61A Lecture 5

Wednesday, September 7

Office Hours: You Should Go!

You are not alone!

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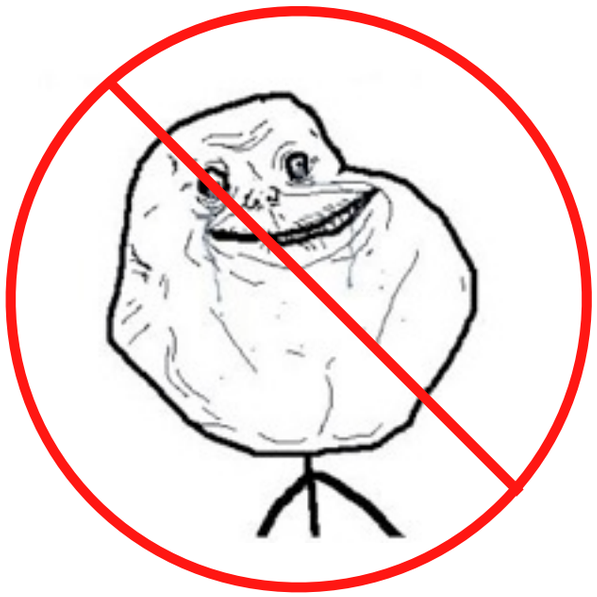

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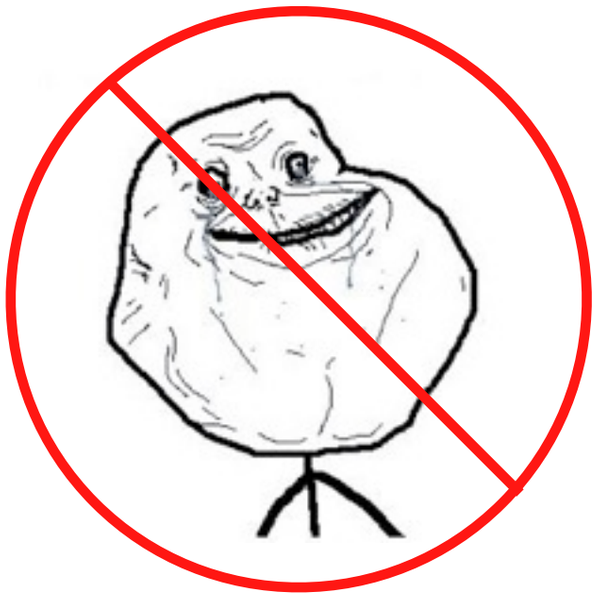

http://inst.eecs.berkeley.edu/~cs61a/fa11/www/staff.html

Reminder: Multiple Assignment & Return Values

```
from operator import floordiv, mod

def divide_exact(n, d):
    """Return the quotient and remainder of dividing n by d.

    >>> q, r = divide_exact(13, 5)
    >>> q
    2
    >>> r
    3
    """
    return floordiv(n, d), mod(n, d)
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Reminder: Multiple Assignment & Return Values

Integer division, which rounds down

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Multiple return values, separated by commas

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Multiple assignment to two names

Multiple return values, separated by commas

The Structure of Project 1

Two functions implement the game simulation

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Pseudo-code
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def play(...):
    while game is not over:
        get a plan (from the current player's strategy)
        call take_turn with a dice and plan
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Pseudo-code
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    while game is not over:
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def take_turn(...):
    while turn is not over:
        get an action (from plan) and outcome (from dice)
        call an action
    return points scored during the turn
```

The Structure of Project 1

Four types of functions are involved in simulating game

Domain **Range**

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Action	(integer, integer)	(integer, integer, boolean)

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Action	`(integer, integer)`	`(integer, integer, boolean)`
Plan	`integer`	`Action`

Two arguments

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	Domain	**Range**
Action	`(integer, integer)`	`(integer, integer, boolean)`
	Two arguments	Three return values
Plan	integer	Action
Strategy	(integer, integer)	Plan

The Structure of Project 1

Four types of functions are involved in simulating game

	Domain	**Range**
Action	`(integer, integer)`	`(integer, integer, boolean)`
Plan	`integer`	`Action`
Strategy	`(integer, integer)`	`Plan`
Dice	*No arguments*	`integer`

Two arguments

Three return values

The Purpose of Higher-Order Functions

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- Remove repetition from programs
- Separate concerns among functions

Review: Summation Example

```
def cube(k):
    return pow(k, 3)

def summation(n, term):
    """Sum the first n terms of a sequence.

    >>> summation(5, cube)
    225
    """
    total, k = 0, 1
    while k <= n:
        total, k = total + term(k), k + 1
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The cube function is passed as an argument value

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$0 + 1^3 + 2^3 + 3^3 + 4^3 + 5^5$

Environments Enable Higher-Order Functions!

Functions as arguments:

Functions as return values:

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Our current environment model handles that!

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We need to extend the model a little

Functions need to know where they were defined

Almost everything stays the same

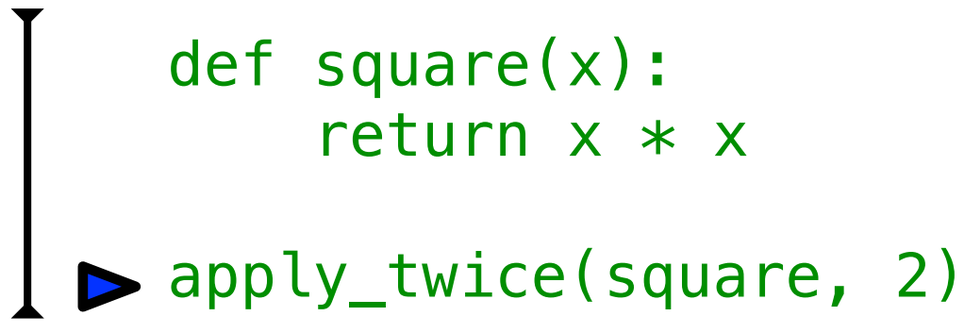
Names and Environments with Functional Values

```
def apply_twice(f, x):
    return f(f(x))

