CS 188 Fall 2007 — Introduction to Artificial Intelligence — Midterm

You have 80 minutes. The exam is closed book, closed notes except a one-page crib sheet, basic calculators only. 80 points total. Don't panic!

Mark your answers ON THE EXAM ITSELF. Write your name, SID, login, and section number at the top of each page.

For true/false questions, CIRCLE *True* OR *False*.

If you are not sure of your answer you may wish to provide a *brief* explanation. All short answer sections can be successfully answered in a few sentences at most.

For staff use only

Q. 1	Q. 2	Q. 3	Q. 4	Q. 5	Total
/18	/18	/16	/15	/13	/80

1. **(18 points.) True/False**

Each problem is worth 2 points. Incorrect answers are worth 0 points. Skipped questions are worth 1 point.

(a) *True/False*: If one search heuristic $h_1(s)$ is admissible and another one $h_2(s)$ is inadmissible, then $h_3(s) = \min(h_1(s), h_2(s))$ will be admissible.

(b) *True/False*: Greedy search has the same worst-case number of node expansions as DFS.

(c) *True/False*: In A*, the first path to the goal which is added to the fringe will always be optimal.

(d) *True/False*: If a CSP is arc consistent, it can be solved without backtracking.

(e) *True/False*: A CSP with only binary constraints can be solved in time polynomial in n and d, the number of variables and size of the domains.

(f) *True/False*: The minimax value of a state is always less than or equal to the expectimax value of that state.

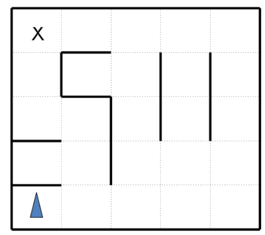
(g) *True/False*: Alpha-beta pruning can alter the computed minimax value of the root of a game search tree.

(h) *True/False*: When doing alpha-beta pruning on a game tree which is traversed from left to right, the leftmost branch will never be pruned.

(i) *True/False*: Every search problem can be expressed as an MDP with at most as many states as the original search problem.

2. (18 points.) Search and Heuristics

Imagine a car-like agent wishes to exit a maze like the one shown below:



The agent is directional and at all times faces some direction $d \in (N,\ S,\ E,\ W)$. With a single action, the agent can *either* move forward at an adjustable velocity v *or* turn. The turning actions are *left* and *right*, which change the agent's direction by 90 degrees. Turning is only permitted when the velocity is zero (and leaves it at zero). The moving actions are *fast* and *slow*. *Fast* increments the velocity by 1 and *slow* decrements the velocity by 1; in both cases the agent then moves a number of squares equal to its NEW adjusted velocity. Any action which would collide with a wall crashes the agent and is illegal. Any action which would reduce v below 0 or above a maximum speed V_{max} is also illegal. The agent's goal is to find a plan which parks it (stationary) on the exit square using as few actions (time steps) as possible.

As an example: if the agent shown were initially stationary, it might first turn to the east using (*right*), then move one square east using *fast*, then two more squares east using *fast* again. The agent will of course have to *slow* to turn.

(a) (3 points) If the grid is M by N, what is the size of the state space? Justify your answer. You should assume that all configurations are reachable from the start state.

(b) (2 points) What is the maximum branching factor of this problem? You may assume that illegal actions are simply not returned by the successor function. Briefly justify your answer.

(c) (3 points) Is the Manhattan distance from the agent's location to the exit's location admissible? Why or why not?

(d) (4 points) State and justify a non-trivial admissible heuristic for this problem which is not the Manhattan distance to the exit.

(e) (2 points) If we used an inadmissible heuristic in A* tree search, could it change the completeness of the search?

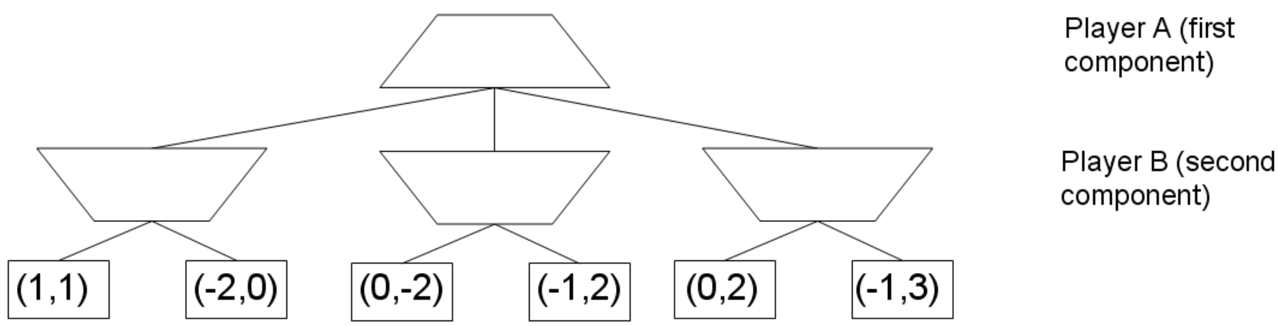
(f) (2 points) If we used an inadmissible heuristic in A* tree search, could it change the optimality of the search?

