# CS-184: Computer Graphics

Lecture #24: Global Illumination

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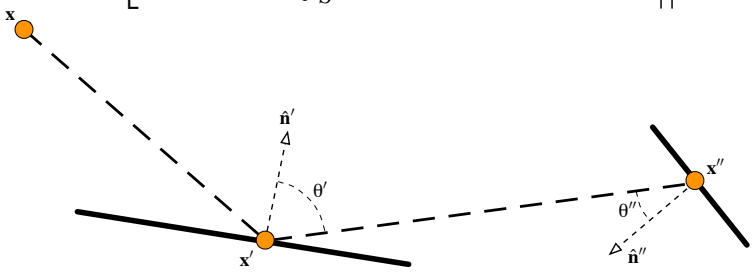
V2006-F-24-1.0

# Today

- Radiosity Method
- Photon Mapping

# The Rendering Equation

$$L_s(\mathbf{x},\mathbf{x}') = \delta(\mathbf{x},\mathbf{x}')\left[E(\mathbf{x},\mathbf{x}') + \int_S \rho_{x'}(\mathbf{x},\mathbf{x}'')L_s(\mathbf{x}',\mathbf{x}'')\frac{\cos(\theta')\cos(\theta'')}{||\mathbf{x}'-\mathbf{x}''||^2}d\mathbf{x}''\right]$$




- Slightly different notation from last class
  - Assumes "smart" functions convert positions to angles
  - Includes *emissive* term $E(\mathbf{x},\mathbf{x}')$



# Radiosity

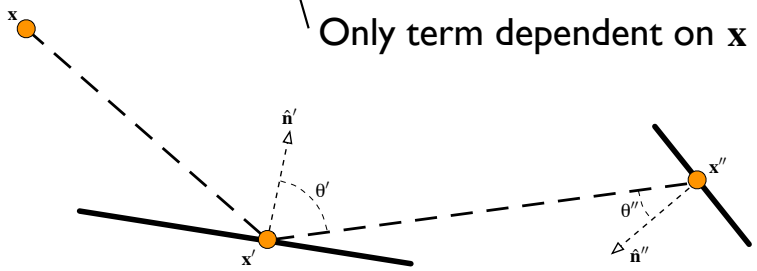
- Assume all materials are perfectly Lambertian
  - Removes all dependance on directions
  - Reduces dimensionality of lightfield
  - Allows a FEM solution
- Can also relax assumption slightly...

# Assume Lambertian

$$L_s(\mathbf{x},\mathbf{x}') = \delta(\mathbf{x},\mathbf{x}')\left[E(\mathbf{x},\mathbf{x}') + \int_S \rho_{x'}(\mathbf{x},\mathbf{x}'')L_s(\mathbf{x}',\mathbf{x}'')\frac{\cos(\theta')\cos(\theta'')}{||\mathbf{x}'-\mathbf{x}''||^2}d\mathbf{x}''\right]$$

$$L_s(\mathbf{x},\mathbf{x}') = \underline{\delta(\mathbf{x},\mathbf{x}')}\left[E_{x'} + \int_S \rho_{x'}L_s(\mathbf{x}',\mathbf{x}'')\frac{\cos(\theta')\cos(\theta'')}{||\mathbf{x}'-\mathbf{x}''||^2}d\mathbf{x}''\right]$$