def square(x):
    return x * x

apply_twice(square, 2)
```

Names and Environments with Functional Values



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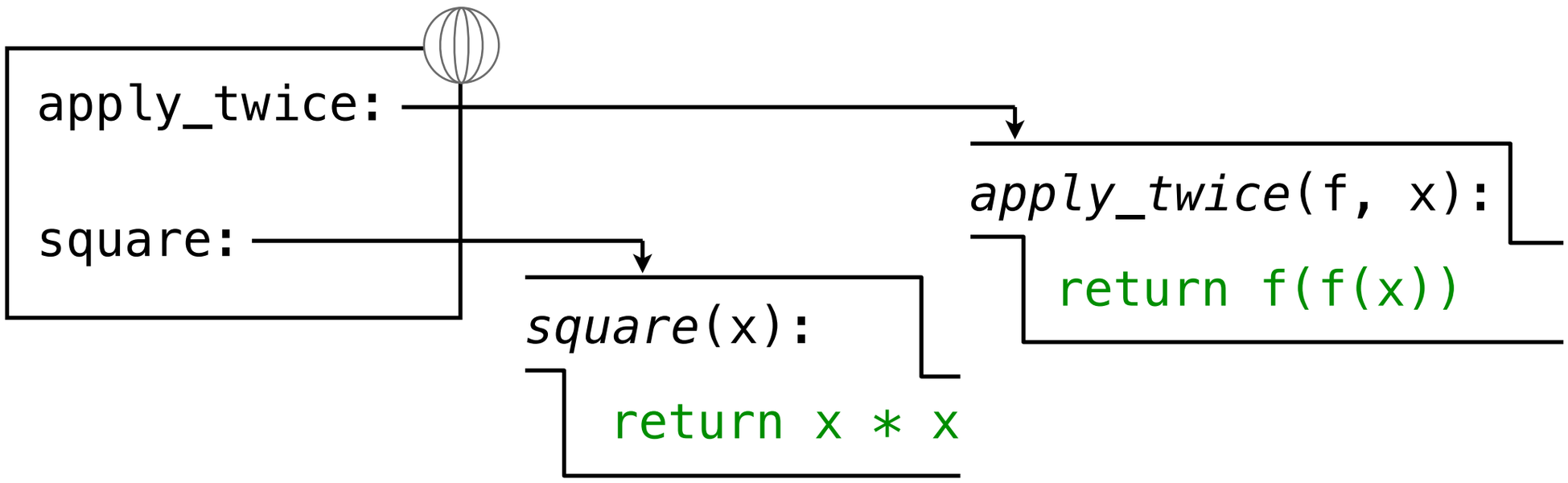


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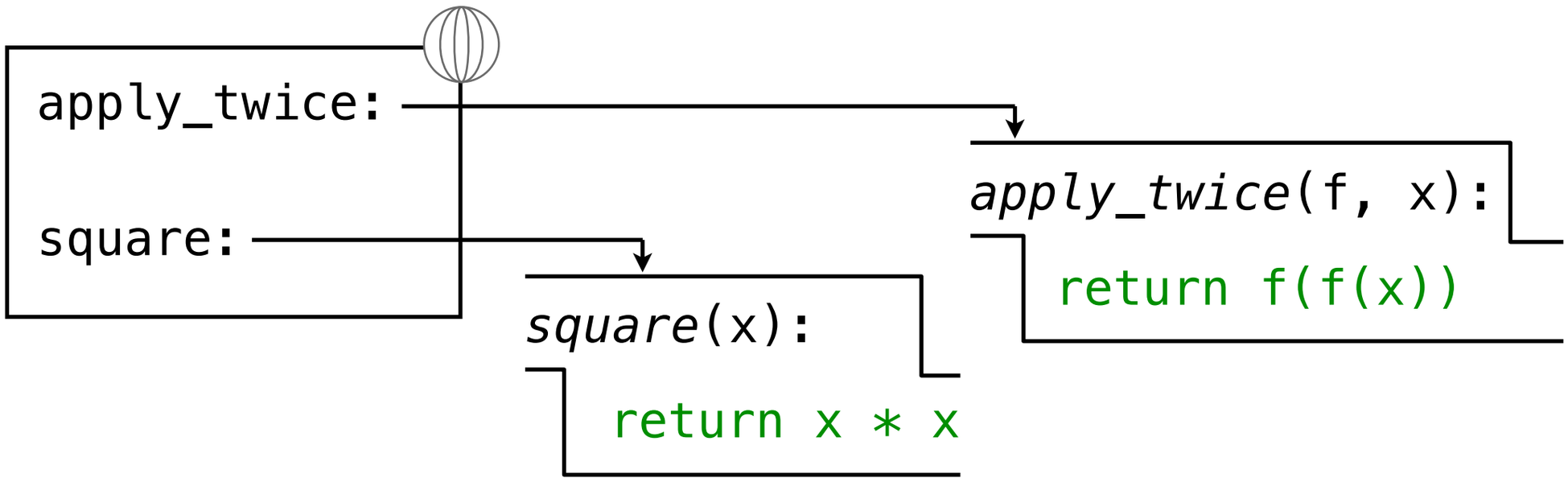

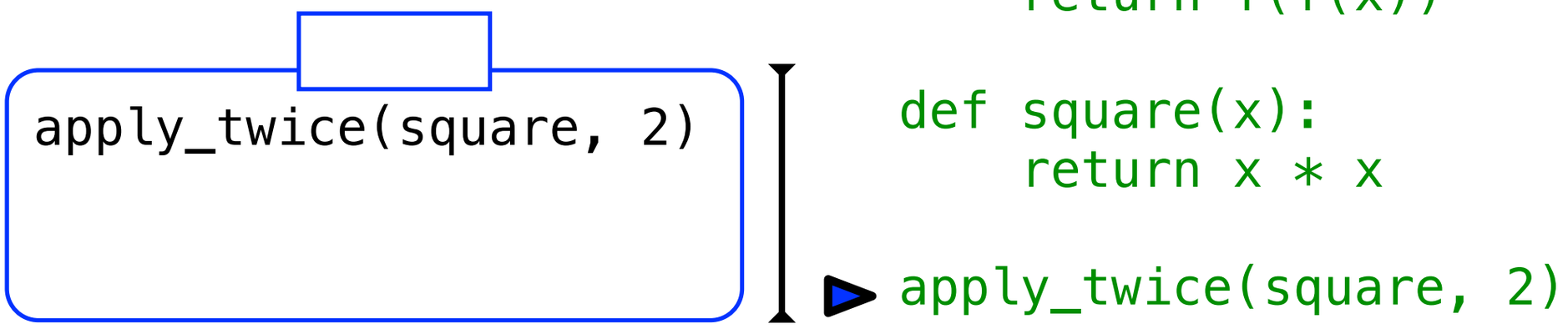

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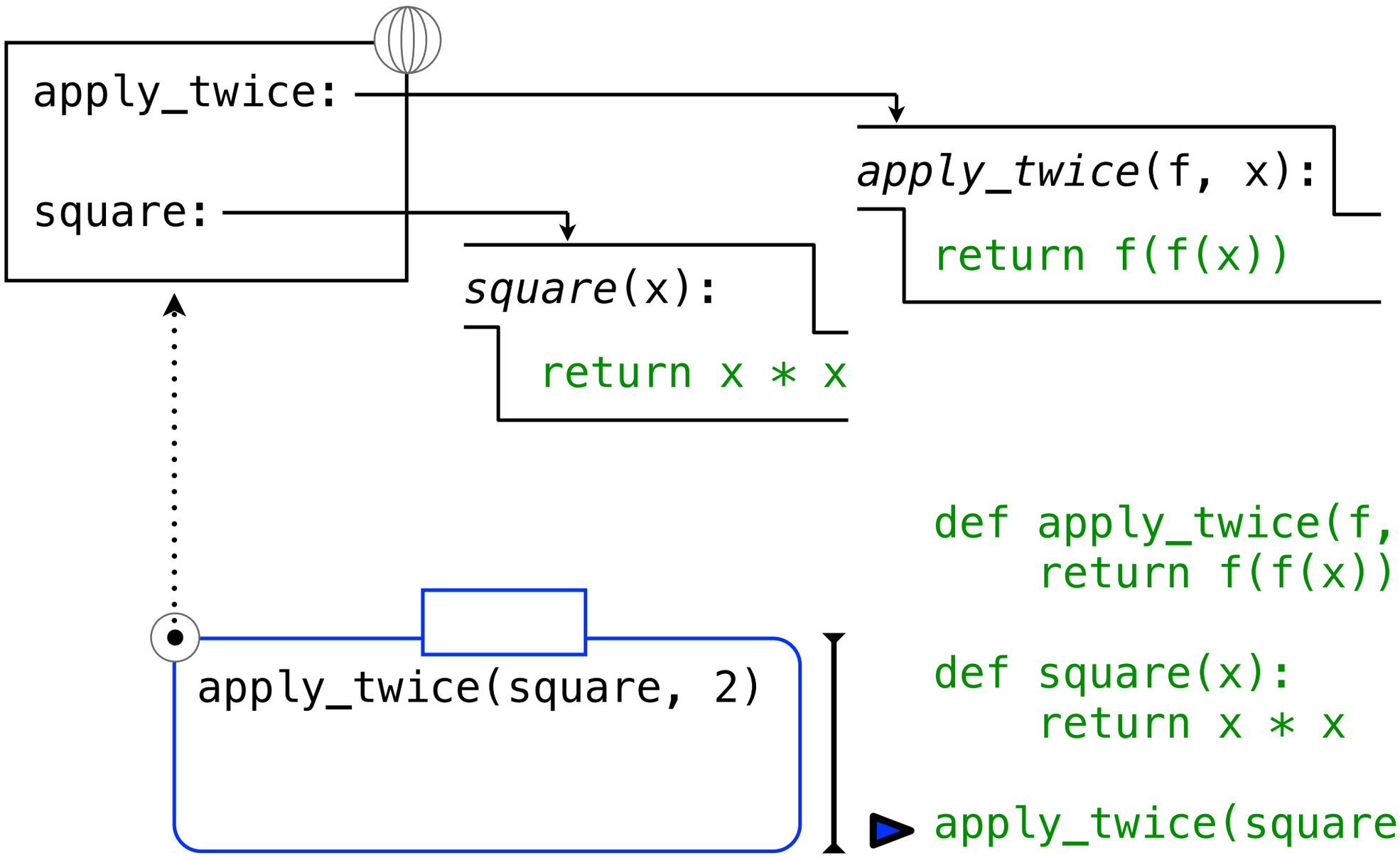

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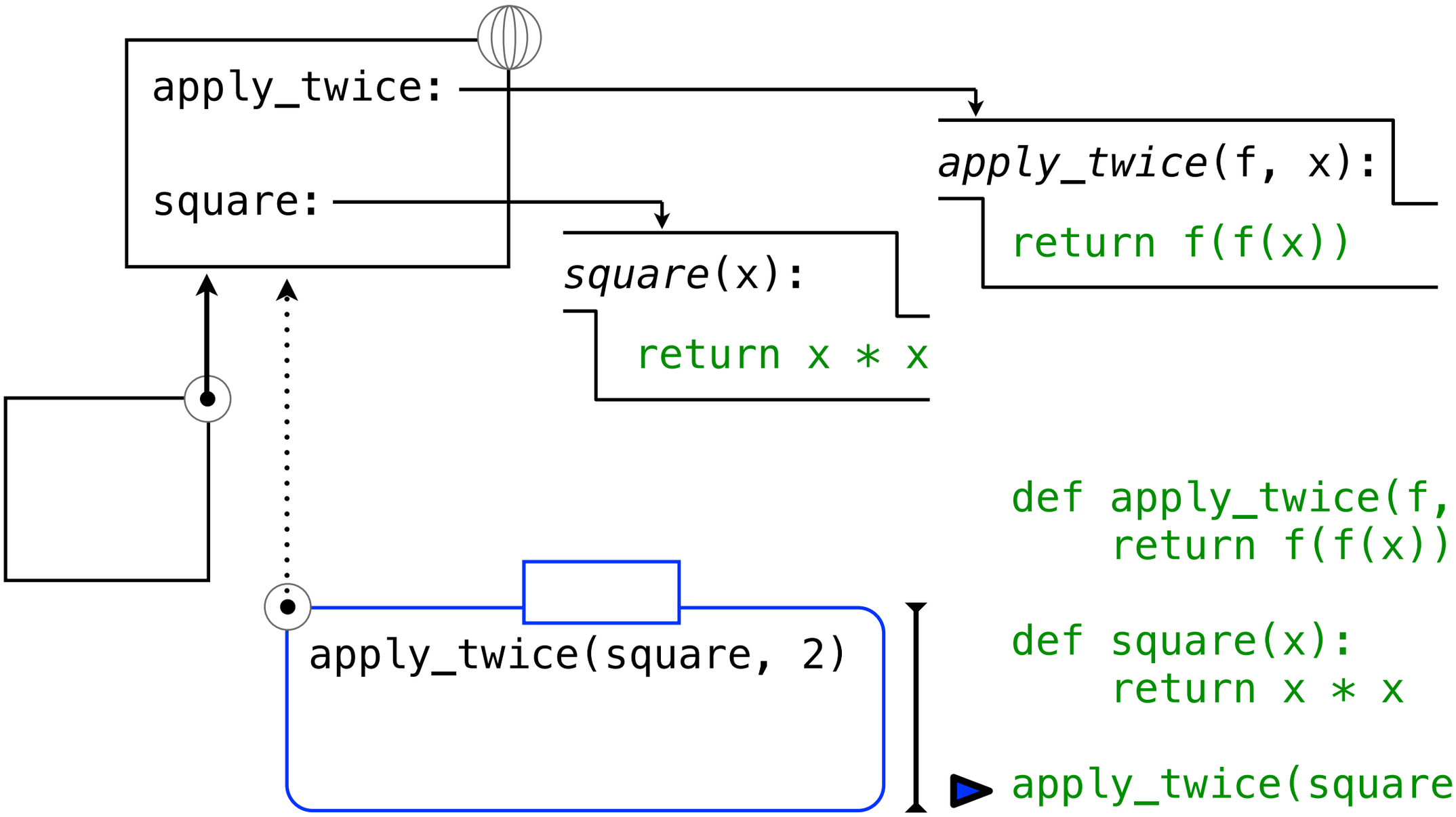

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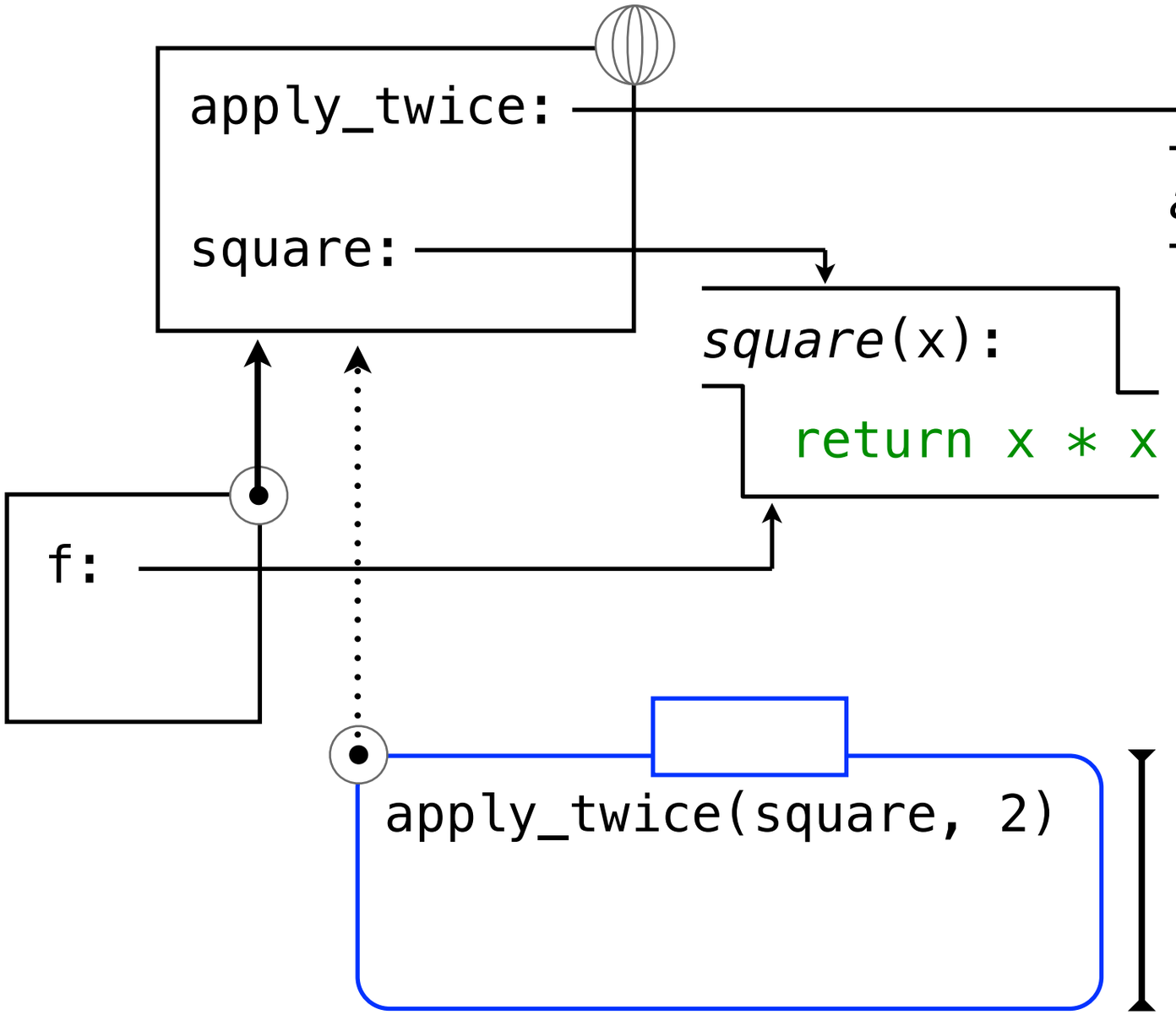


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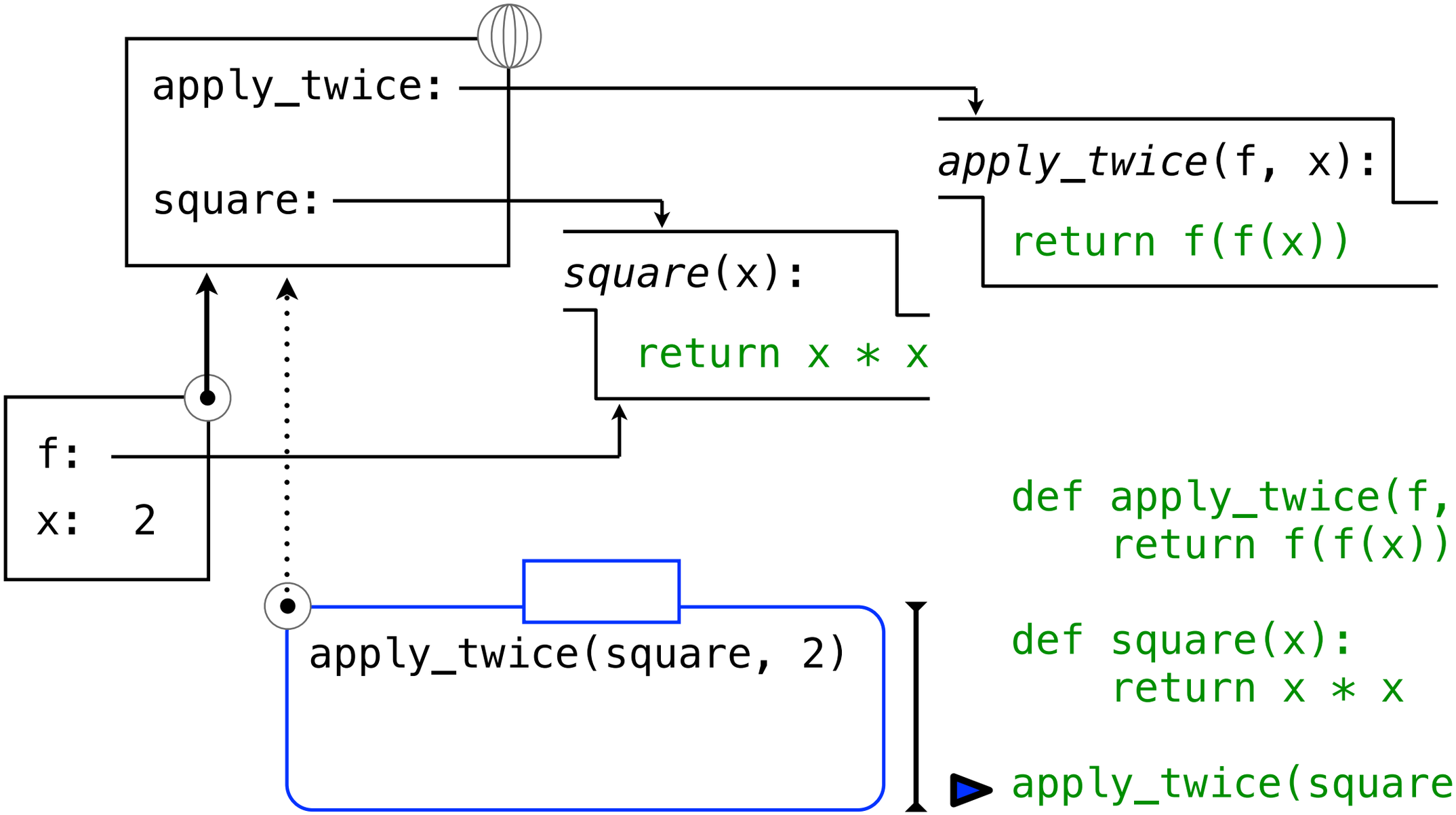


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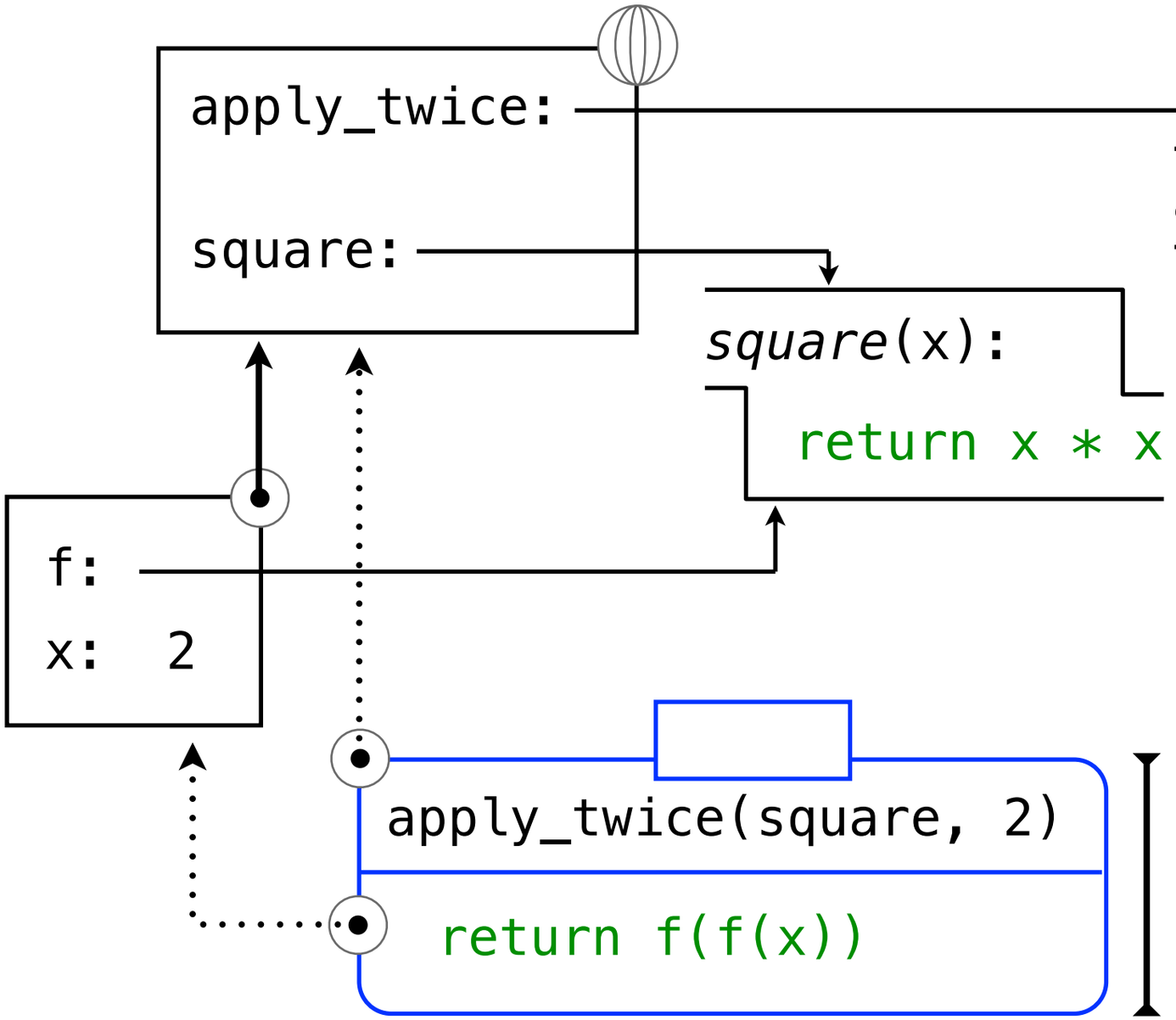


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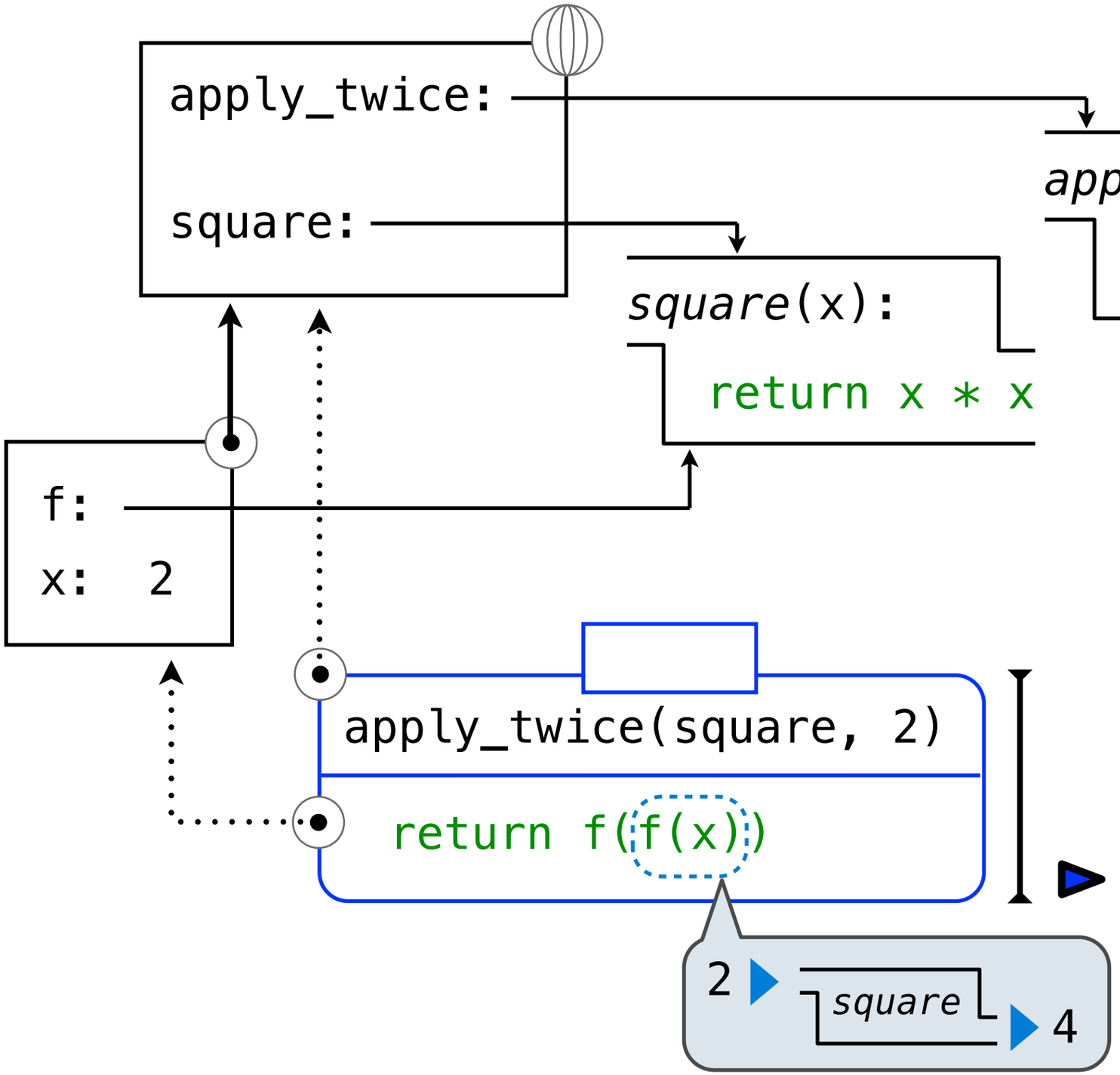


