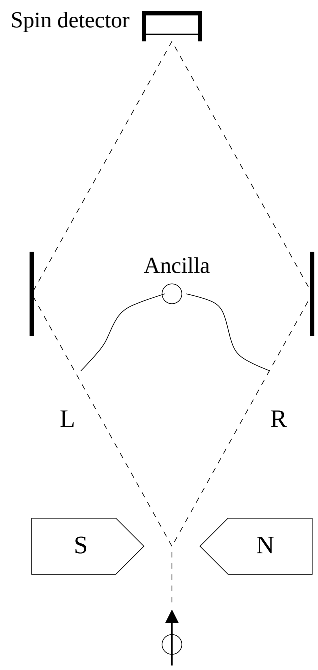
Entanglement, measurements and garbage ancillas

In the lectures it was pointed out that measurement and entanglement with the environment are equivalent things, and also that we want to make sure that ancillas are reset to their initial states, since leaving them in unknown 'garbage' states could destroy interference effects between our computational qubits. Here we will explore why this is so:

Suppose we have a spin-$\frac{1}{2}$ particle in the state $|\uparrow\rangle$, i.e. its spin is pointing up along the z-axis. Now send it flying through between two magnets which pull spins to the left if their spin points along the negative x-axis, and right if their spins point along the positive x-axis, so that the particle can take two different paths L and R after passing through the magnet.



1. If we make a measurement to see which path the spin took (by placing a detector somewhere along L and/or R), what is the spin state of the particle if it is found to be in path L? What if it is found in path R? If we measure the spin along the z-axis while the particle is on its way in either path L or path R, what are the probabilities that we will measure spin up and spin down?

2. What is the state of the particle while it is en route along paths L and/or R if we *don't* measure it?

3. Now assume we put up some kind of mirrors which bring the two paths back together again and focus them onto a spin detector which can measure the spin of the particle along the z-axis (assume that we can do this without changing the spin state of the particle in any way). We assume that the two paths have the same lengths and there is no relative phase shift between the two paths. What is the state of the particle when it reaches the detector? What is the probability that the detector will detect spin up, and what is the probability for spin down? This was not the same as if we had measured while the particle was still on its way through one of the two paths. What effect is responsible for this?

4. Now we introduce an ancilla qubit which is initially in some (unimportant but known) state $|\psi\rangle$, and which interacts with the particle in such a way that its state is turned into $|0\rangle$ if the particle goes to the left, and $|1\rangle$ if it goes to the right. Once the particle reaches the detector at the far end (but before it is actually detected), what is the joint state of the particle and the ancilla?

5. What are *now* the probabilities that the detector will measure spin up and spin down? (**Note**, we do not look at the ancilla or measure it in any way, it just sits there after the interaction with the particle)

6. What happened here seems similar to what happens when we measure the spin while it is underway, even though we did *not* make any measurements on the ancilla. Why, do you think?