Only term dependent on $\mathbf{x}$



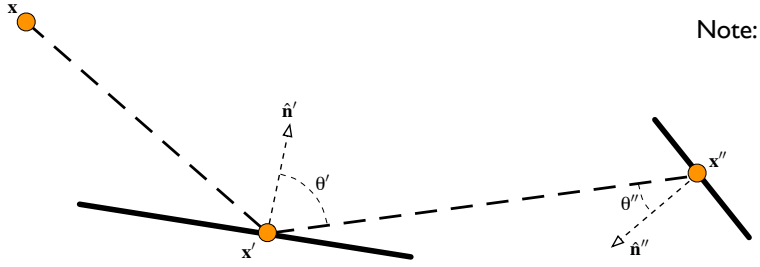



# Rewrite in Terms of Radiosity

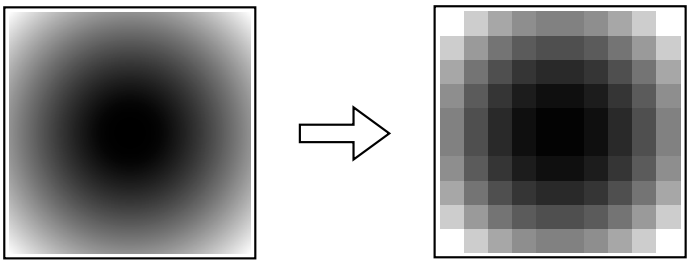
$$L_s(\mathbf{x},\mathbf{x}') = \delta(\mathbf{x},\mathbf{x}')\left[E_{x'} + \int_S \rho_{x'}L_s(\mathbf{x}',\mathbf{x}'')\frac{\cos(\theta')\cos(\theta'')}{||\mathbf{x}'-\mathbf{x}''||^2}d\mathbf{x}''\right]$$

$$H_{x'} = E_{x'} + \rho_{x'}\int_S \delta(\mathbf{x}',\mathbf{x}'')\frac{H_{x''}}{2\pi}\frac{\cos(\theta')\cos(\theta'')}{||\mathbf{x}'-\mathbf{x}''||^2}d\mathbf{x}''$$

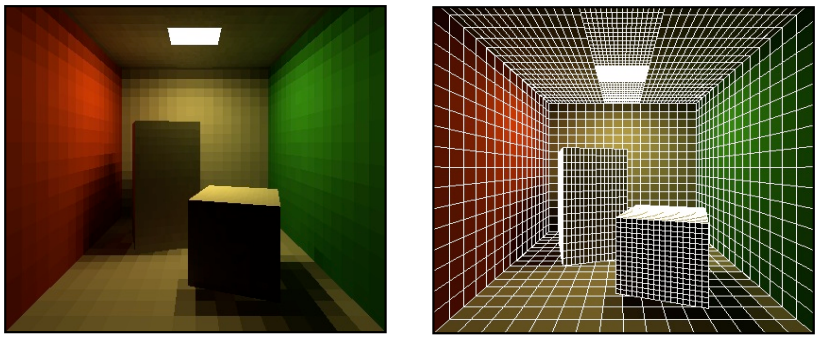
Note: we changed defn of $E$ here.



# Discretize into Patches



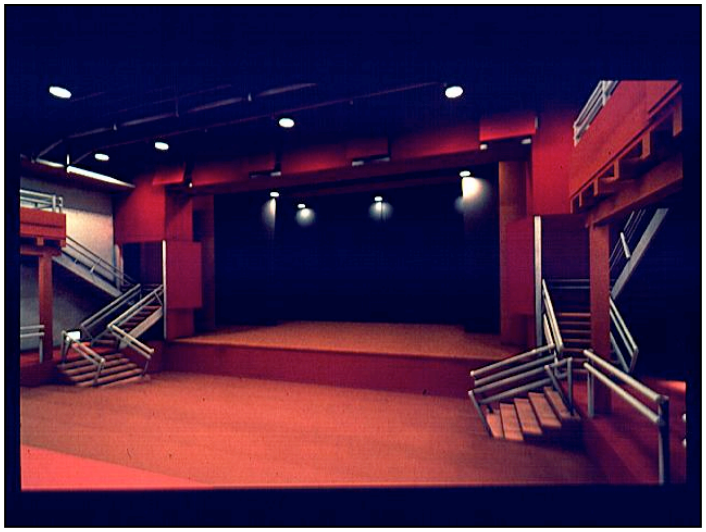
Piece-wise
constant patches



Example mesh for Cornell Box
by Mark Schmelzenbach



# Discretize into Patches



The Candlestick Theater,
Mark Mack Architects.

# Discretize into Patches

The Candlestick Theater,
Mark Mack Architects.