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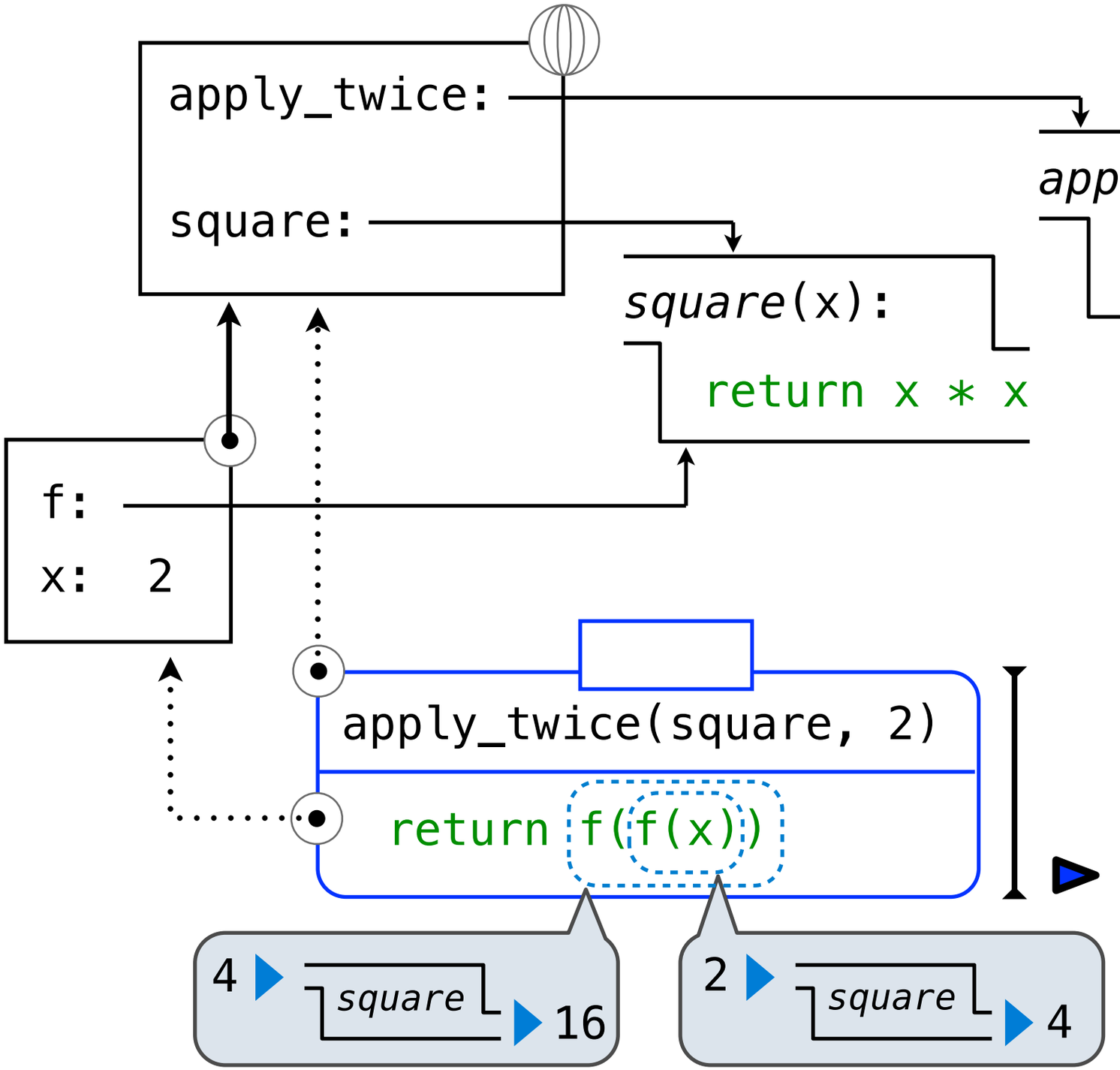


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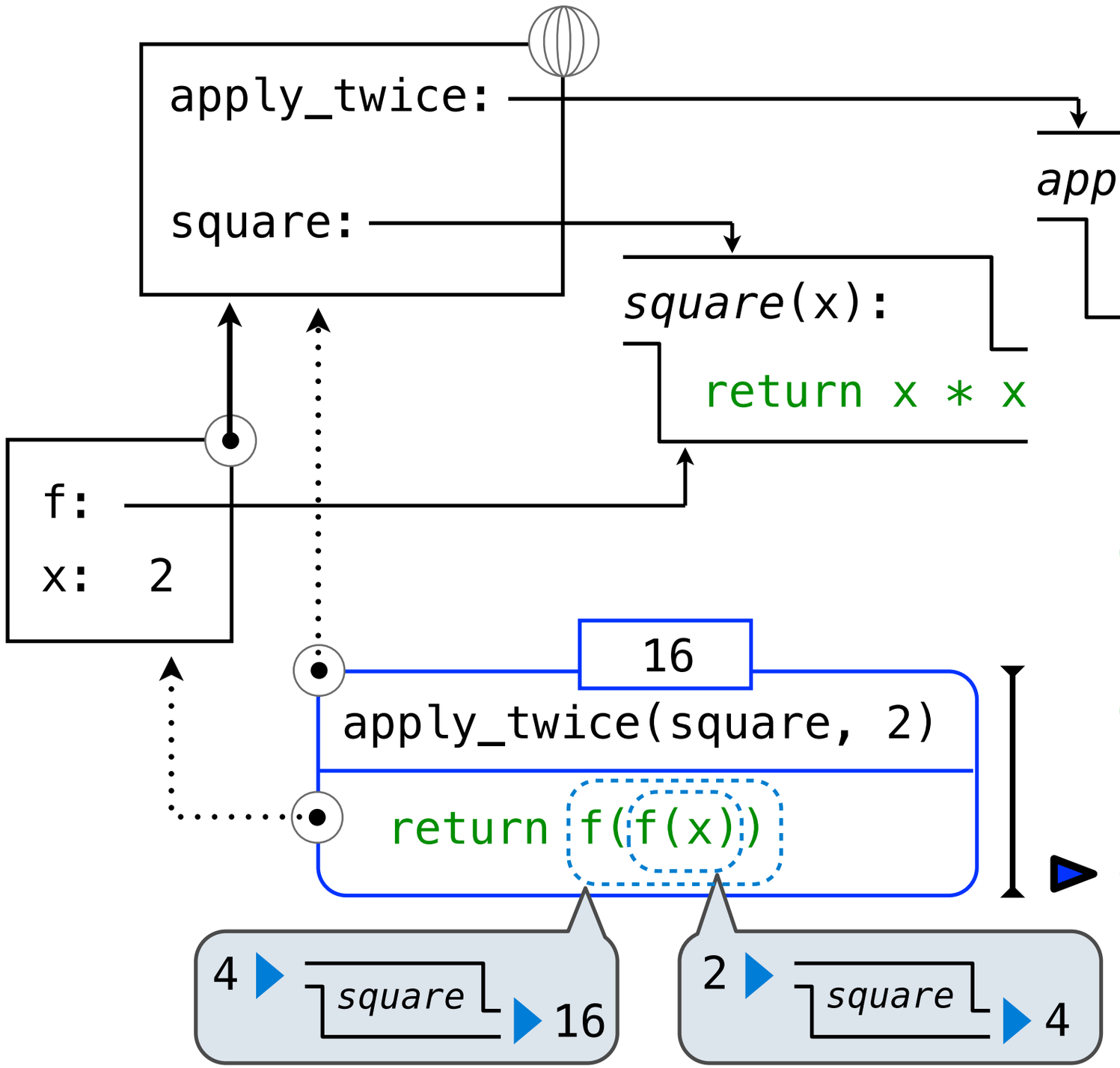


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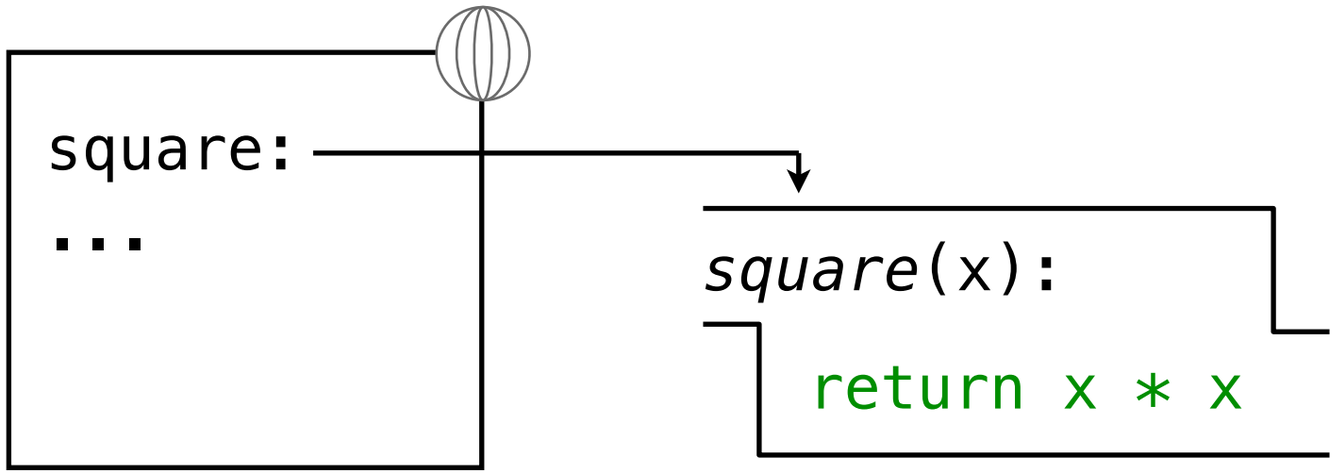


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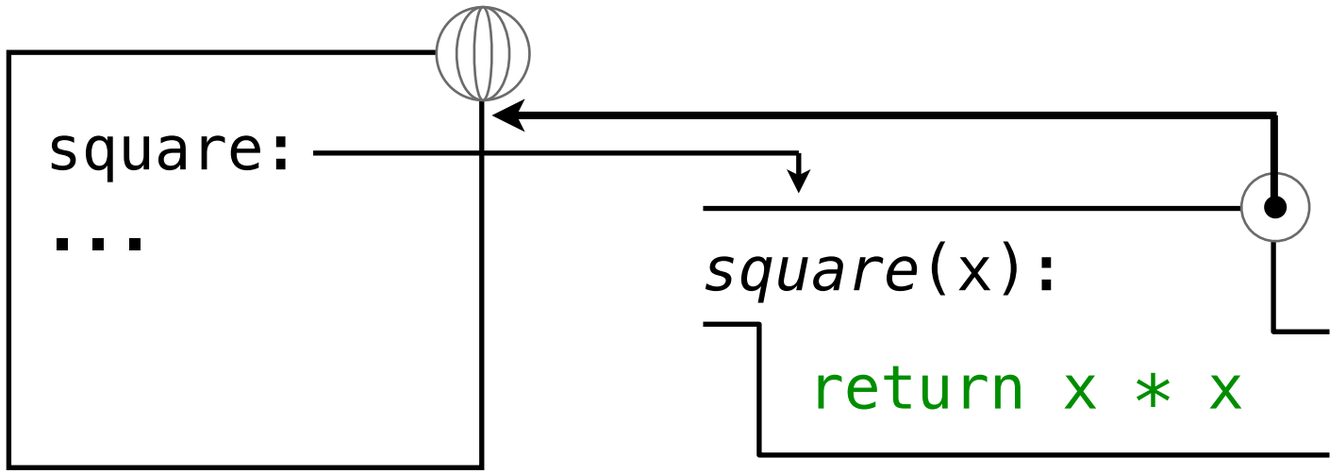


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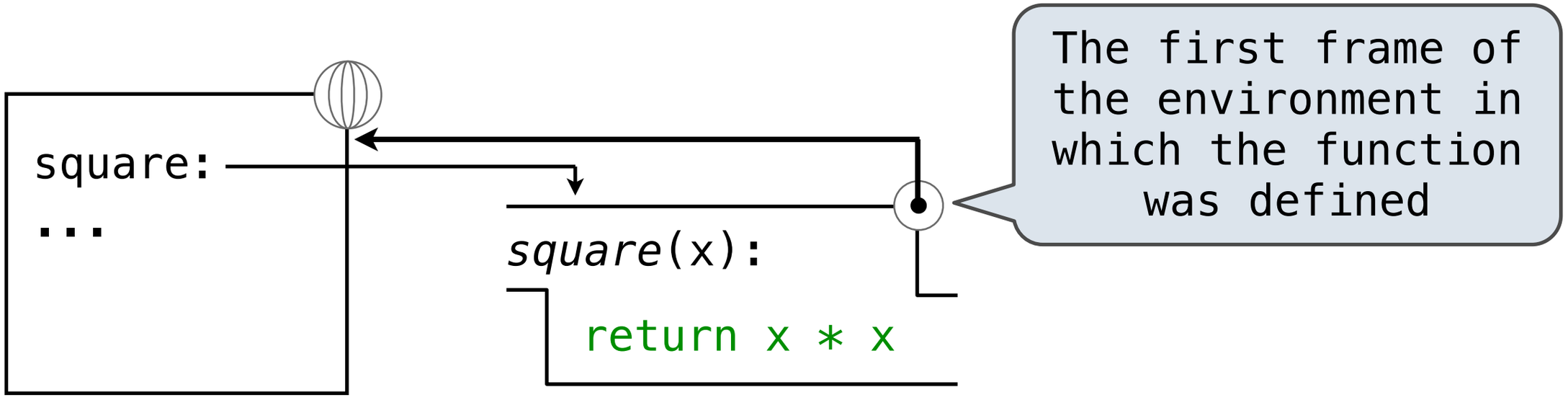


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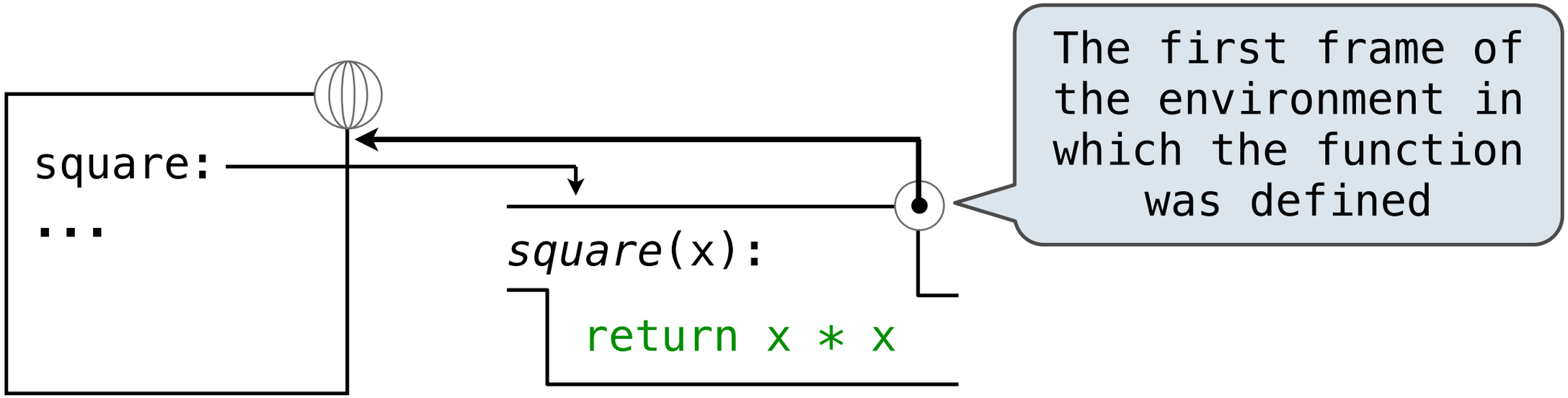


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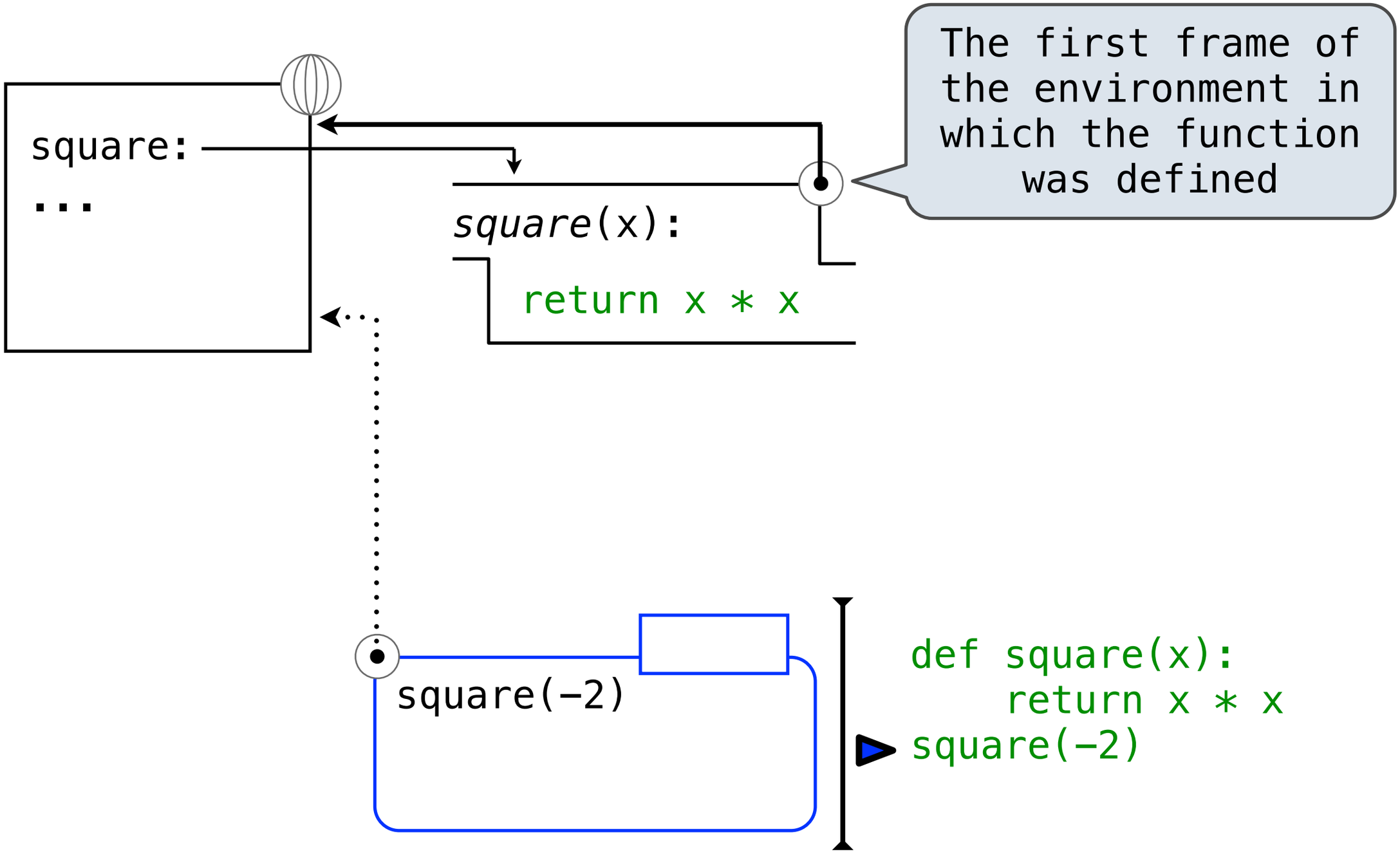

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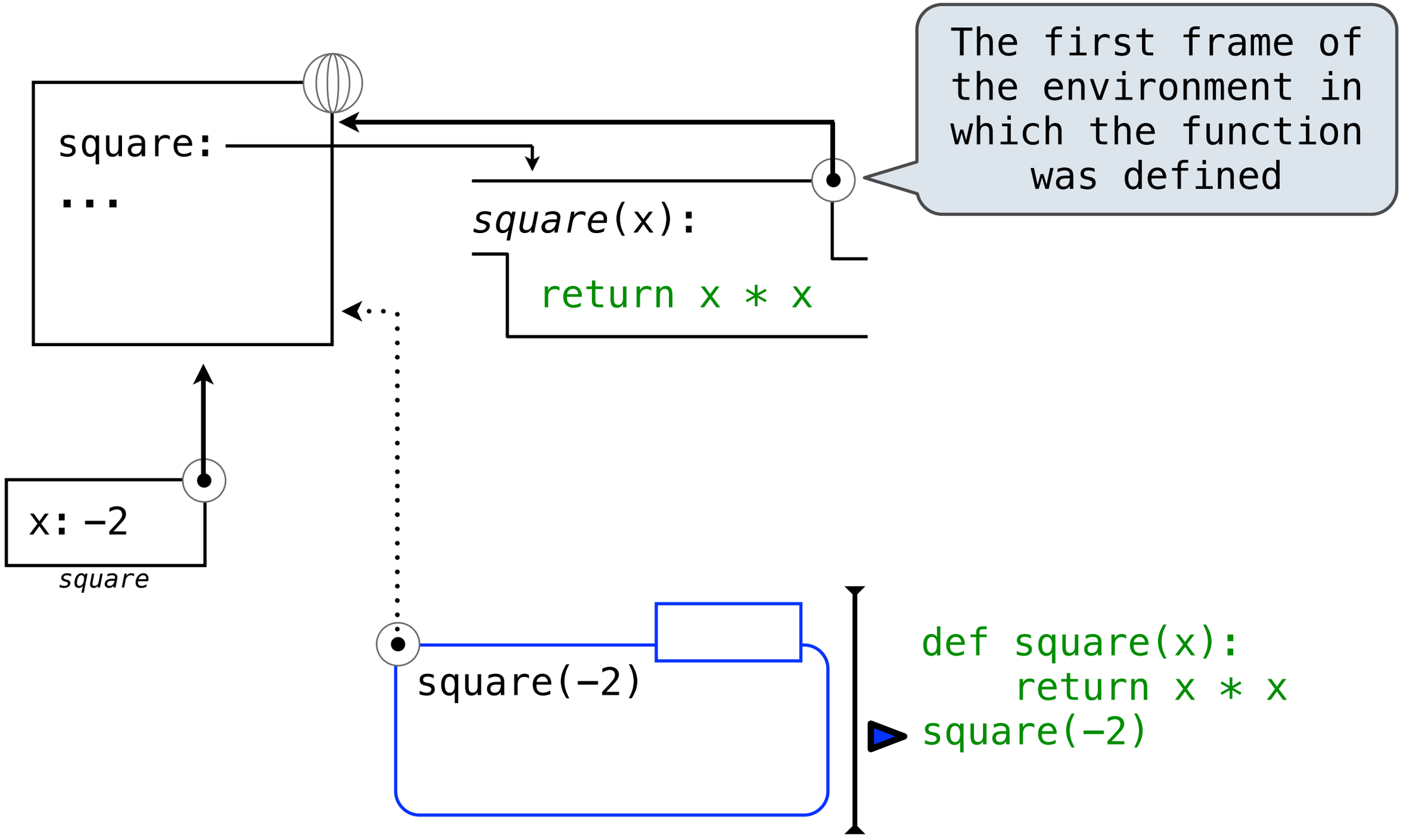

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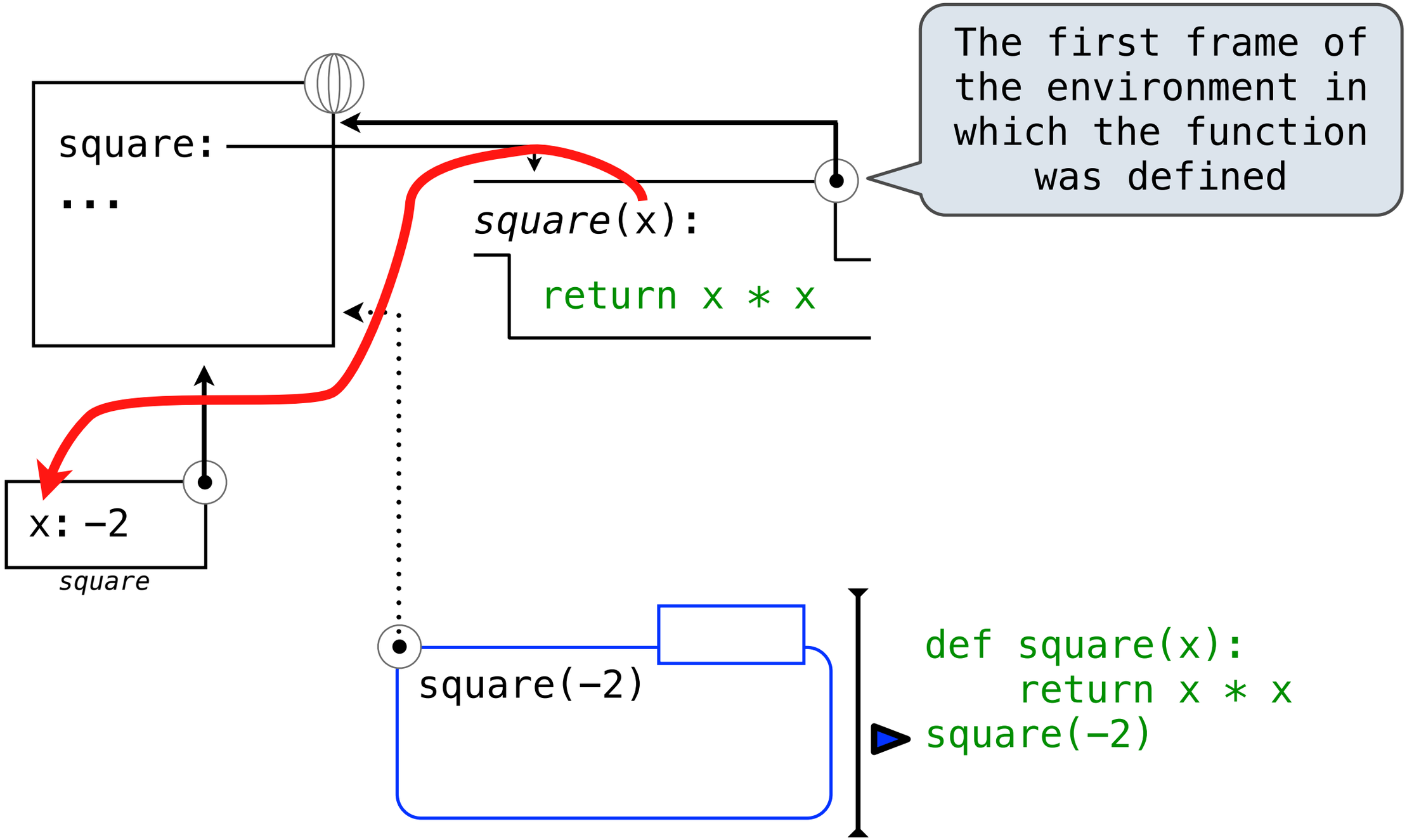

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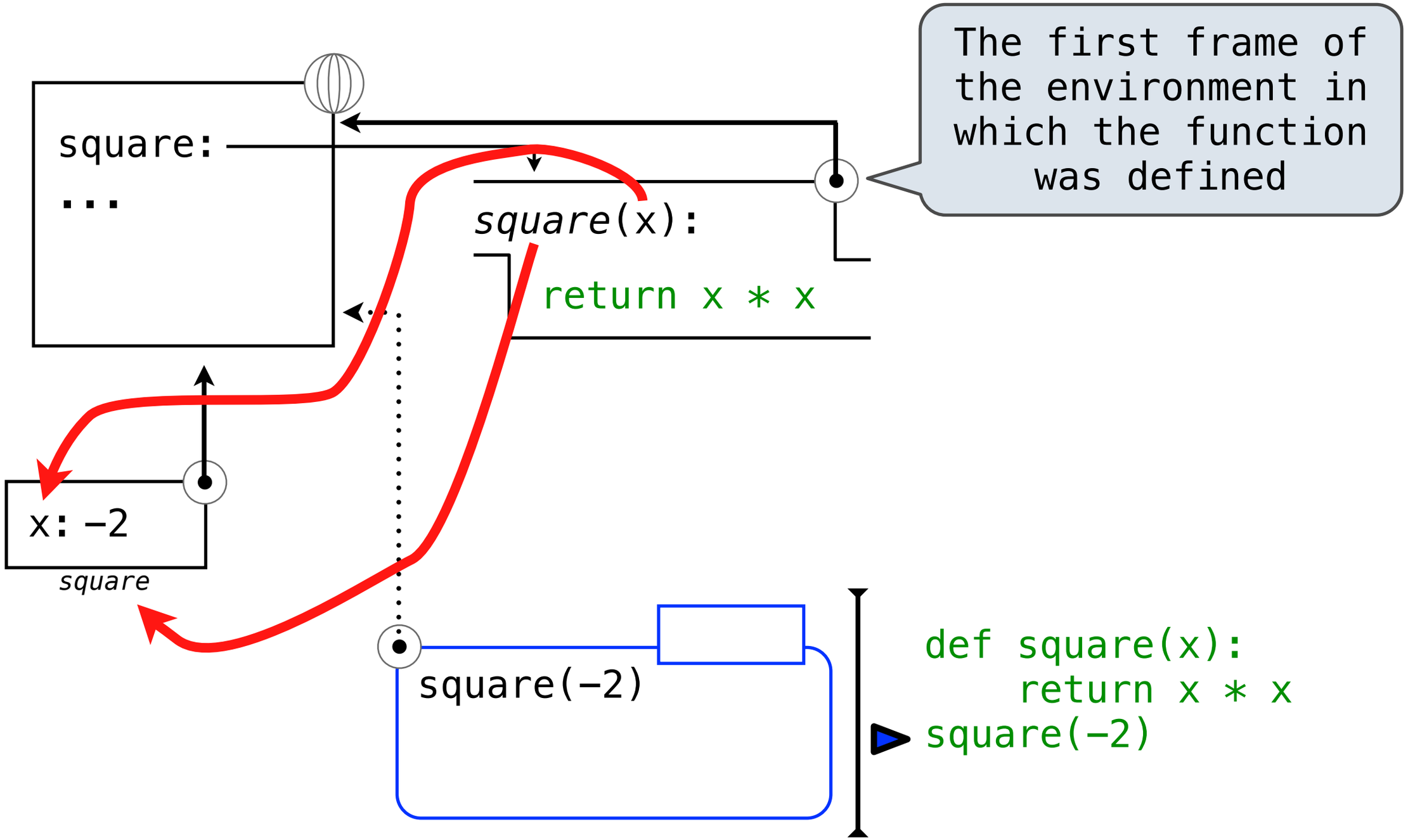