(g) (2 points) Give a general advantage that an inadmissible heuristic might have over admissible one.

3. (16 points.) Game Search

The standard Minimax algorithm calculates worst-case values in a *zero-sum* two player game, i.e. a game in which for all terminal states s, the utilities for players A (MAX) and B (MIN) obey $U_A(s) + U_B(s) = 0$. In the zero sum case, we know that $U_A(s) = -U_B(s)$ and so we can think of player B as simply minimizing $U_A(s)$.

In this problem, you will consider the *non zero-sum* generalization in which the sum of the two players' utilities are not necessarily zero. Because player A's utility no longer determines player B's utility exactly, the leaf utilities are written as pairs (U_A, U_B), with the first and second component indicating the utility of that leaf to A and B respectively. In this generalized setting, A seeks to maximize U_A, the first component, while B seeks to **maximize** U_B, the second component.



(a) (4 points) Propagate the terminal utility pairs up the tree using the appropriate generalization of the minimax algorithm on this game tree. Fill in the values (as pairs) at each of the internal node. Assume that each player maximizes their own utility. *Hint:* just as in minimax, the utility pair for a node is the utility pair of one of its children.

(b) (2 points) Briefly explain why no alpha-beta style pruning is possible in the general non-zero sum case. *Hint:* think first about the case where $U_A(s) = U_B(s)$ for all nodes.

For minimax, we know that the value v computed at the root (say for player A = MAX) is a worse-case value, in the sense that, if the opponent MIN doesn't act optimally, the actual outcome v' for MAX can only be better, never worse, than v.

(c) (3 points) In the general non-zero sum setup, can we say that the value U_A computed at the root for player A is also a worst-case value in this sense, or can A's outcome be worse than the computed U_A if B plays suboptimally? Briefly justify.

Now consider the *nearly zero sum* case, in which $|U_A(s) + U_B(s)| < \epsilon$ at all terminal nodes s for some ϵ which is known in advance. For example, the previous game tree is nearly zero sum for $\epsilon = 2$.

(d) (3 points) In the nearly zero sum case, pruning is possible. Draw an X in each node in this game tree which could be pruned with the appropriate generalization of alpha-beta pruning. Assume that the exploration is being done in the standard left to right depth-first order and the value of ϵ is known to be 2. Make sure you make use of ϵ in your reasoning.

(e) (2 points) Give a general condition under which a child n of a B node (MIN node) b can be pruned. Your condition should generalize α-pruning and should be stated in terms of quantities such as the utilities $U_A(s)$ and/or $U_B(s)$ of relevant nodes s in the game tree, the bound ϵ, and so on. Do not worry about ties.

(f) (3 points) In the nearly zero sum case with bound ϵ, what guarantee, if any, can we make for the actual outcome u' for player A (in terms of the value U_A of the root) in the case where player B acts suboptimally?

4. (15 points.) MDPs and Reinforcement Learning

In Flipper's Folly, a player tries to predict the total number of heads in two coin flips. The game proceeds as follows (also shown below):

(a) From the start state (XX), choose the special action *begin* (only possible action)