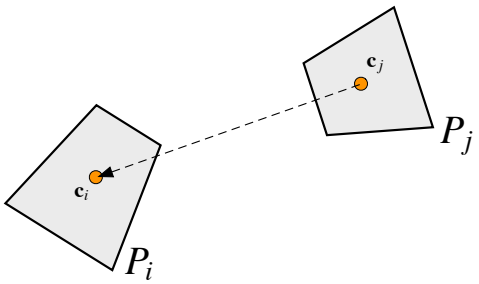


# Rewrite in Terms of Patches

$$H_{x'} = E_{x'} + \rho_{x'} \int_S \delta(\mathbf{x}', \mathbf{x}'') H_{x''} \frac{\cos(\theta') \cos(\theta'')}{||\mathbf{x}' - \mathbf{x}''||^2} d\mathbf{x}''$$

$$H_i = E_i + \rho_i \sum_j H_j F_{ij}$$

Form factor from $j$ to $i$, $F_{ij}$




Example of a rough approximation:

$$F_{ij} \approx \delta_{ij} \frac{\cos(\theta_i) \cos(\theta_j)}{2\pi ||\mathbf{c}_i - \mathbf{c}_j||^2} A_j$$

# Radiosity Method

- Given the $E_i$ and $\rho_i$
- First compute $F_{ij}$
- Then solve $H_i = E_i + \rho_i \sum_j H_j F_{ij}$ $\Rightarrow$ $\mathbf{h} = \mathbf{e} + \mathbf{A}\mathbf{h}$, $(\mathbf{I} - \mathbf{A})\mathbf{h} = \mathbf{e}$
- Comments:
  - The matrix $\mathbf{A}$ is typically very large
  - It is also sparse (why?)
  - Should be solved with an iterative method
    - *e.g.:* Jacobi or Gauss-Seidel
  - **Solution is view independent**



# Progressive Radiosity

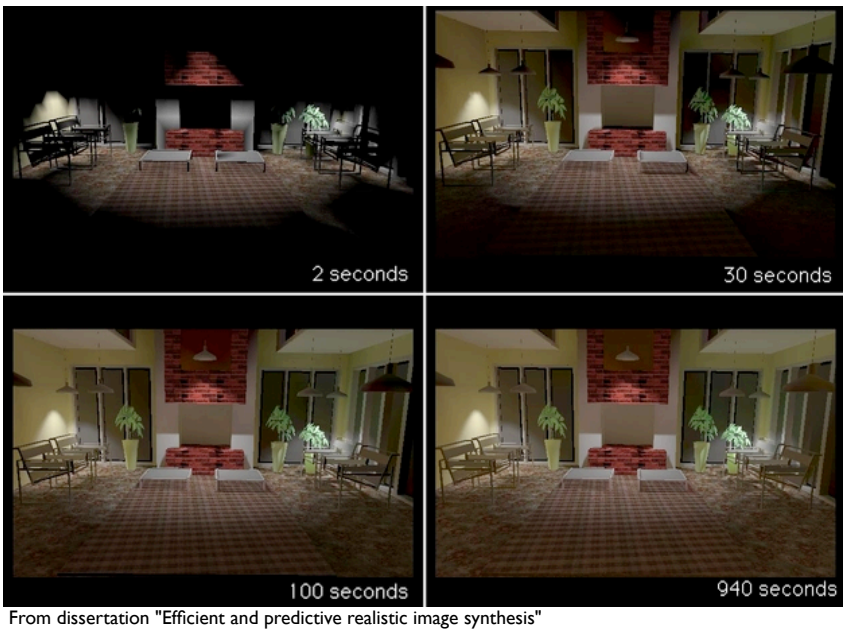
- If magnitude of eigenvalues of $\mathbf{A} < 1$

$$(\mathbf{I} - \mathbf{A})^{-1} = \mathbf{I} + \mathbf{A} + \mathbf{A}^2 + \mathbf{A}^3 + \cdots$$

  - True for form-factor matrices
  - $\mathbf{h}^{k+1} = \mathbf{h}^k + \mathbf{u}^{k+1}$
    $\mathbf{u}^{k+1} = \mathbf{A}\,\mathbf{u}^k$
    $\mathbf{h}^0 = 0 \quad \mathbf{u}^0 = \mathbf{e}$
  - Use Gauss-Seidel-like iteration but reorder by priority

Southwell Relaxation

# Progressive Radiosity

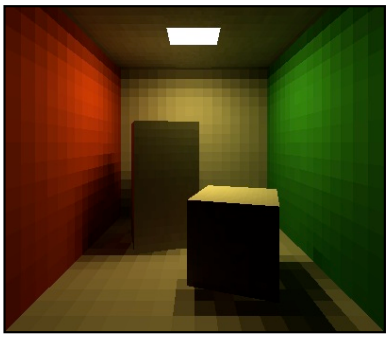



From dissertation "Efficient and predictive realistic image synthesis"
by Karol Myszkowski