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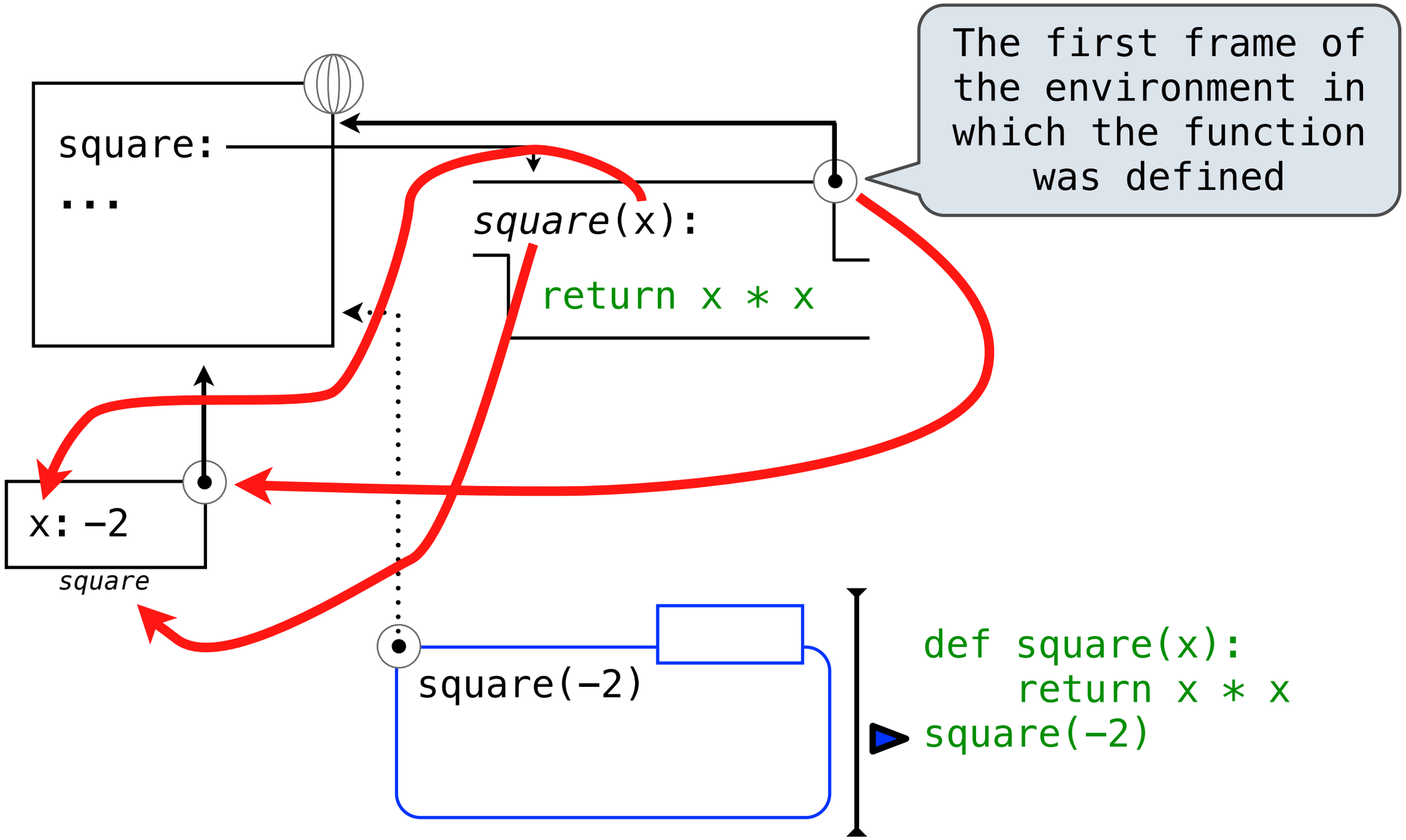

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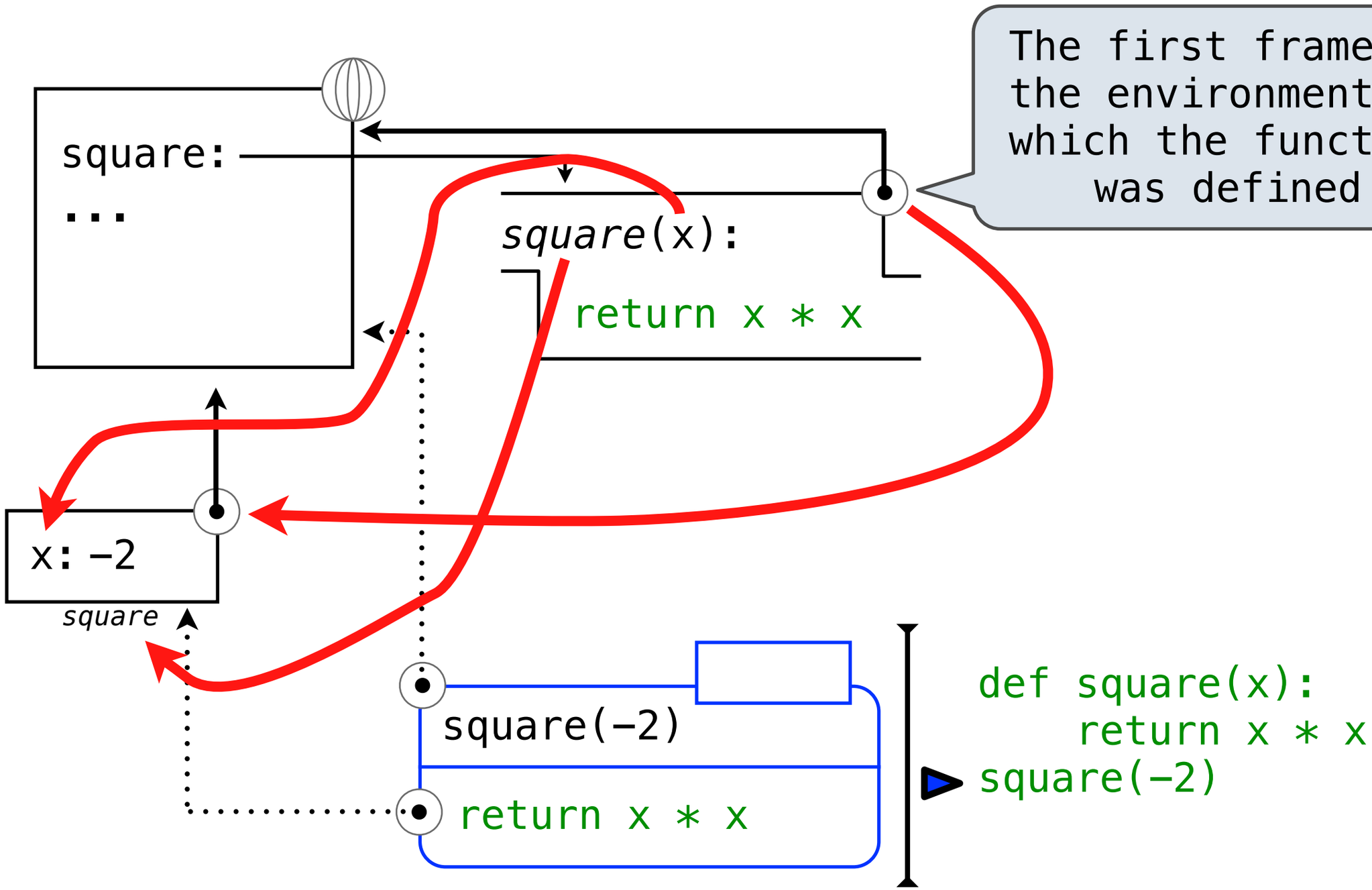

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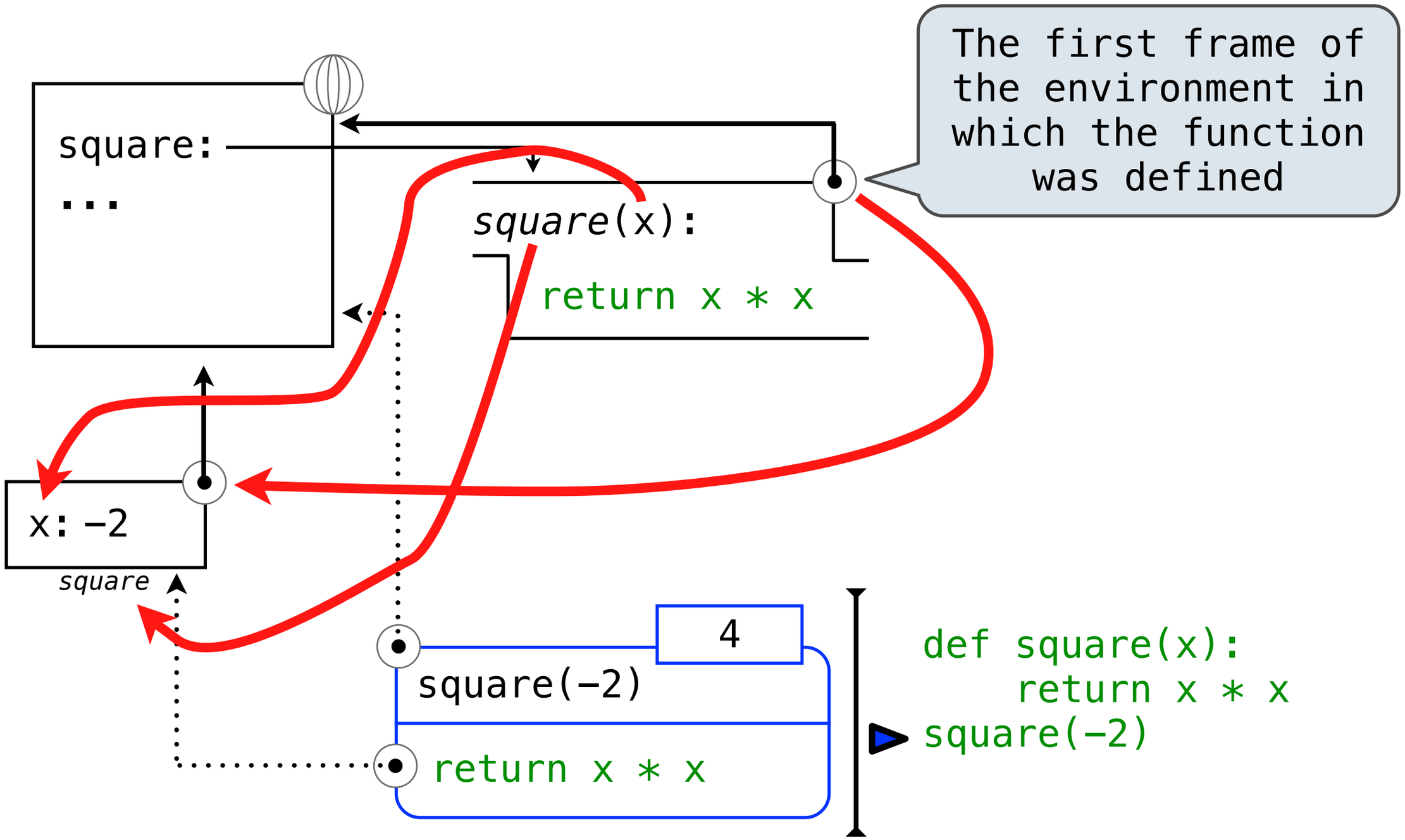
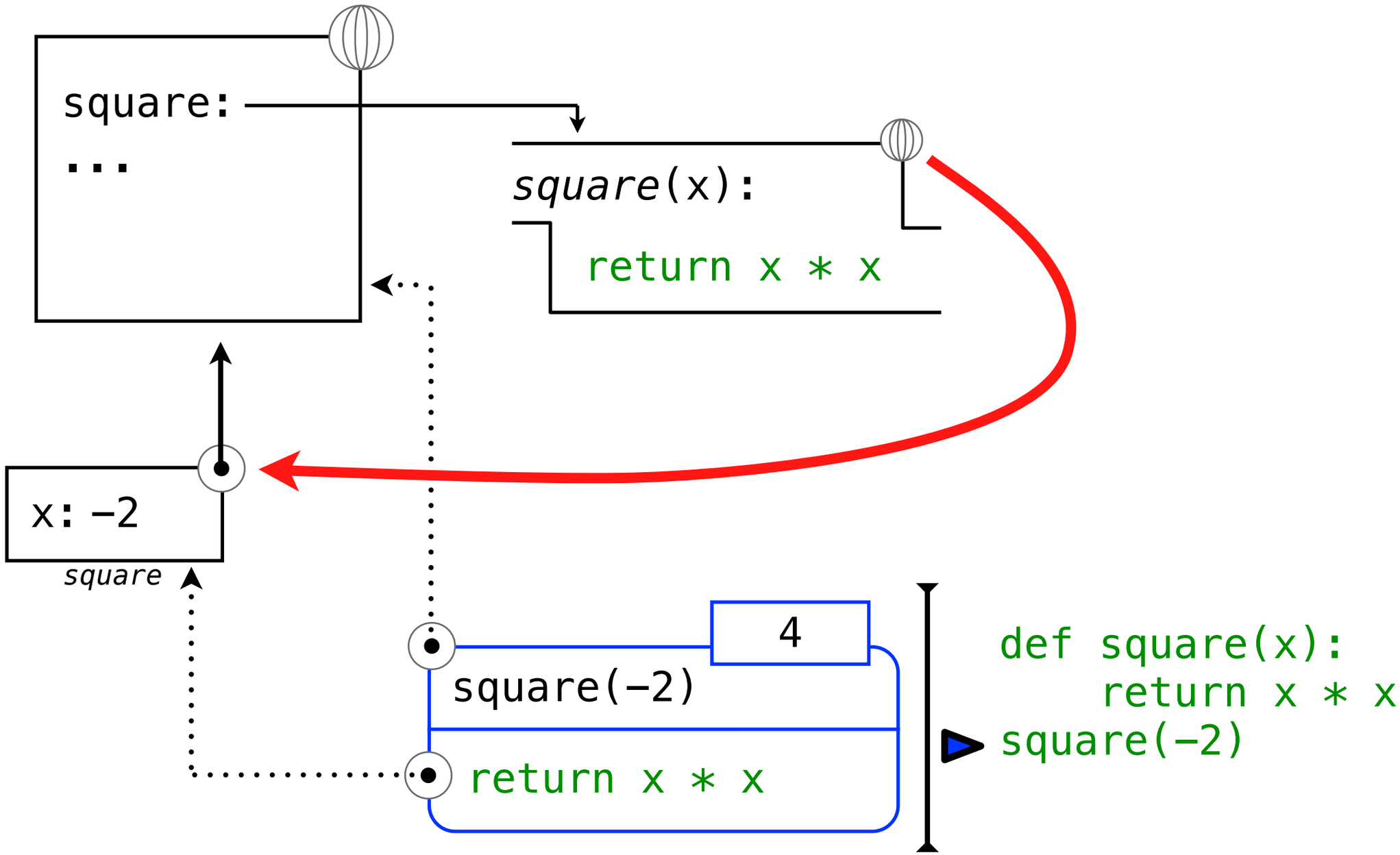


Functions Associated with the Global Frame



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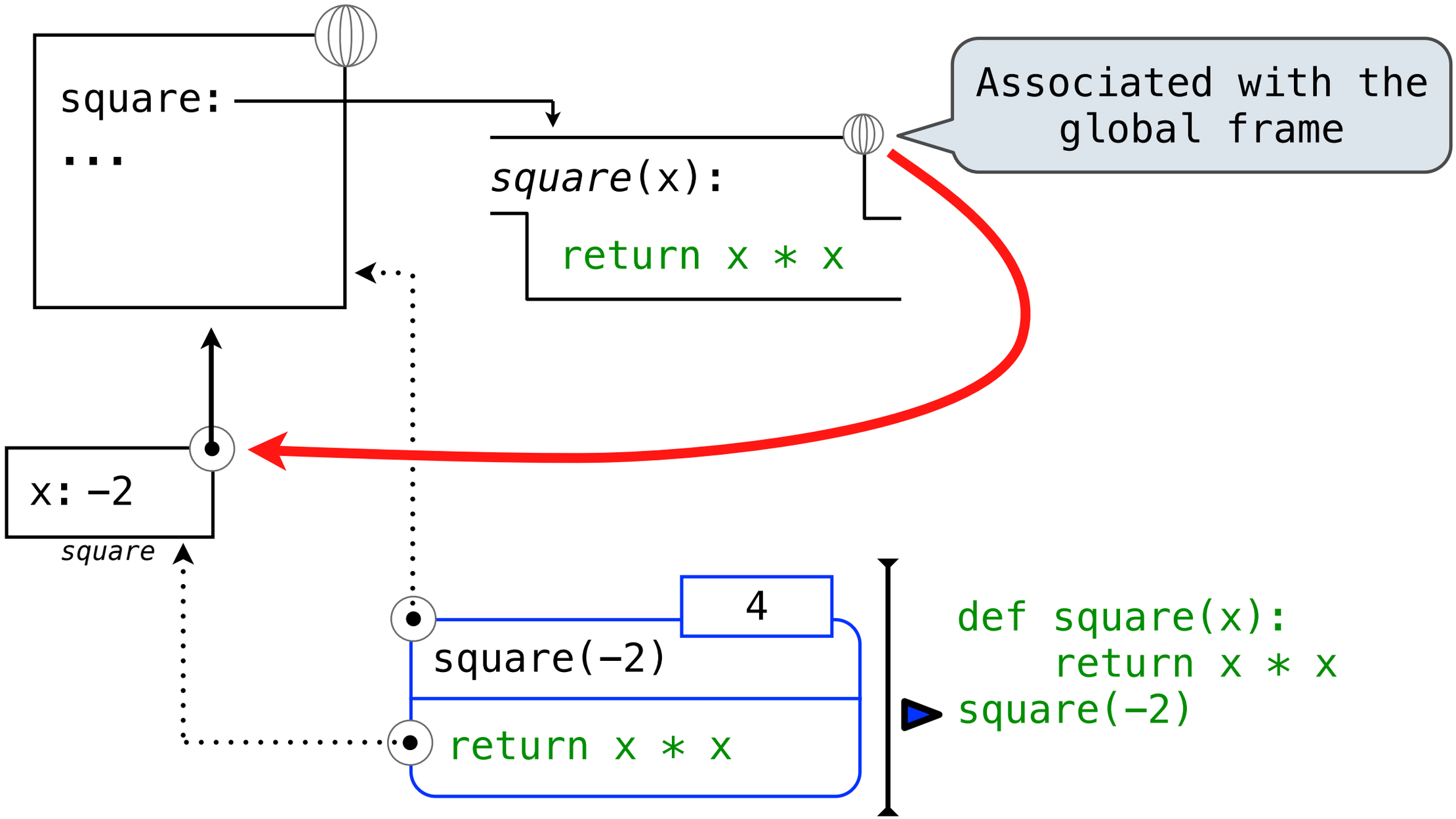

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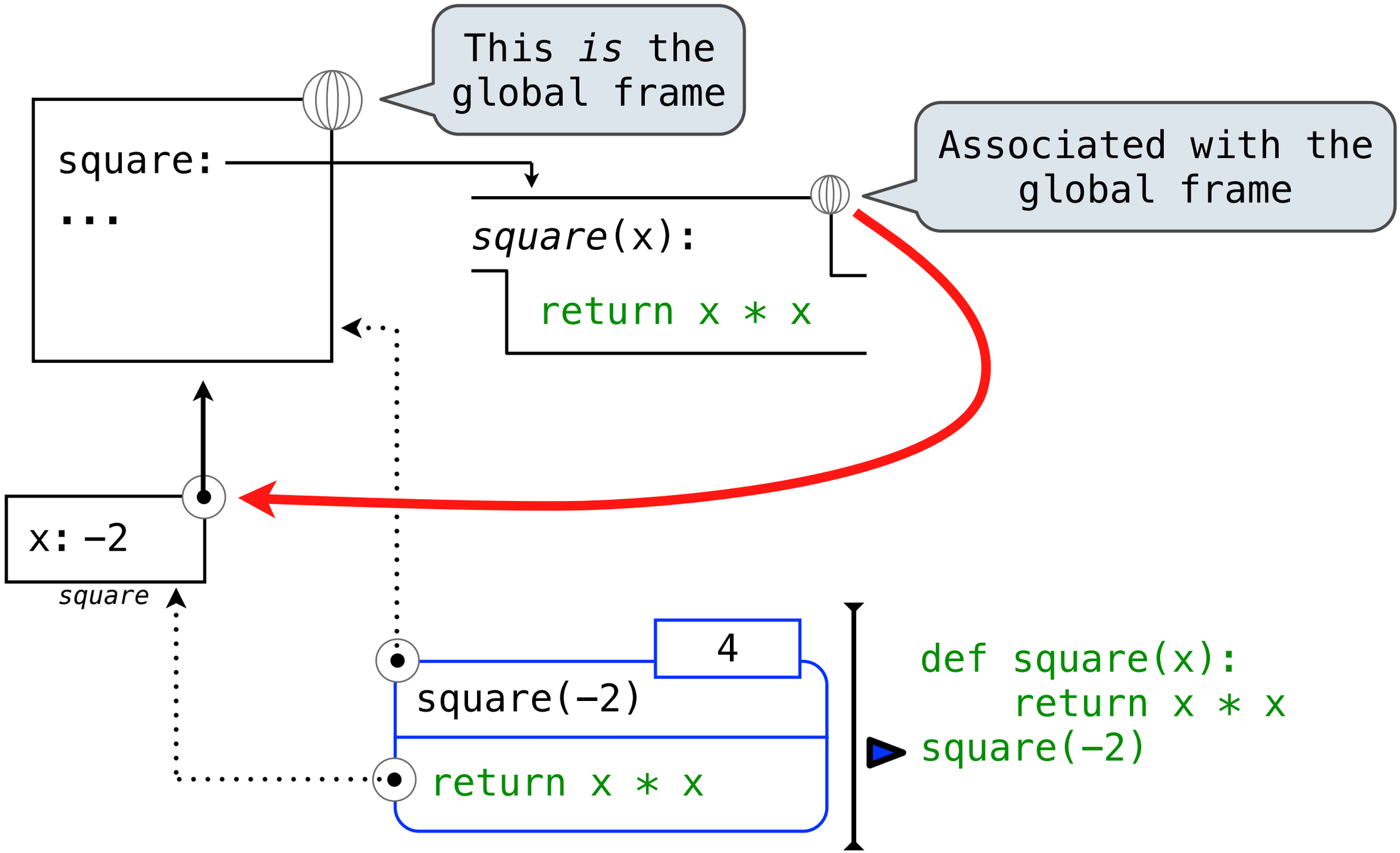

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    >>> add_three = make_adder(3)
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    """
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A local def statement

Can refer to names in the enclosing function

Locally Defined Functions: Call Expressions