(b) Flip a coin and observe the result, arriving in the state HX or TX

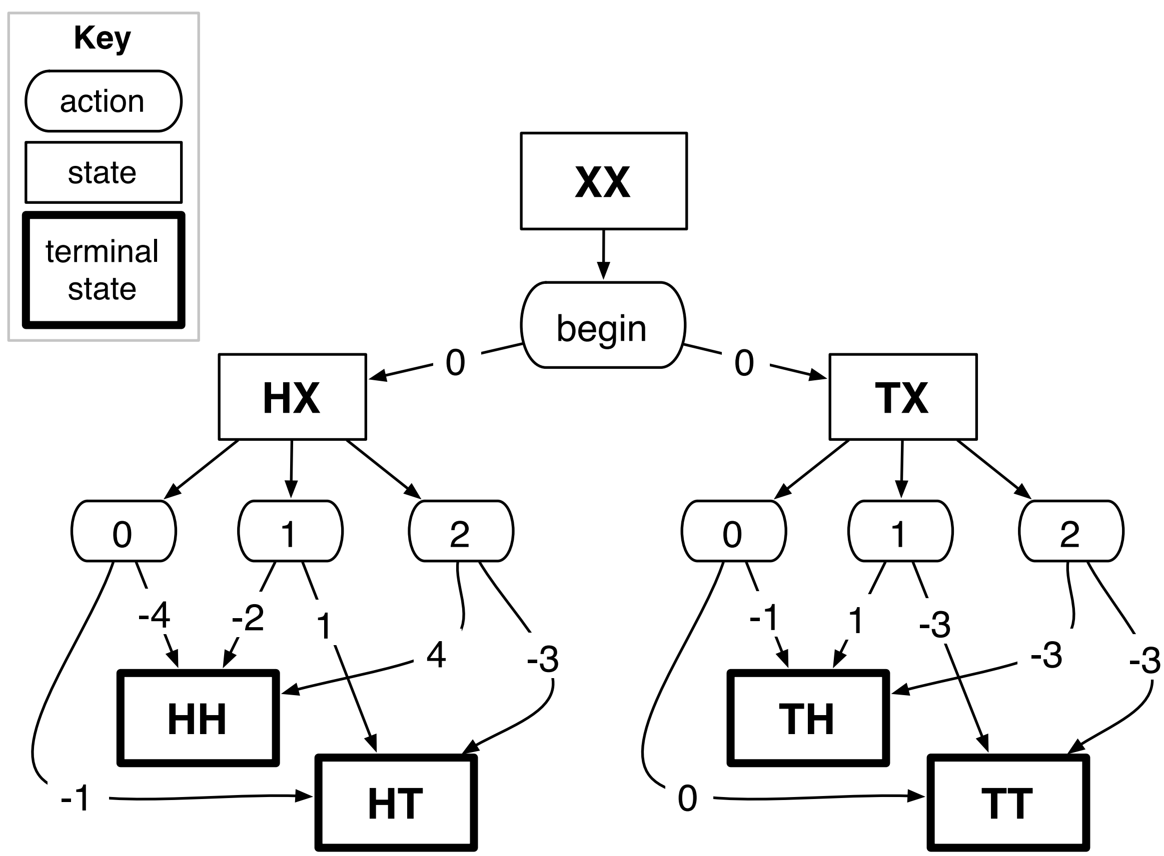
(c) Guess what the total number of heads will be: $a \in \{0, 1, 2\}$

(d) Flip a coin and observe the result, arriving in one of the states HH, HT, TH, TT.

(e) Count the total number of heads in the two flips; $c \in \{0, 1, 2\}$

(f) Receive reward $R(s, a, s') = \begin{cases} 2 \cdot a^2 - c^2 & \text{if } c \geq a \\ -3 & \text{if } c < a \end{cases}$ where c is the total number of heads in s'

Note that the rewards depend only on the action and the landing state, and that all rewards for leaving the start state are zero. The MDP for this game has the following structure, where all legal transitions have probability $\frac{1}{2}$. **Assume a discount rate of 1.**



(a) (3 points) What is the value of the start state under the policy of always guessing $a = 2$?

(b) (5 points) Run value iteration on this MDP until convergence. *Hint*: values and q-values of terminal states are always 0.

	$V_k^*(s)$		
k	XX	HX	TX
0	0	0	0
1			
2			
3			
4			
5			

(c) (2 points) What is the optimal policy for this MDP?

(d) (5 points) Run q-learning in this MDP with the following (s, a, s', r) observations. Use a learning rate of $\frac{1}{2}$. Leave zero entries blank.

				$Q(s,a)$						
s	a	s'	r	(XX,begin)	(HX,0)	(HX,1)	(HX,2)	(TX,0)	(TX,1)	(TX,2)
				0	0	0	0	0	0	0
XX	begin	HX	0							
HX	0	HT	-1							
XX	begin	HX	0							
HX	2	HH	4							
XX	begin	HX	0							

5. (13 points.) Short Answer

Each question can be answered in a single sentence!

(a) (2 pts) For A* search, why might we prefer a heuristic which expands more nodes over one which expands fewer nodes?

(b) (2 pts) Why is minimax less reasonable than expectimax as a practical decision-making principle for a complex agent acting in the real world?

(c) (4 pts) An agent prefers to be given an envelope containing \$4 rather than one containing either \$0 and \$10 (with equal probability). Give a justification for how the agent could be acting in accordance with the principle of maximum expected utility.

(d) (4 pts) A *stochastic policy* π is one which does not recommend a single, deterministic action for each state s, but rather gives each possible action a a probability. Let $\pi(s, a)$ be the probability that the policy assigns to action a from state s. State a one-step lookahead Bellman equation for $V^{\pi}(s)$ for the case of stochastic policies π.

(e) (3 pts) Under what conditions can an MDP be solved using standard state space search techniques (DFS, BFS, etc.)?