression itself. Parsers must validate that expressions are well-formed.

A Parser takes a sequence of lines and returns an expression.

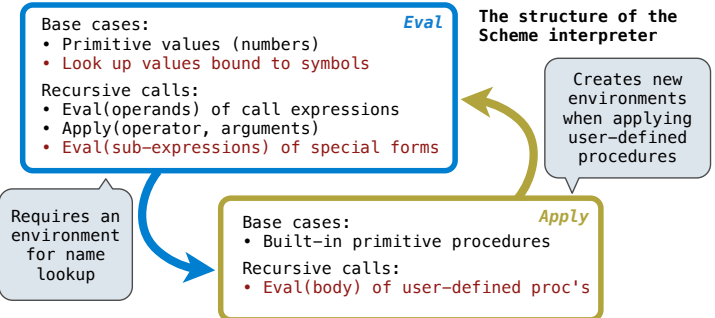


Syntactic analysis identifies the hierarchical structure of an expression, which may be nested.

Each call to `scheme_read` consumes the input tokens for exactly one expression.

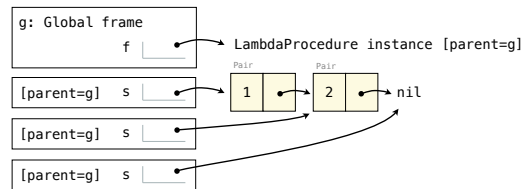
Base case: symbols and numbers

Recursive call: `scheme_read` sub-expressions and combine them



To apply a user-defined procedure, create a new frame in which formal parameters are bound to argument values, whose parent is the **env** of the procedure, then evaluate the body of the procedure in the environment that starts with this new frame.

```
(define (f s) (if (null? s) '(3) (cons (car s) (f (cdr s)))))
(f (list 1 2))
```



A procedure call that has not yet returned is *active*. Some procedure calls are *tail calls*. A Scheme interpreter should support an unbounded number of active tail calls. A tail call is a call expression in a *tail context*, which are:

- The last body expression in a **lambda** expression
- Expressions 2 & 3 (consequent & alternative) in a tail context **if** expression

```
(define (factorial n k)
  (if (= n 0) k
      (factorial (- n 1) (* k n))))

(define (length s)
  (if (null? s) 0
      (+ 1 (length (cdr s)))))

(define (length-tail s)
  (define (length-iter s n)
    (if (null? s) n
        (length-iter (cdr s) (+ 1 n))))
  (length-iter s 0))
```

Not a tail call (length (cdr s))

Recursive call is a tail call (length-iter (cdr s) (+ 1 n))