```
make_adder(1)(2)
```

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Locally Defined Functions: Call Expressions

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```
make_adder(1)     (          2          )
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Locally Defined Functions: Call Expressions

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make_adder(1)(2)
```

```
make_adder(1)     (          2          )
```

Operator

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def make_adder(n):
    def adder(k):
        return k + n
    return adder
make_adder(1)(2)
```
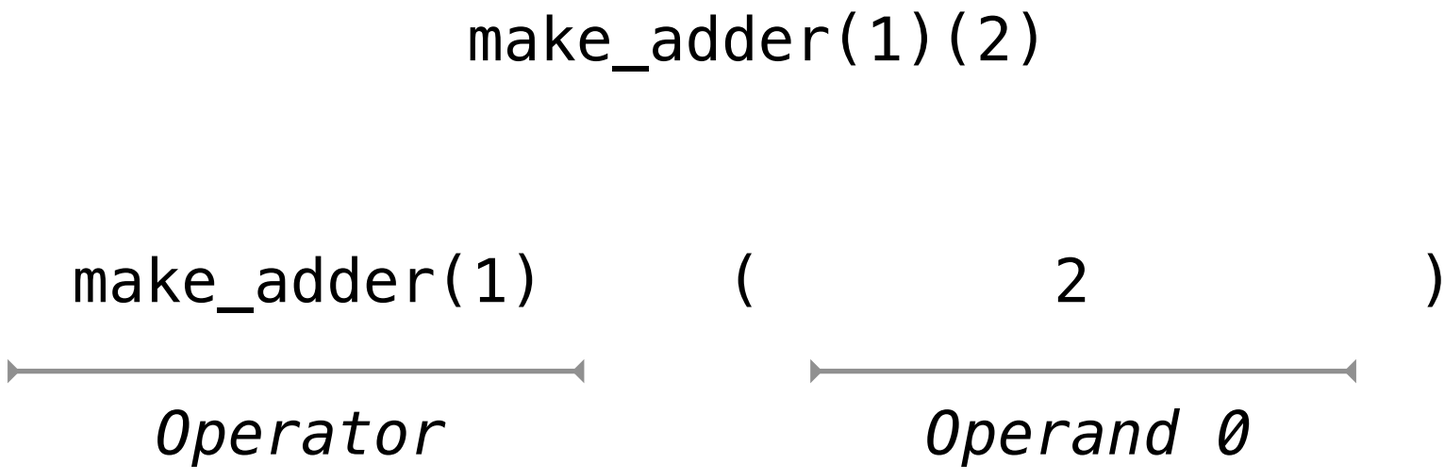

Locally Defined Functions: Call Expressions

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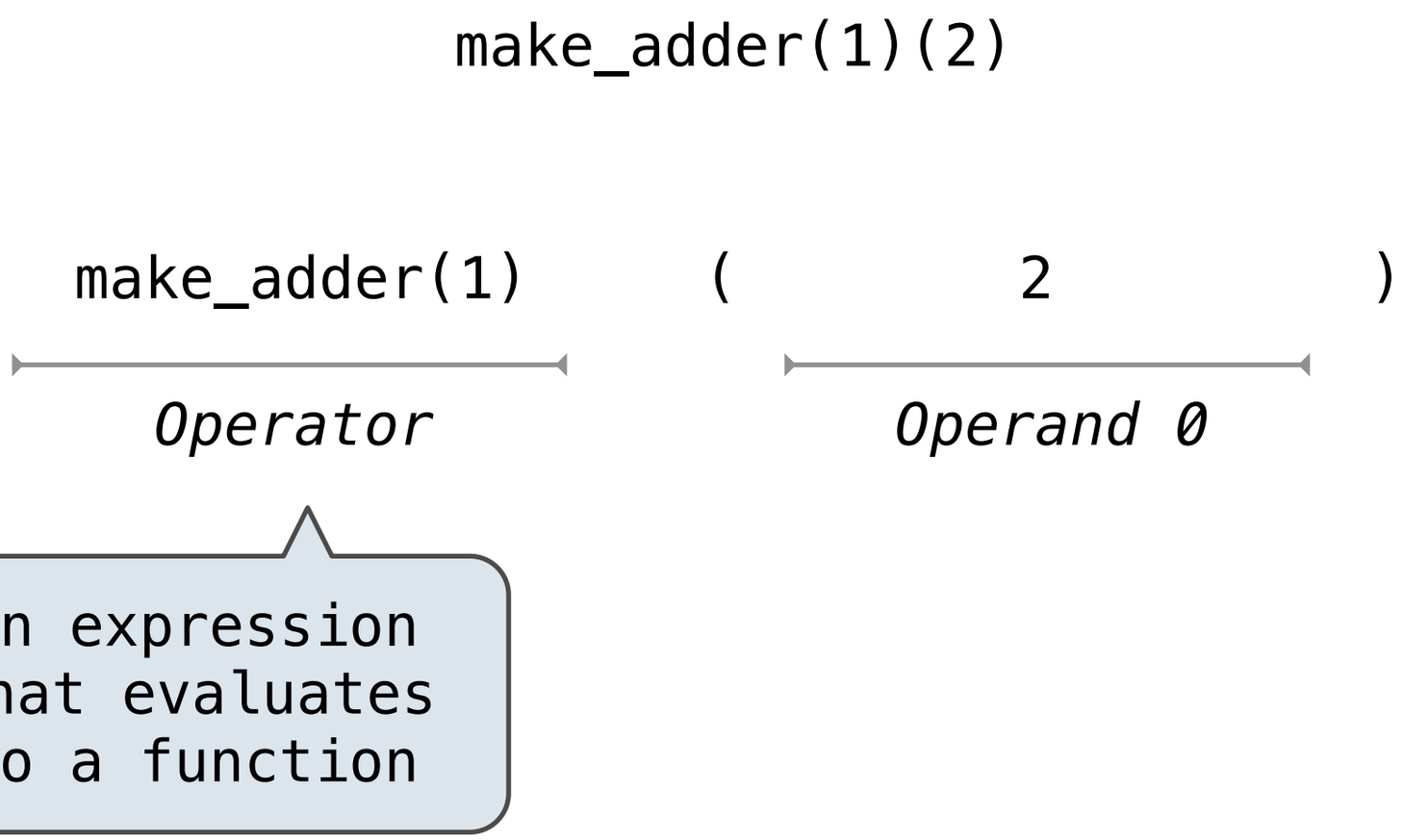


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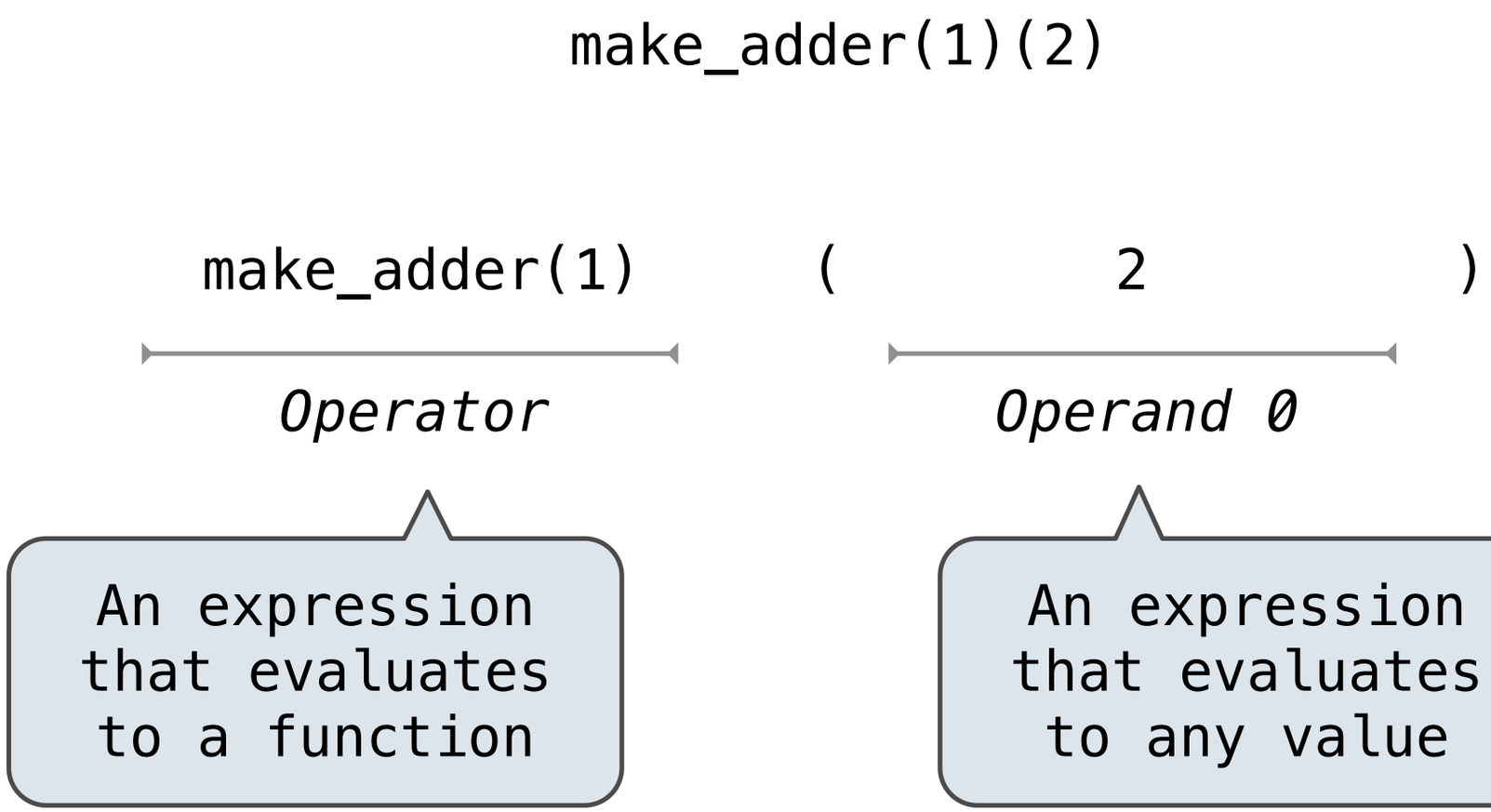