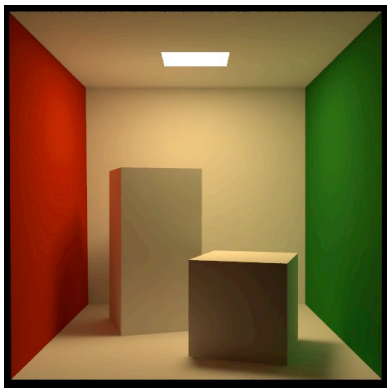


# Touchup

- Each patch will have a constant color
  - Smooth solution (*e.g.* average to vertices)



Example mesh for Cornell Box
by Mark Schmelzenbach



Does not match but you get the idea...

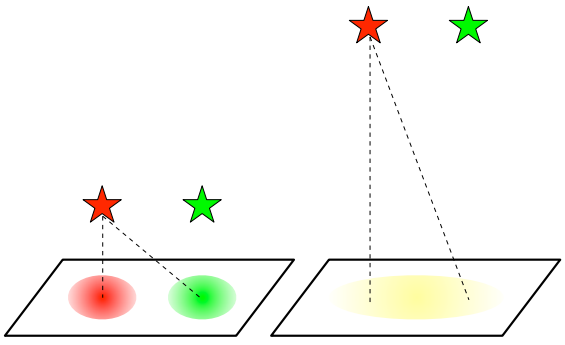
# Other Things

- Each patch will have a constant color
  - Smooth solution (*e.g.* average to vertices)
- No specular reflection
  - Add Phong specular term or raytraced specular reflection
- Grid artifacts
  - Be clever with grid...



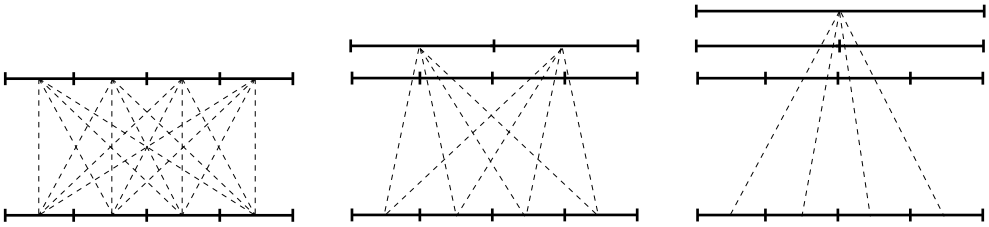
# Hierarchical Radiosity

- Light smoothes with distance
  - Compare $1/h^2$ with $1/(h^2+d^2)$ as $h$ gets large



# Hierarchical Radiosity

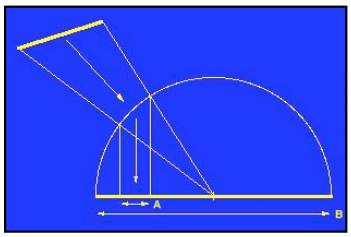
- Light smoothes with distance
  - Compare $1/h^2$ with $1/(h^2+d^2)$ as $h$ gets large
- Group patches into hierarchy
  - Far interactions use lower-res form factors

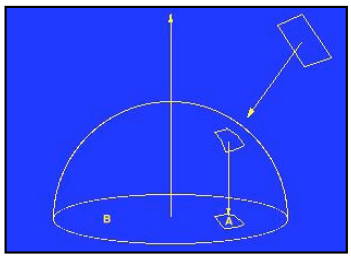




# Computing Form Factors

- Form factors have a geometric meaning

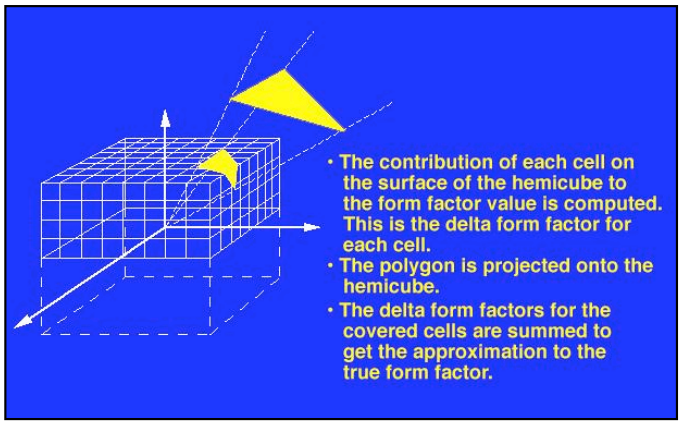



Images from
SIGGRAPH 93 Education Slide Set
by Stephen Spencer

# Computing Form Factors

- Form factors have a geometric meaning
- "Hemicube" algorithm uses regular scan conversion