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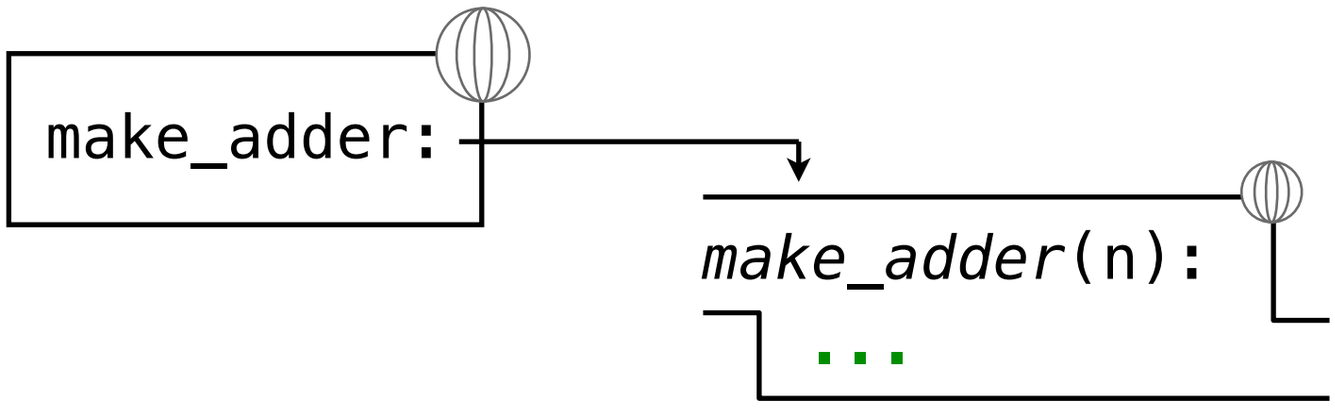


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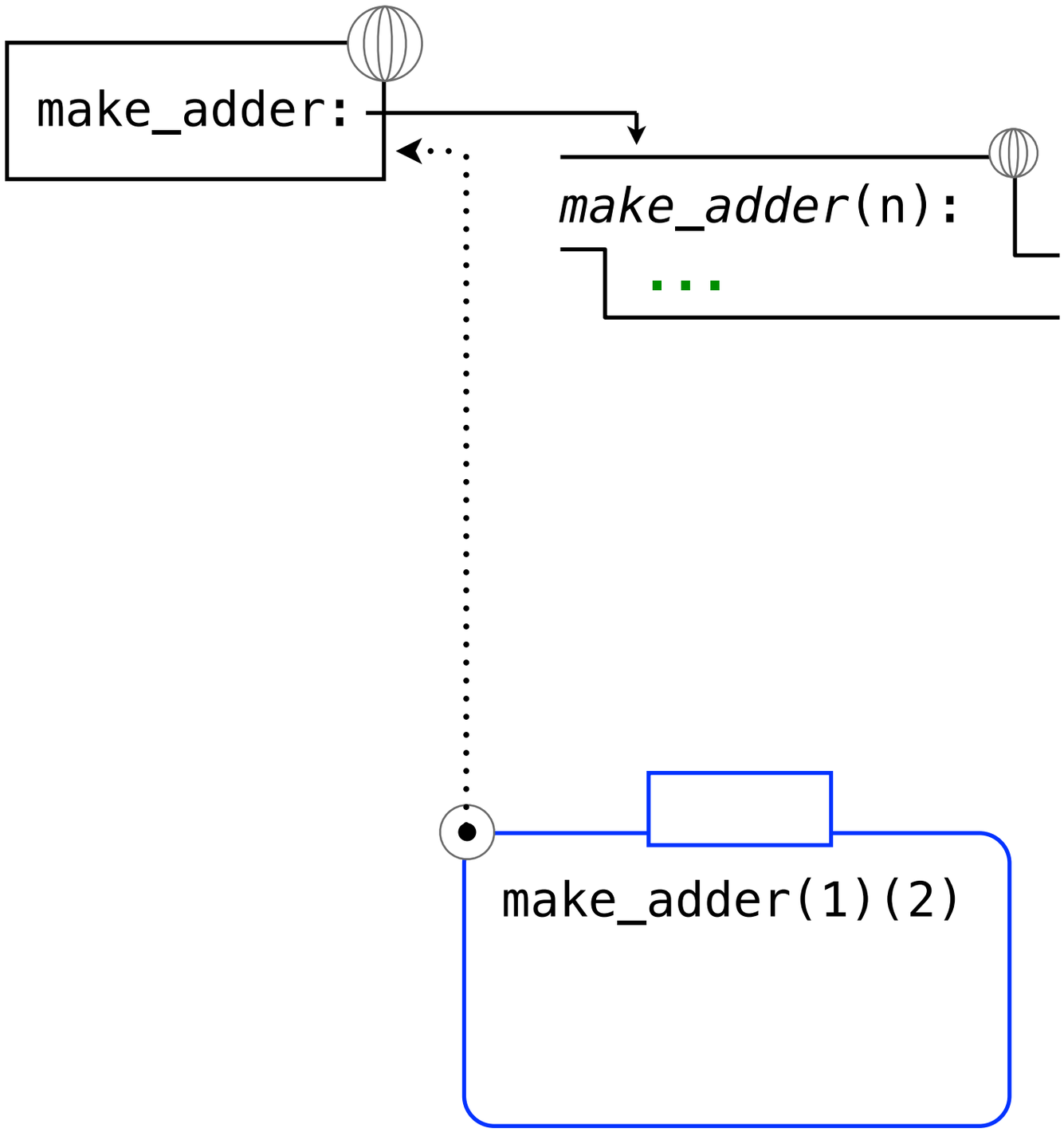


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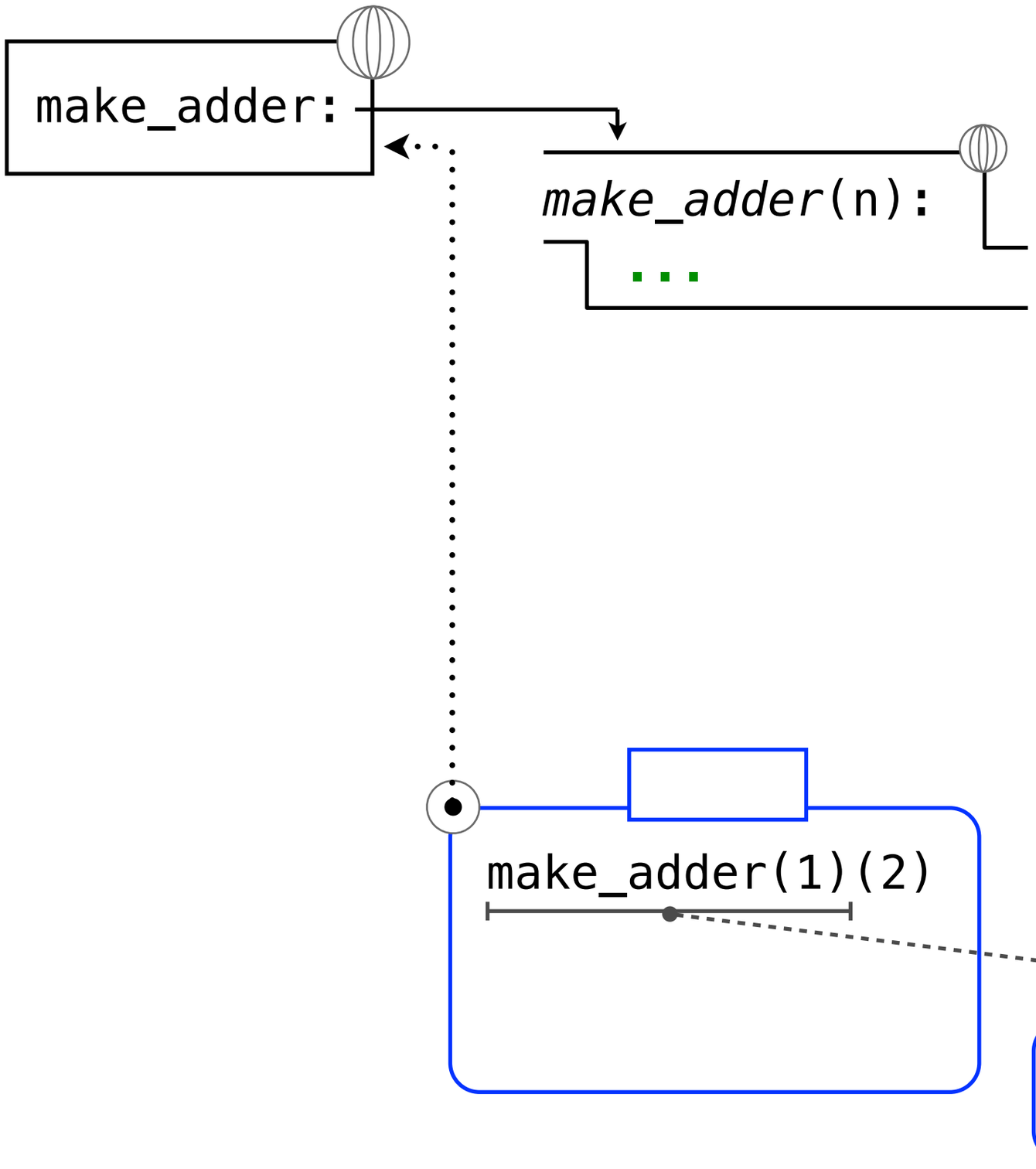


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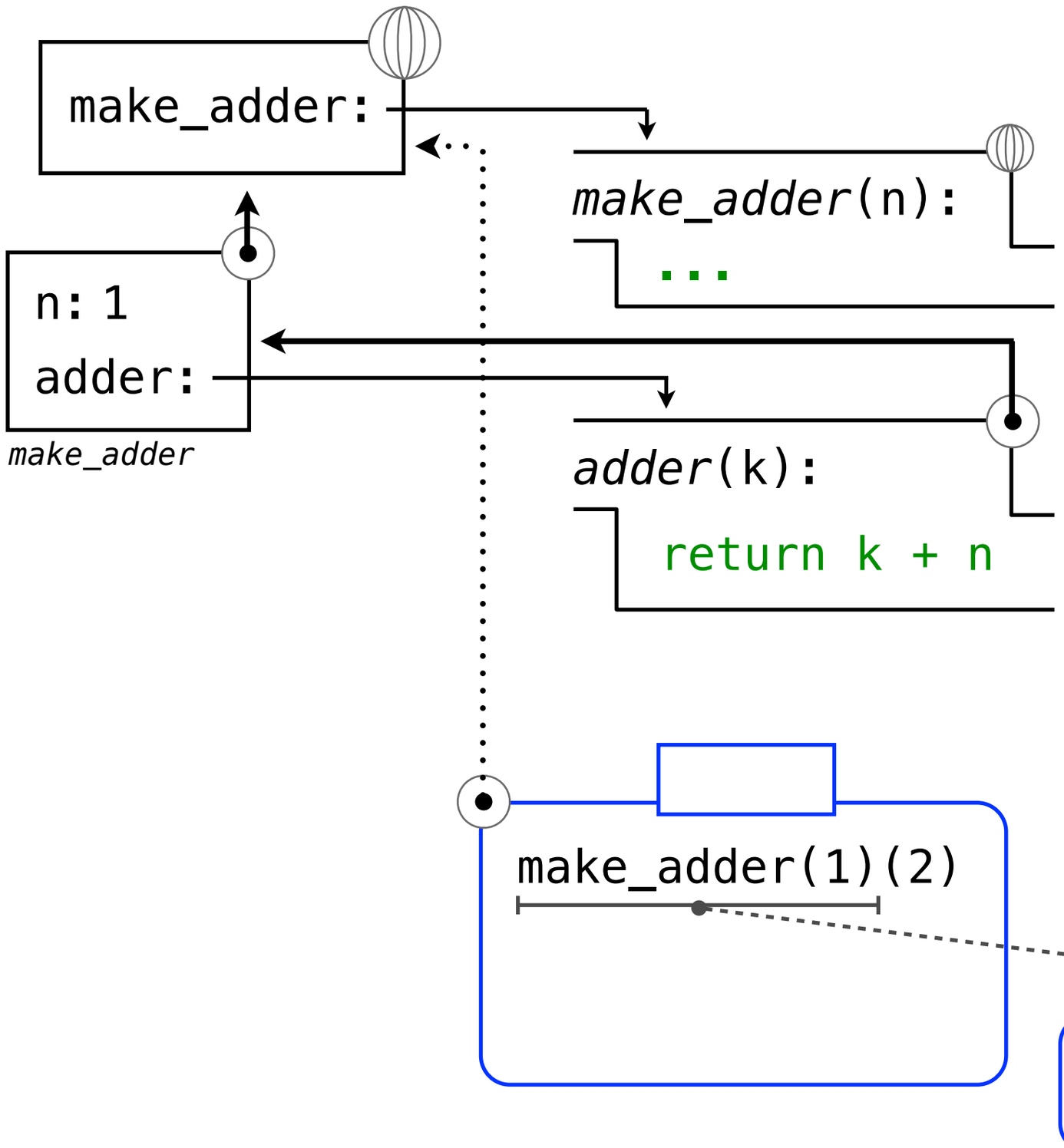


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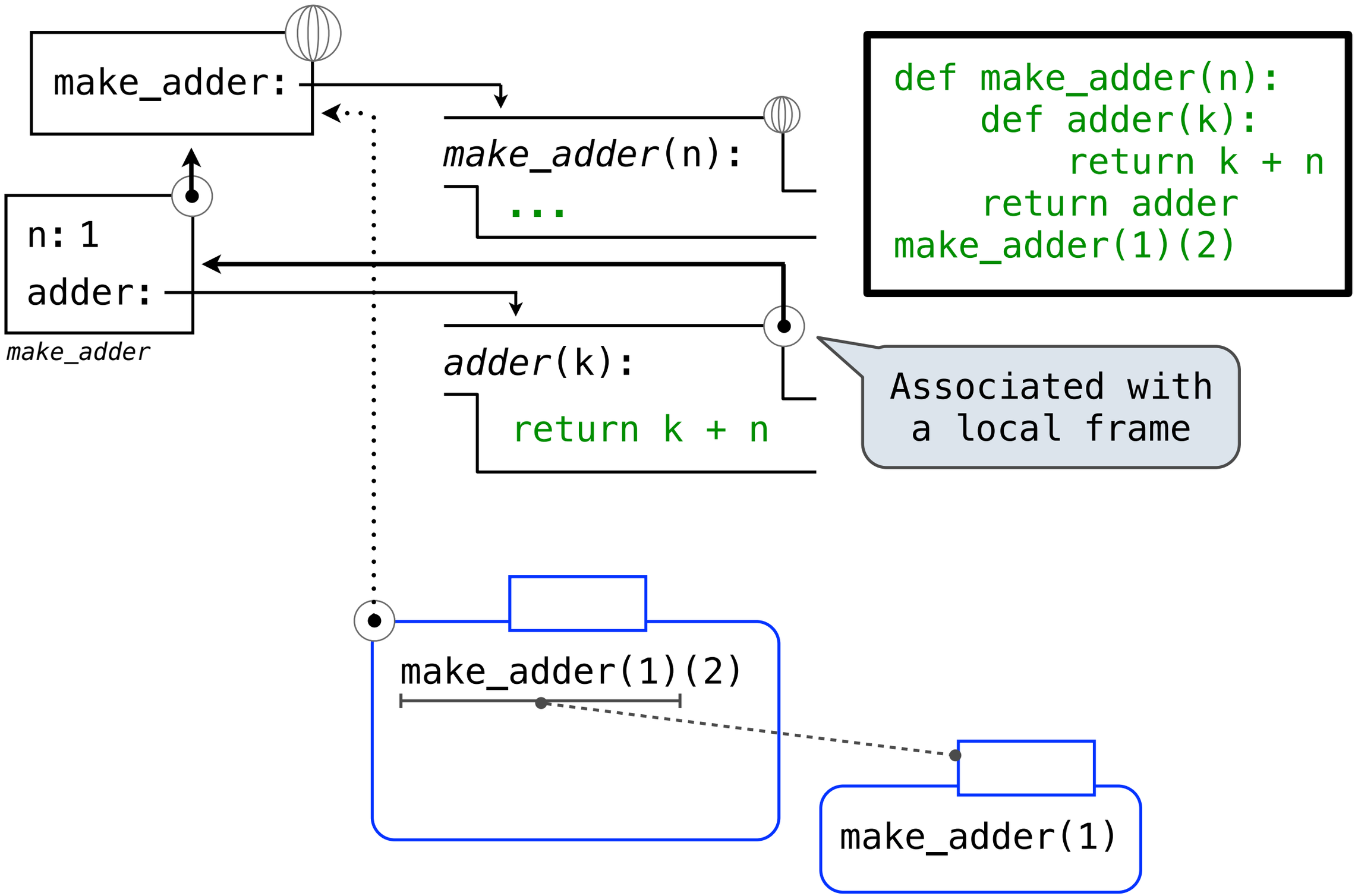

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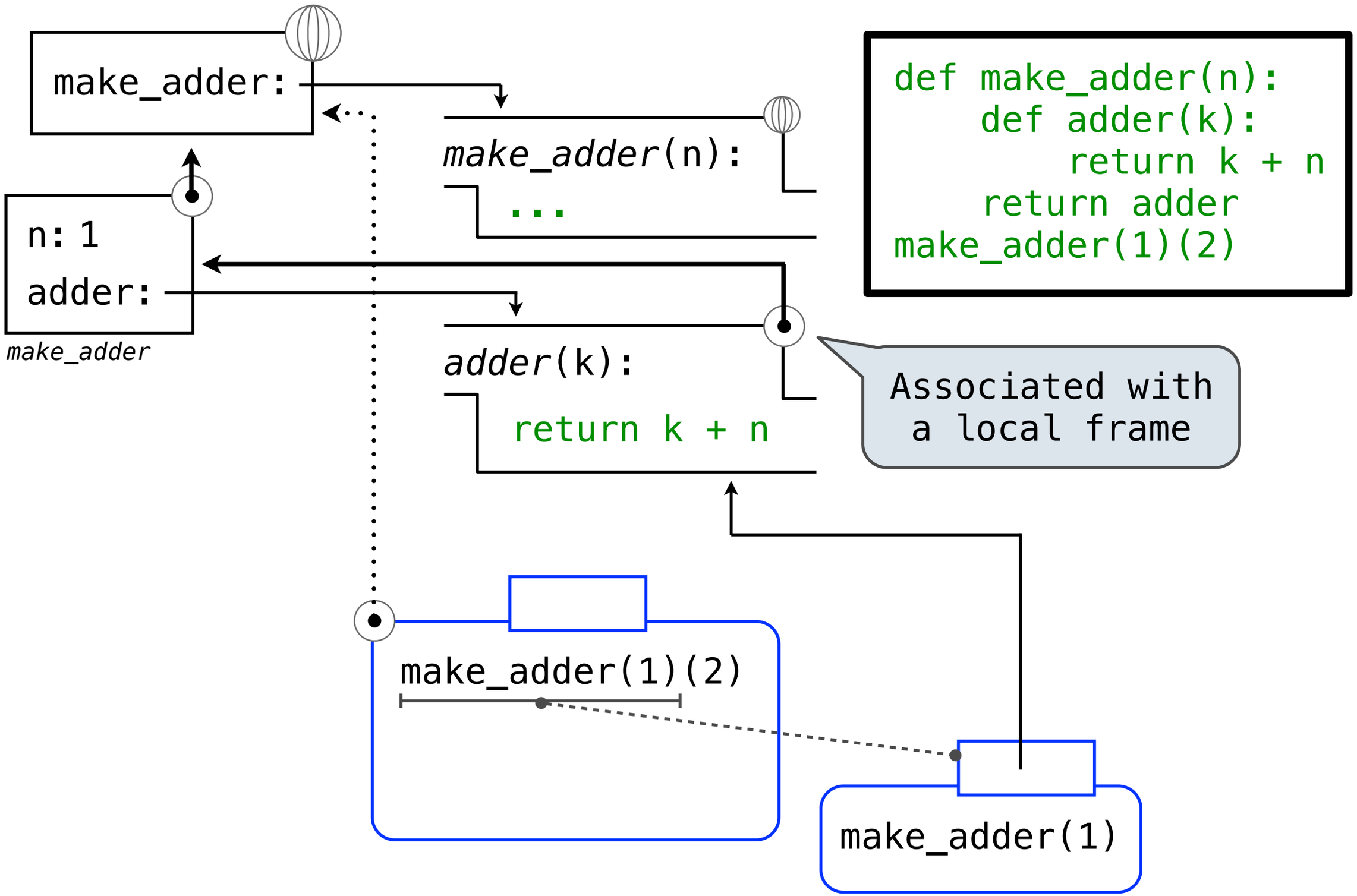

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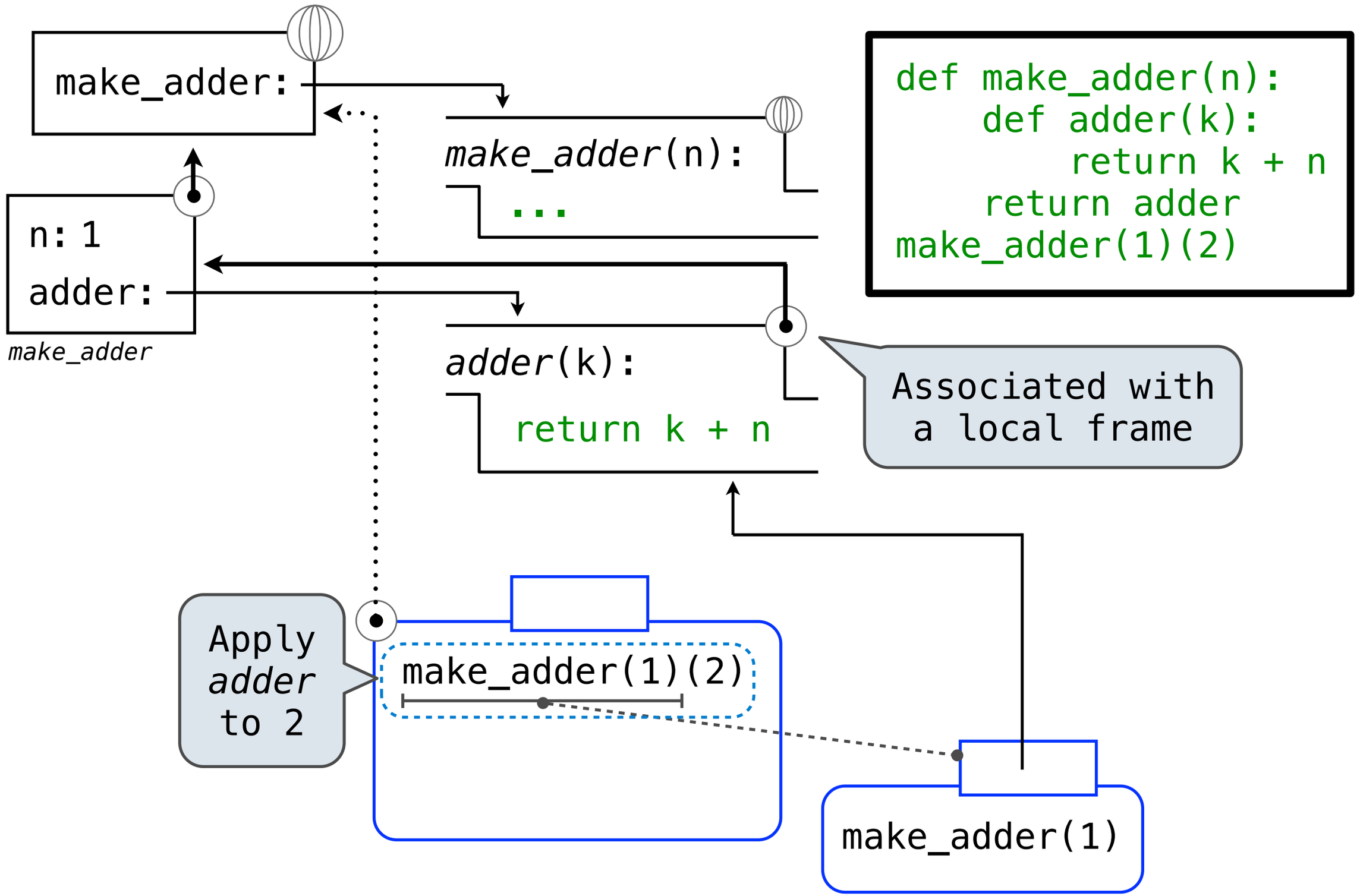

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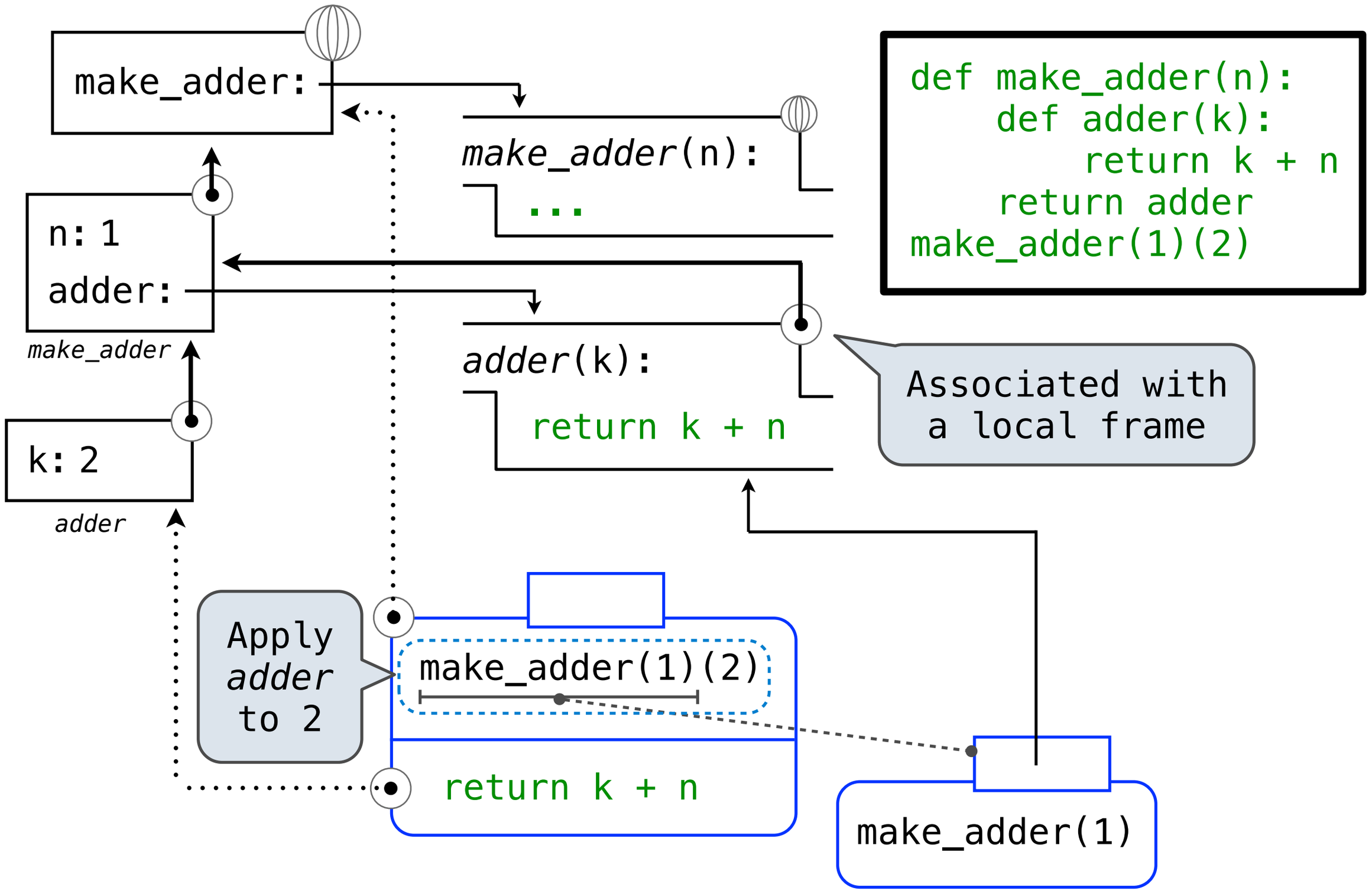

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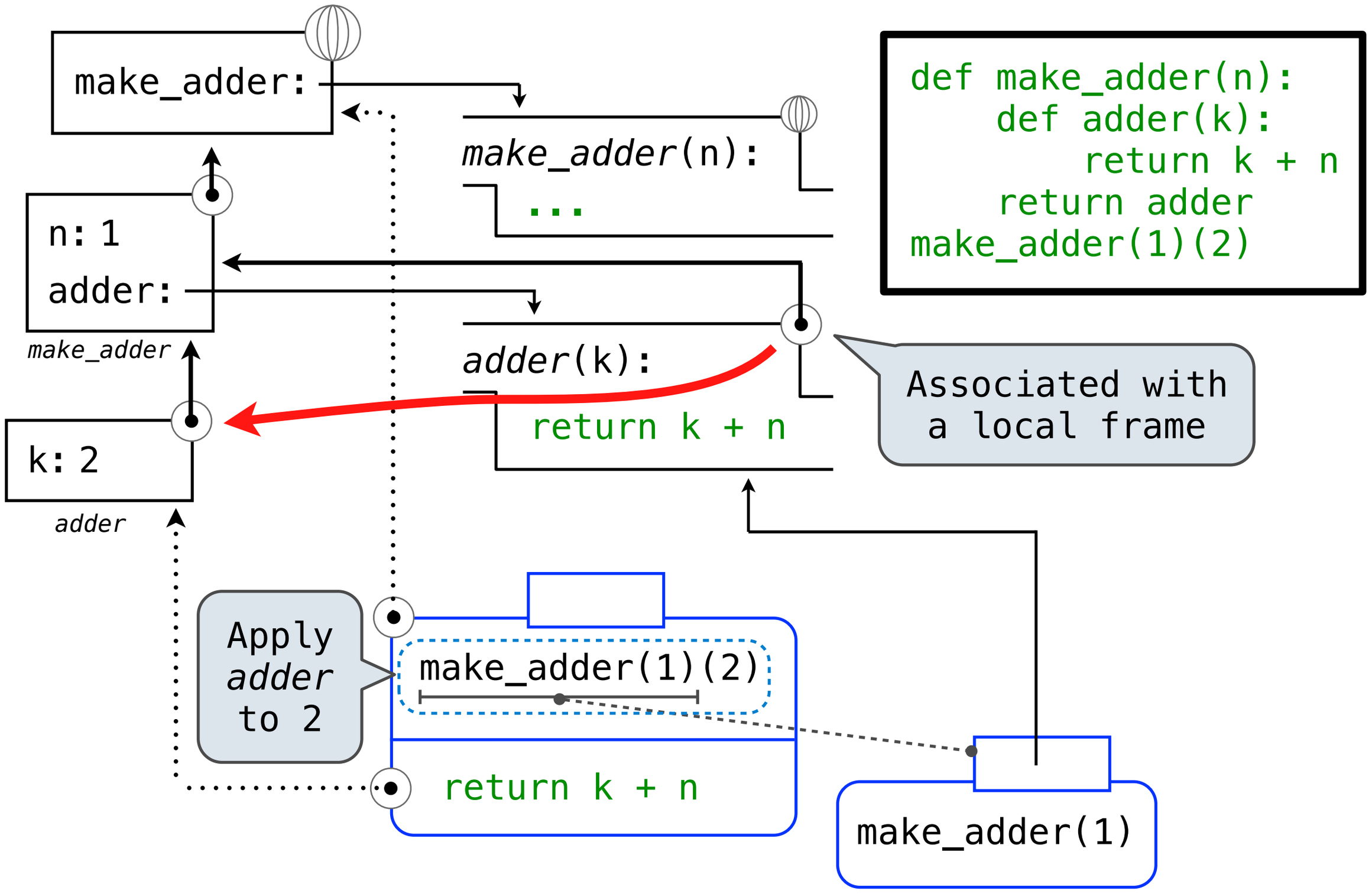

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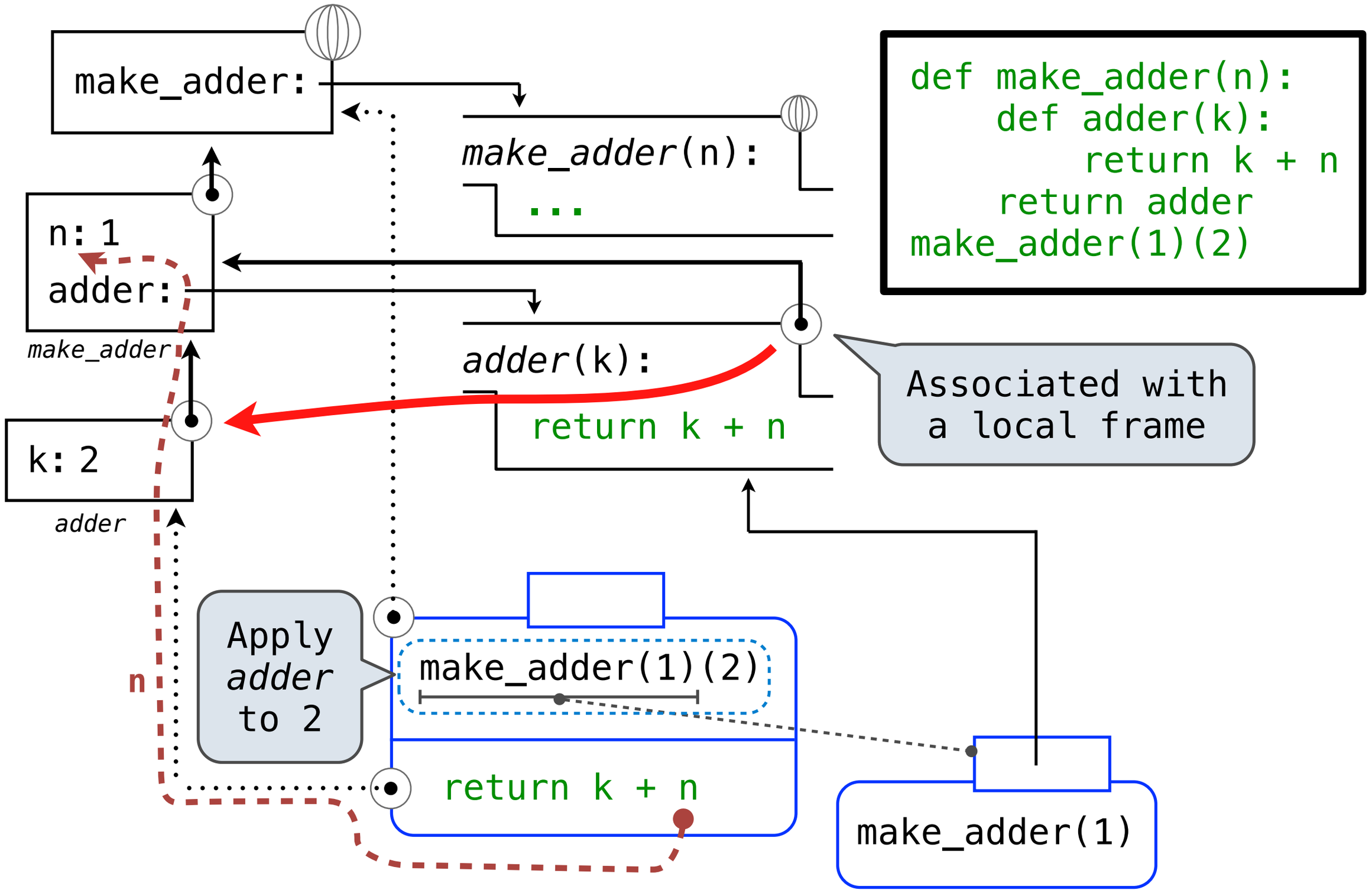

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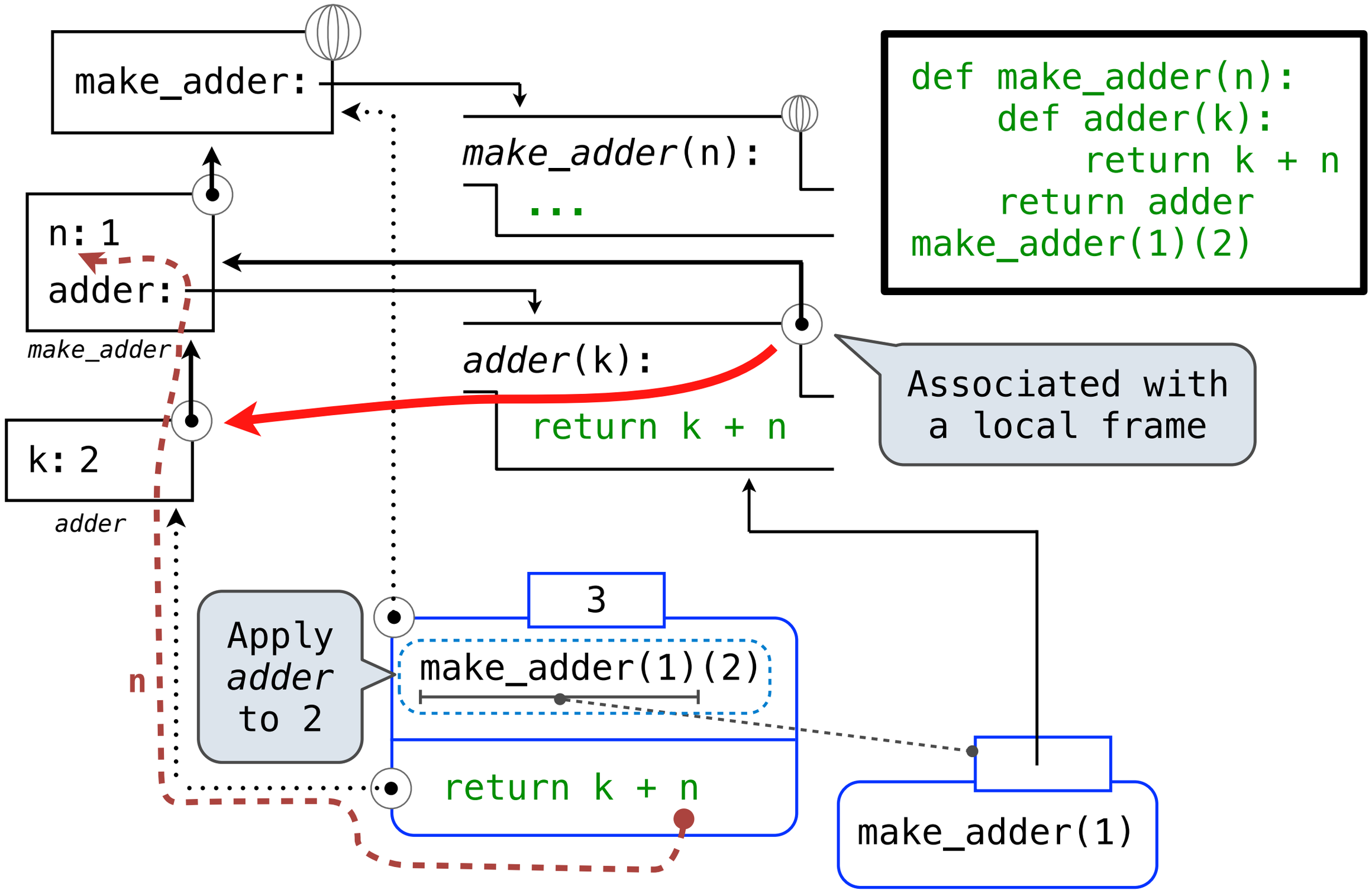

The Environment for Function Composition

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def compose1(f, g):
  def h(x)
    return f(g(x))
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a1 = make_adder(1)
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compose1(a1, a2)(3)
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(Demo)

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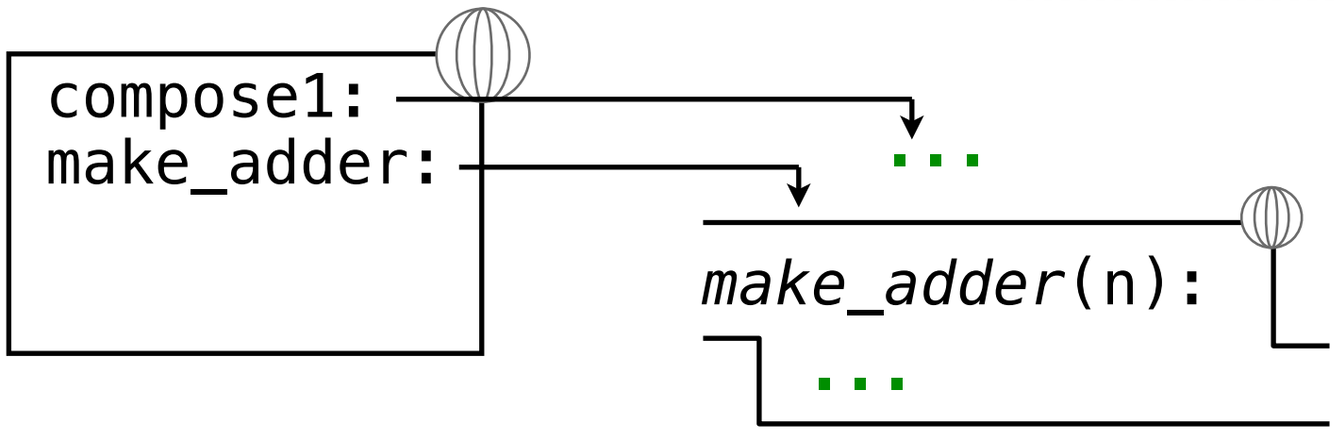


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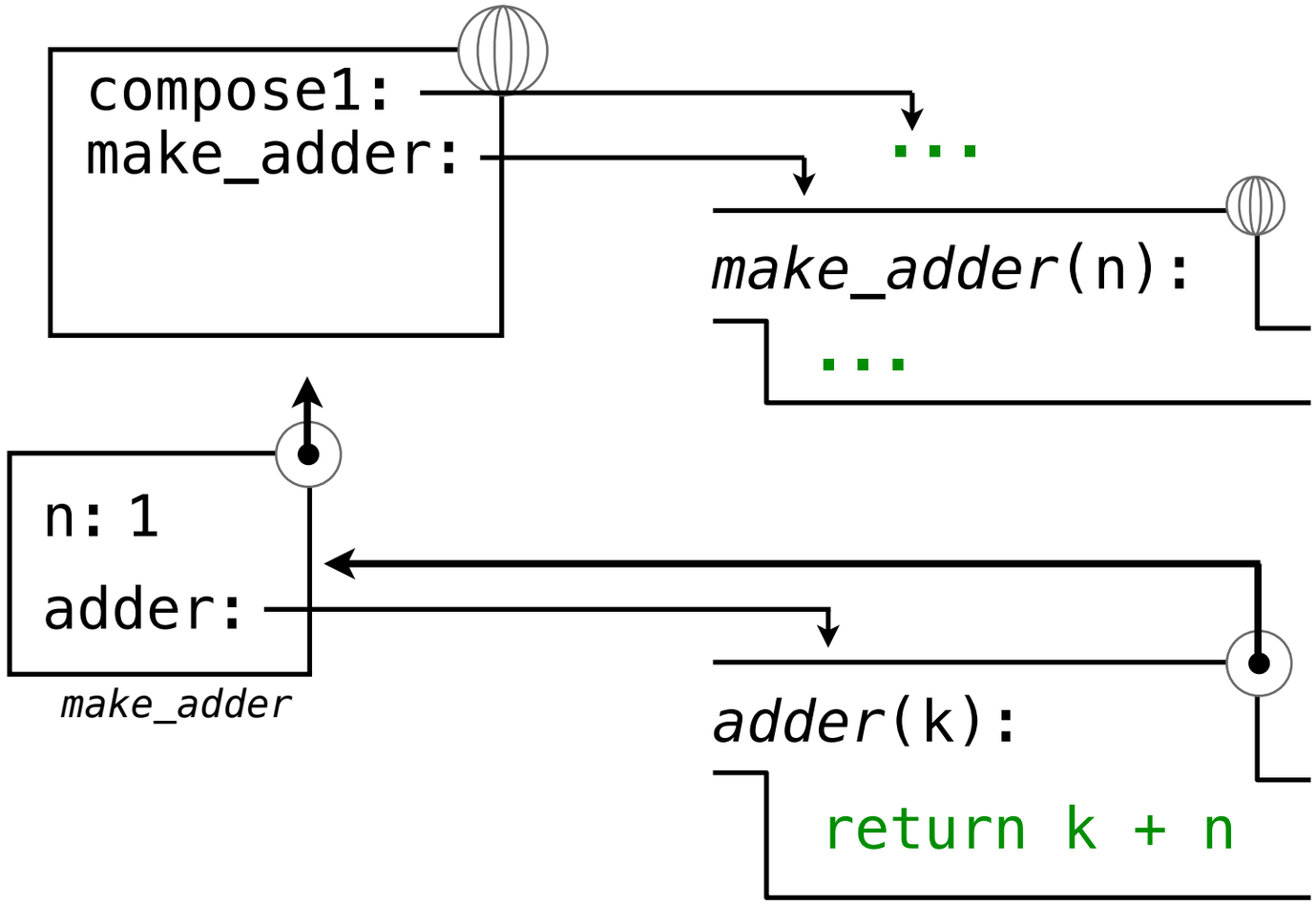


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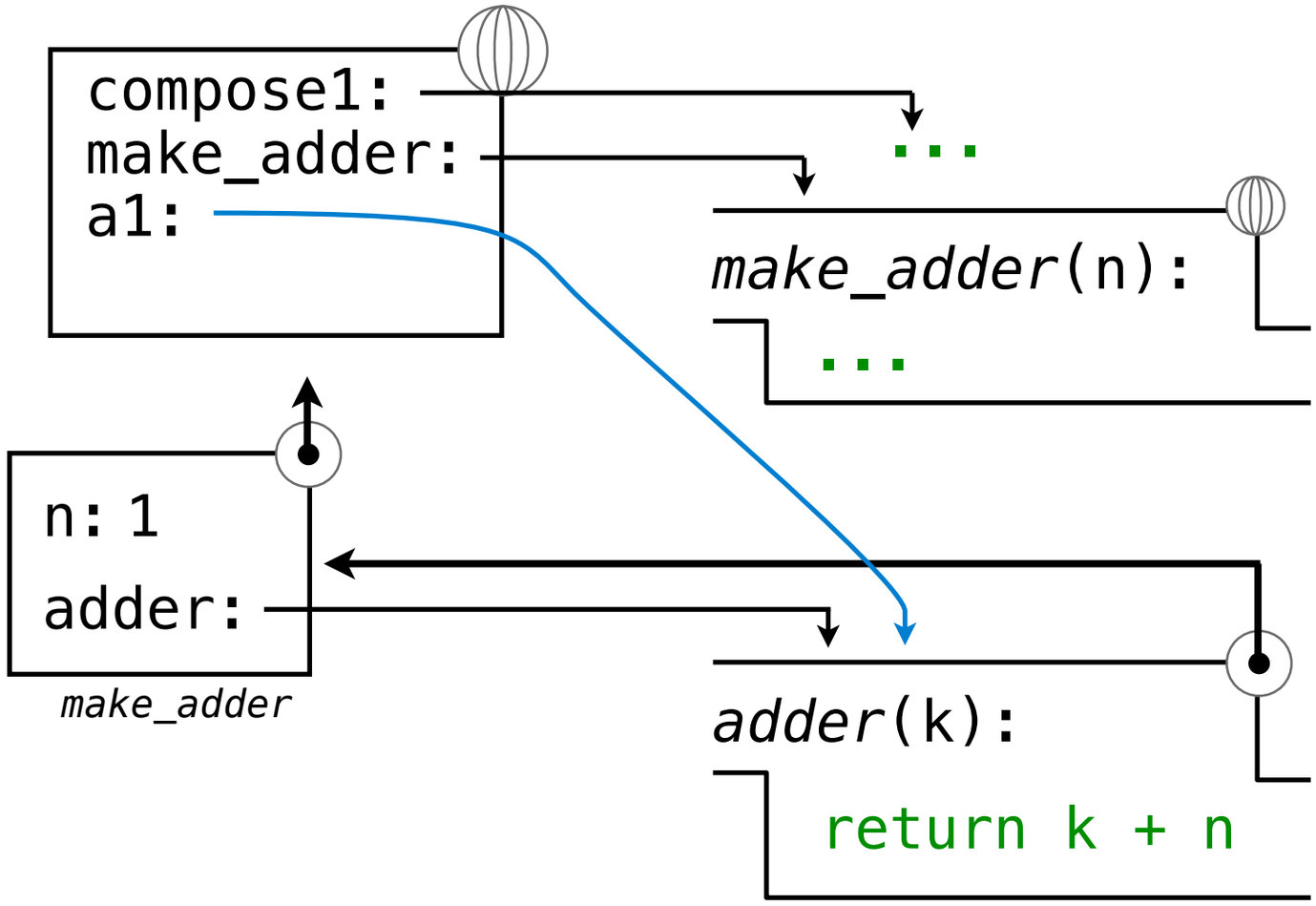