Images from
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by Stephen Spencer

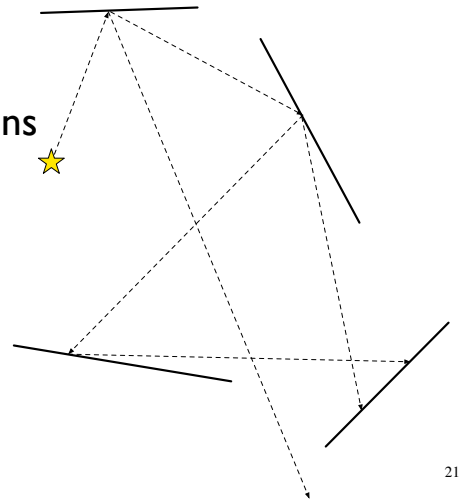


# Computing Form Factors

- Form factors have a geometric meaning
- "Hemicube" algorithm uses regular scan conversion
- Also computed by ray-based sampling
- In practice, computing form factors is the bottleneck

# Photon Mapping

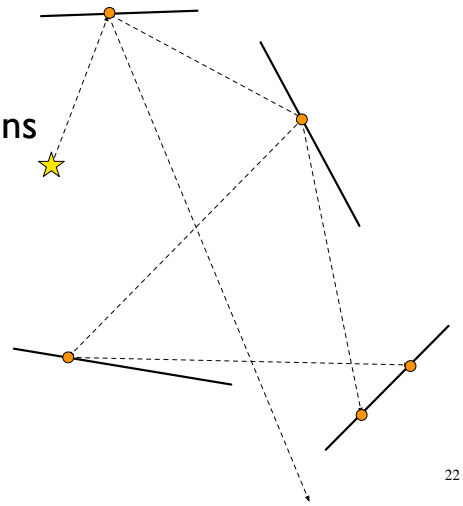
- Lights cast "photons" into environment
  - Cast in random directions
  - Trace into environment
  - Store records at intersections



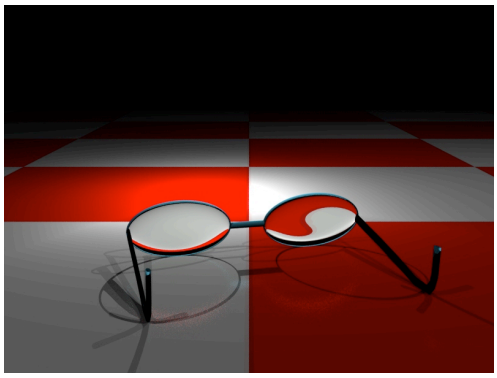


# Photon Mapping

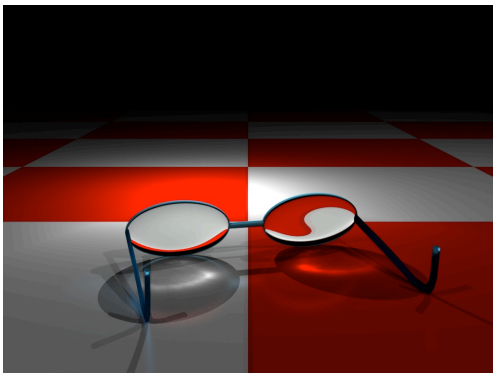
- Lights cast "photons" into environment
  - Cast in random directions
  - Trace into environment
  - Store records at intersections
    - With KD-Trees...