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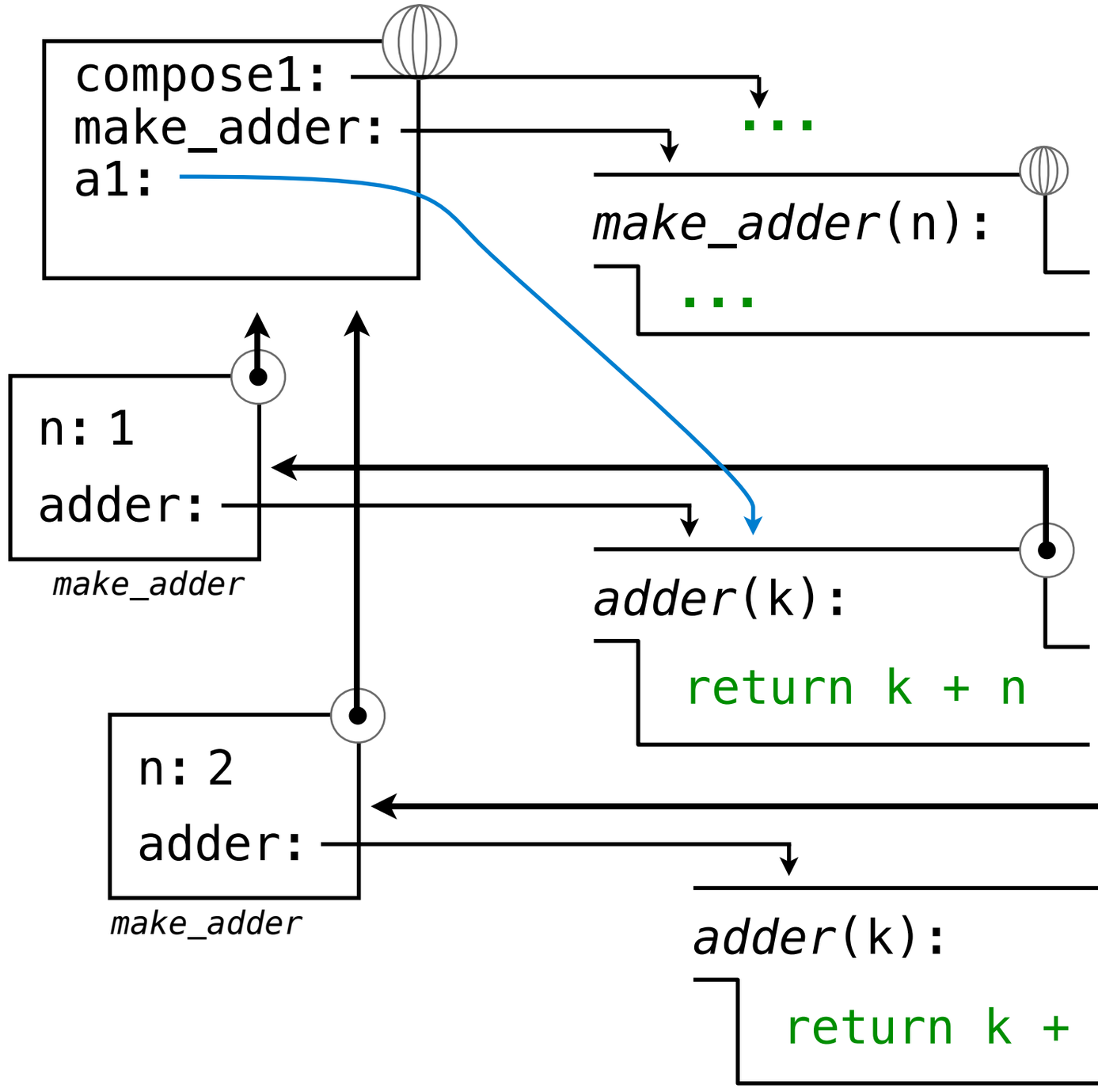


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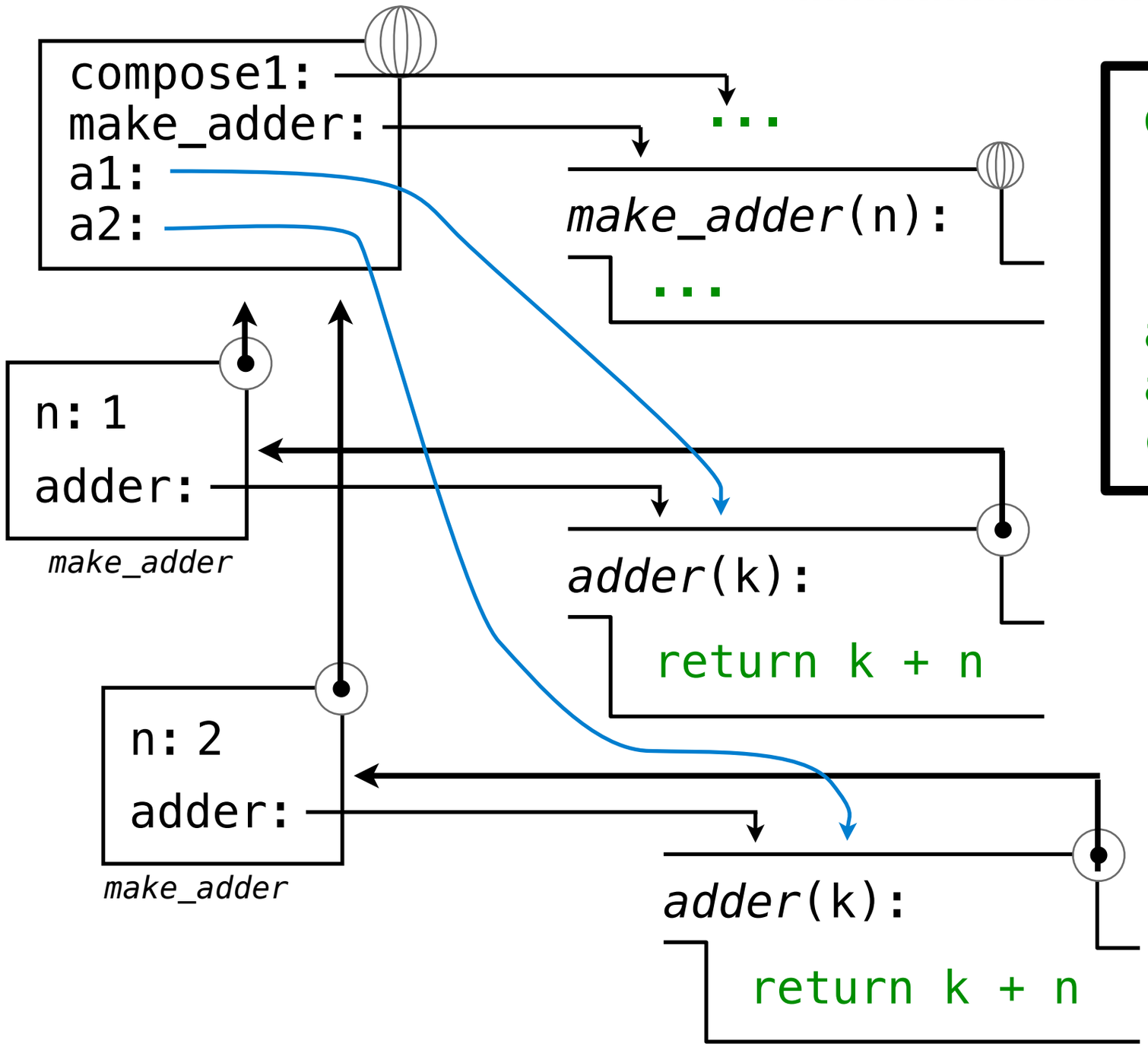


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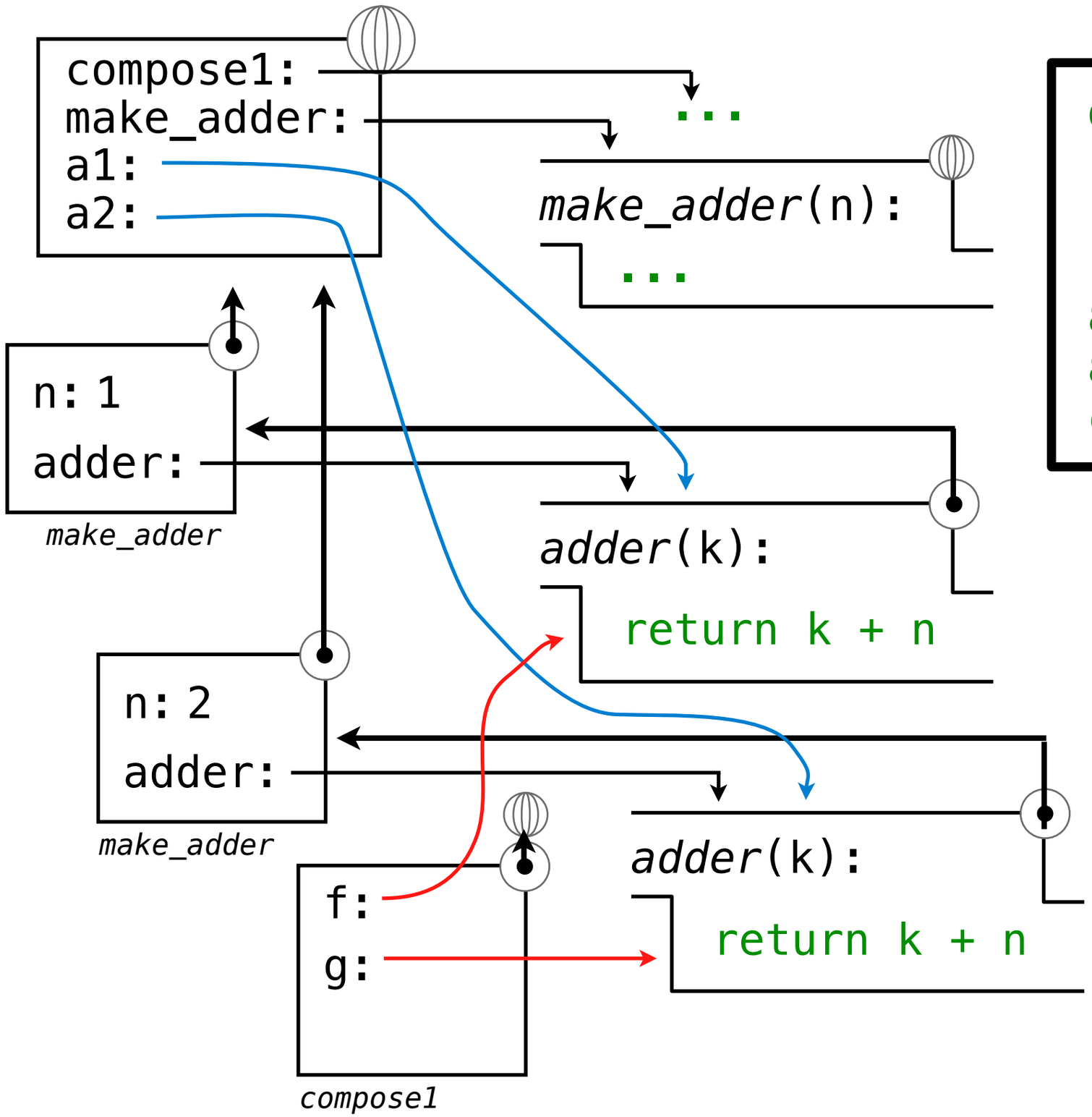


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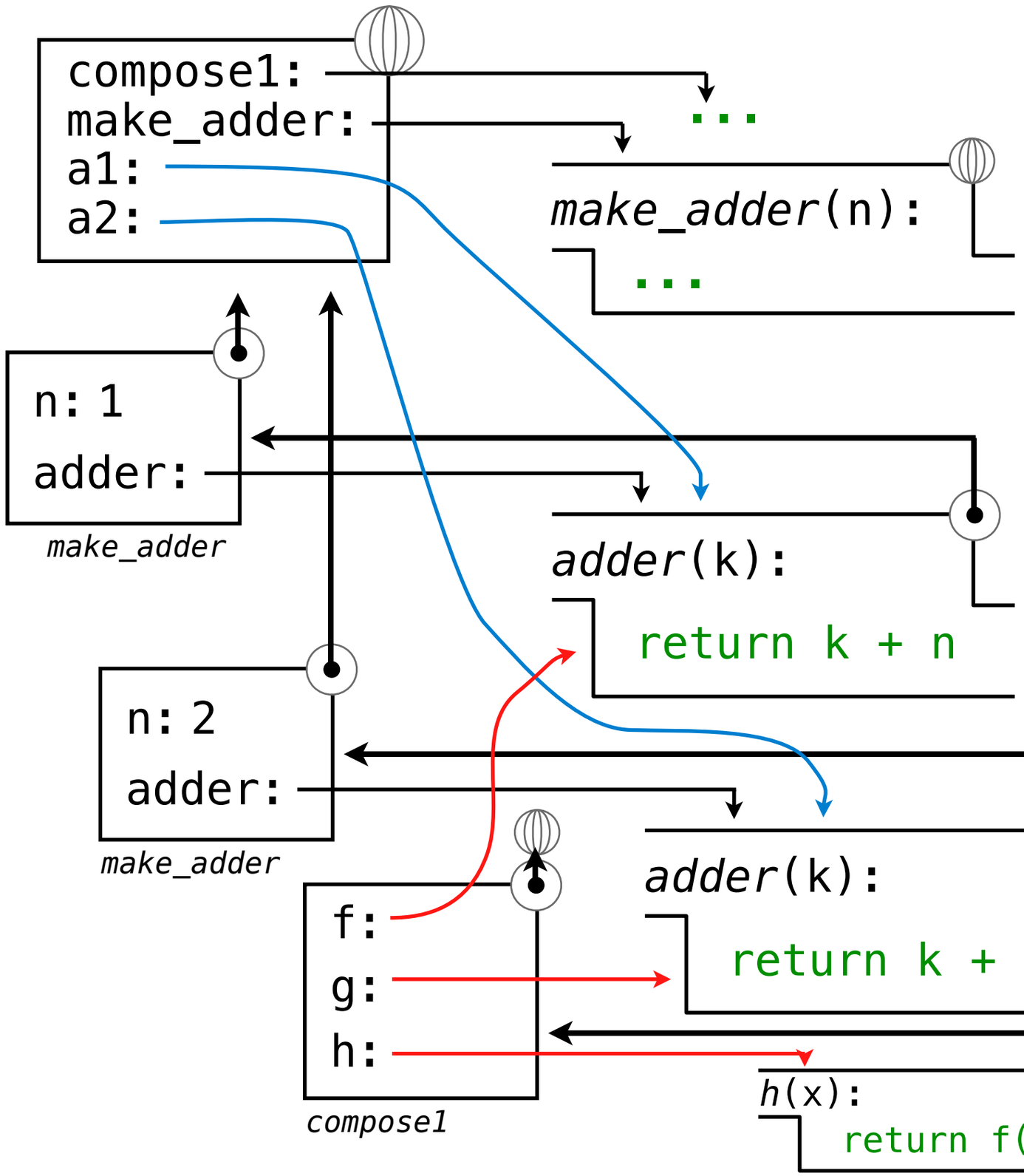


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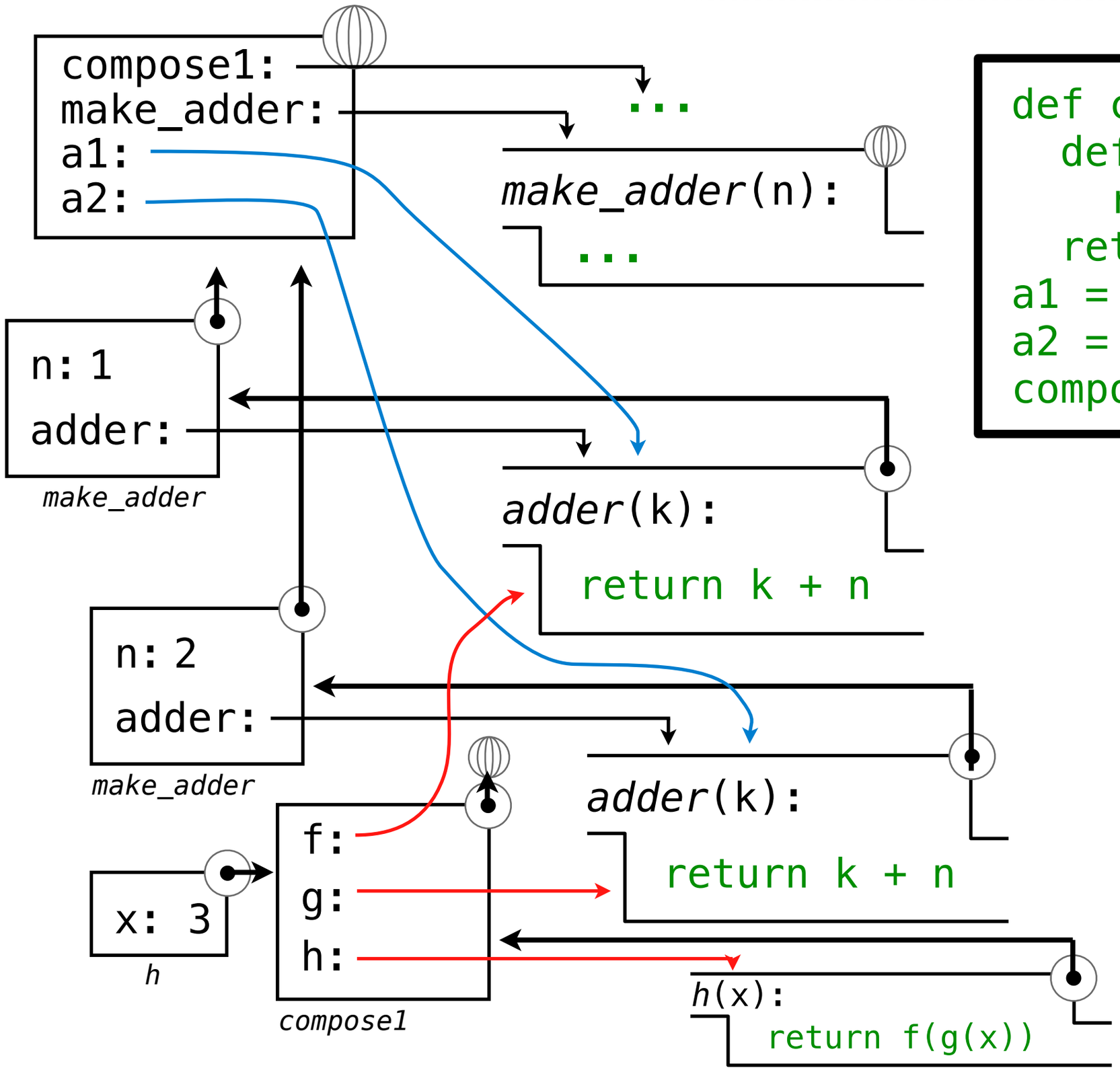


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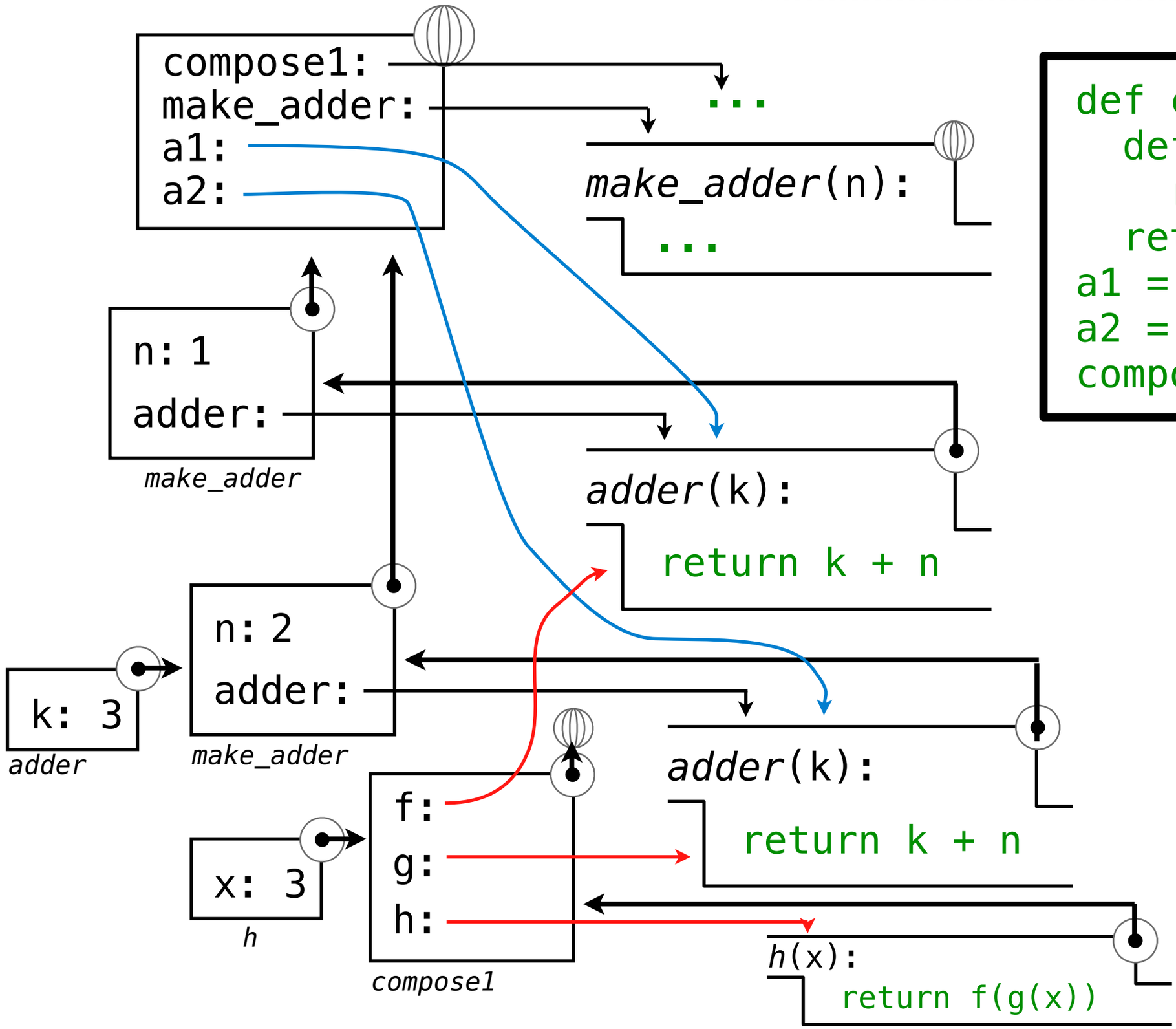


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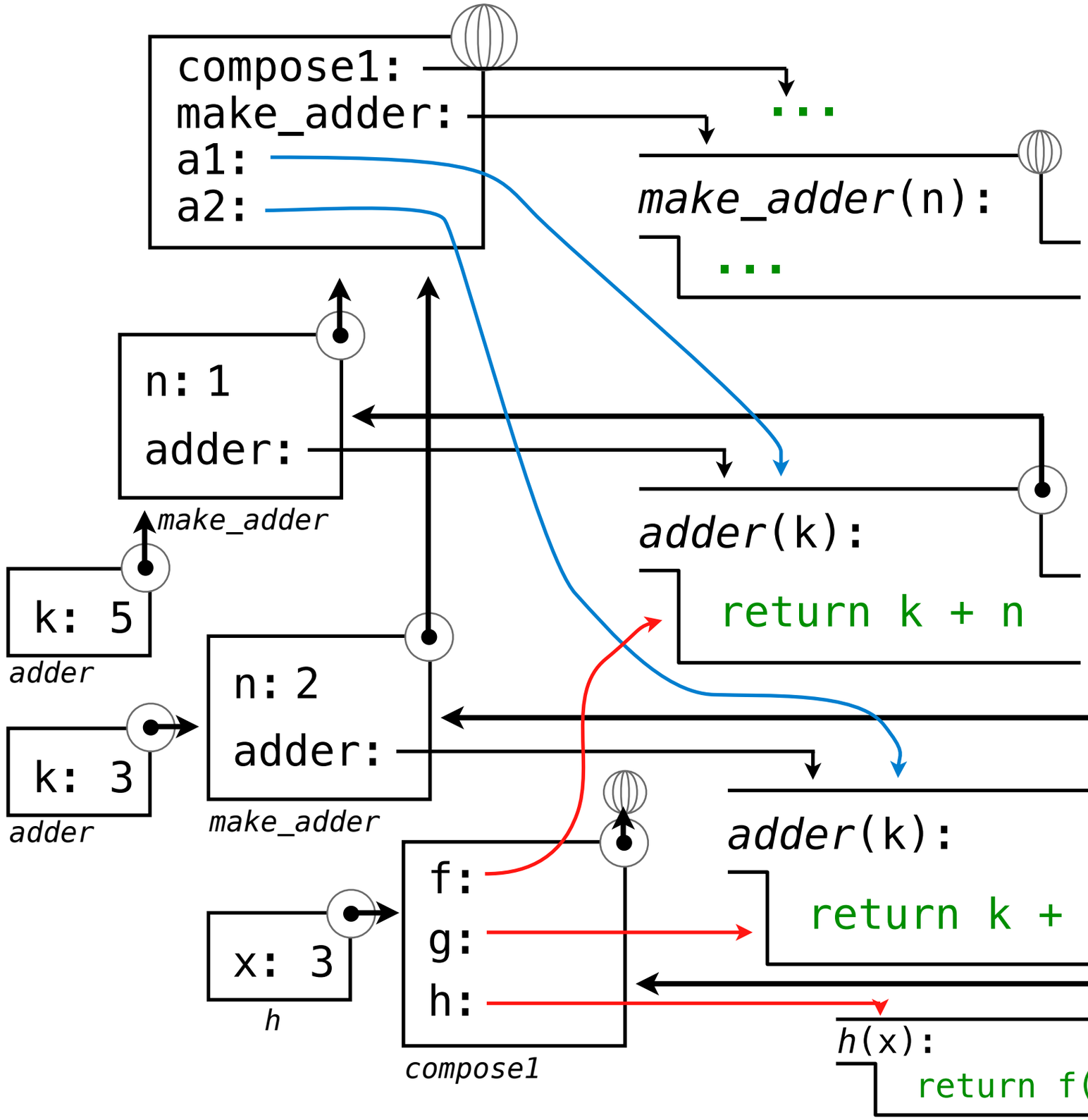


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(Demo)

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Lambda expressions are rare in Python, but important in general

More Higher-Order Function Examples

(Demo)