# Comparison



Ray Tracing



Ray Tracing w/ Photon Map



# Photon Mapping

Image by Per Christensen

A ray traced image

Note:
- Dark shadows
- Unlit corners
- Nice reflections

# Photon Mapping

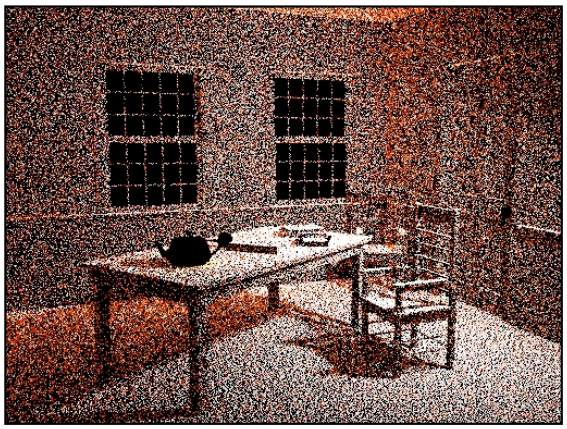


Image by Per Christensen

Raw photons

Note:
- Noisy
- Sparse



# Photon Mapping

Image by Per Christensen

Interpolated Photons

Note:
- Still noisy
- Biased

# Photon Mapping

Image by Per Christensen

Interpolated Photons
(multiplied by diffuse)

Note:
Still noisy
Biased



# Photon Mapping

- Final Gather
  - Ray trace scene
  - Direct and specular rays as normal
  - Diffuse rays traced into photon map
  - *Diffuse reflection smoothes noise*

# Photon Mapping

Image by Per Christensen

Final Image

Note:
- Not noisy
- Nice lighting
- Reflections
- May still be biased

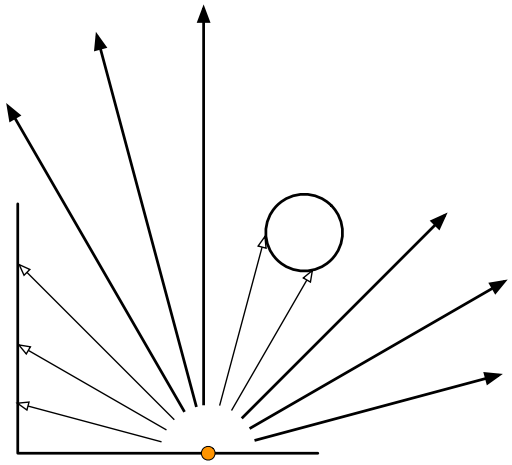
Final gather often bottleneck...



# Ambient Occlusion

- A "hack" to create more realistic ambient illumination cheaply
- Assume light from everywhere is partially blocked by local objects
  - At a point on the surface cast rays at random
  - Ambient term is proportional to percent of rays that hit nothing
  - Weight average by cosine of angle with normal
  - Take into account how far before occluded

# Ambient Occlusion





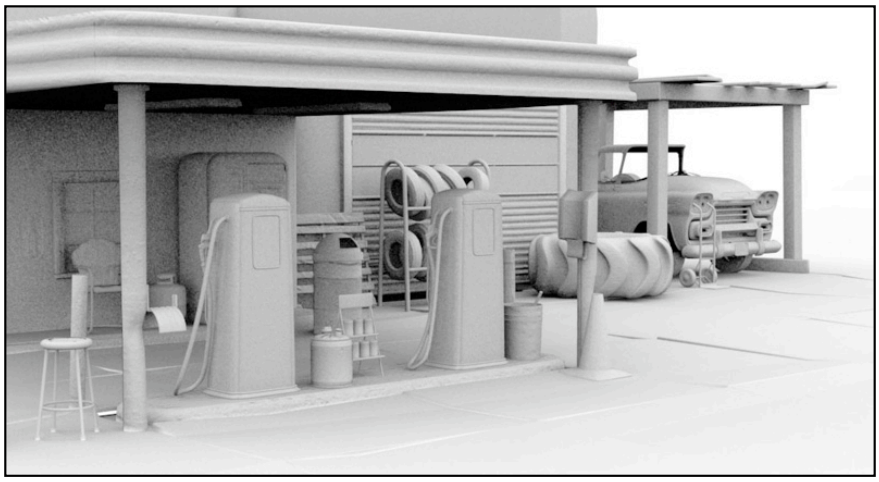
# Ambient Occlusion


Diffuse Only


Ambient Occlusion


Combined

# Ambient Occlusion



nVidia Gelato Demo Image