CS-184: Computer Graphics

Lecture #21: Spring and Mass systems

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Today

- Spring and Mass systems
  - Distance springs
  - Spring dampers
  - Edge springs

A Simple Spring

- Ideal *zero*-length spring

$$\boldsymbol{f}_{a \to b} = k_s(\boldsymbol{b} - \boldsymbol{a})$$

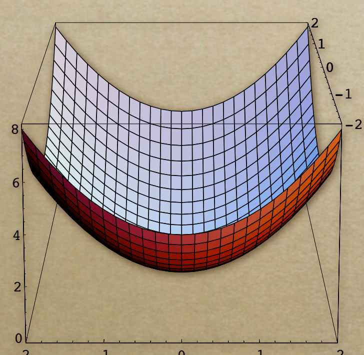
$$\boldsymbol{f}_{b \to a} = -\boldsymbol{f}_{a \to b}$$

- Force pulls points together
- Strength proportional to distance

A Simple Spring

- Energy potential

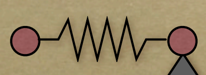
$$E = 1/2\ k_s(\boldsymbol{b} - \boldsymbol{a}) \cdot (\boldsymbol{b} - \boldsymbol{a})$$



$$\boldsymbol{f}_{a \to b} = k_s(\boldsymbol{b} - \boldsymbol{a})$$

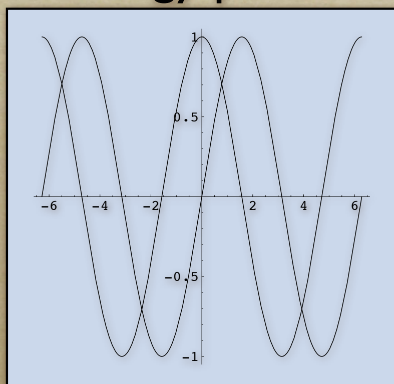
$$\boldsymbol{f}_{b \to a} = -\boldsymbol{f}_{a \to b}$$

$$\boldsymbol{f}_a = -\nabla_a E = -\left[\frac{\partial E}{\partial a_x}, \frac{\partial E}{\partial a_y}, \frac{\partial E}{\partial a_z}\right]$$



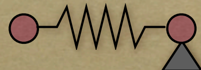
A Simple Spring

- Energy potential: kinetic *vs* elastic



$$E = 1/2\, k_S(\boldsymbol{b} - \boldsymbol{a}) \cdot (\boldsymbol{b} - \boldsymbol{a})$$

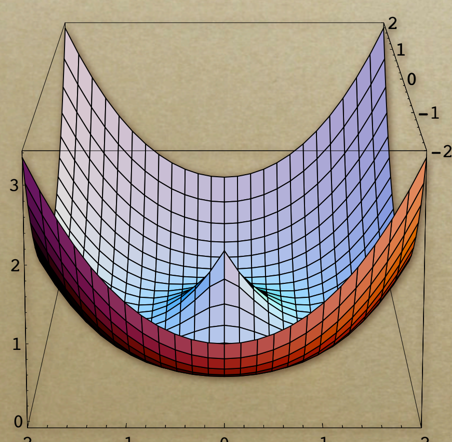
$$E = 1/2\, m(\dot{\boldsymbol{b}} - \dot{\boldsymbol{a}}) \cdot (\dot{\boldsymbol{b}} - \dot{\boldsymbol{a}})$$



Non-Zero Length Springs

$$\boldsymbol{f}_{a \to b} = k_S \frac{\boldsymbol{b} - \boldsymbol{a}}{||\boldsymbol{b} - \boldsymbol{a}||} (||\boldsymbol{b} - \boldsymbol{a}|| - l)$$

Rest length



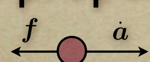
$$E = k_S\, (||\boldsymbol{b} - \boldsymbol{a}|| - l)^2$$

Comments on Springs

- Springs with zero rest length are linear
- Springs with non-zero rest length are nonliner
  - Force *magnitude* linear w/ discplacement (from rest length)
  - Force direction is non-linear
  - Singularity at $||\boldsymbol{b} - \boldsymbol{a}|| = 0$

Damping

- "Mass proportional" damping

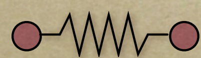


$$\boldsymbol{f} = -k_d \dot{\boldsymbol{a}}$$

  - Behaves like viscous drag on all motion
  - Consider a pair of masses connected by a spring
    - How to model rusty *vs* oiled spring
    - Should internal damping slow group motion of the pair?
  - Can help stability... up to a point

Damping

- "Stiffness proportional" damping

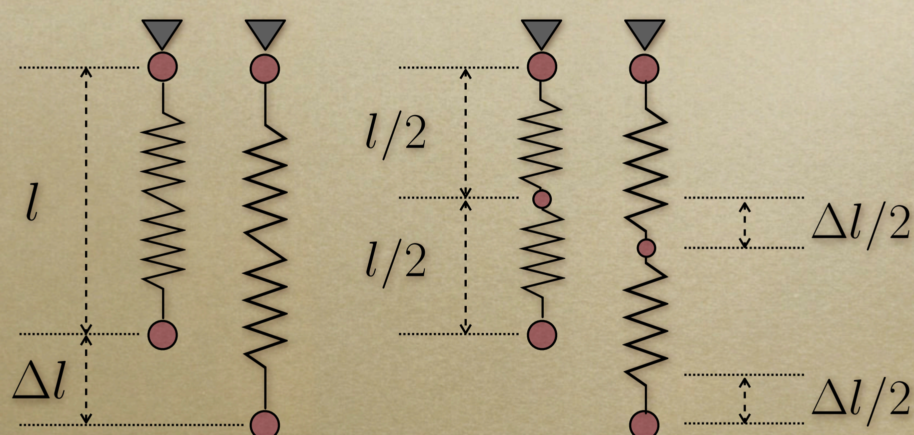


$$\boldsymbol{f}_a = -k_d \frac{\boldsymbol{b}-\boldsymbol{a}}{||\boldsymbol{b}-\boldsymbol{a}||^2}(\boldsymbol{b}-\boldsymbol{a})\cdot(\dot{\boldsymbol{b}}-\dot{\boldsymbol{a}})$$

  - Behaves viscous drag on change in spring length
  - Consider a pair of masses connected by a spring
    - How to model rusty *vs* oiled spring
    - Should internal damping slow group motion of the pair?

Spring Constants

- Two ways to model a single spring



Spring Constants

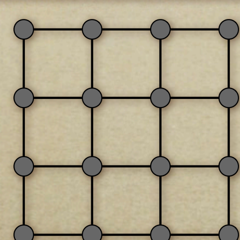
- Constant k_S gives inconsistent results with different discretizations
- Change in length is not what we want to measure
- Strain: change in length as fraction of original length

$$\epsilon = \frac{\Delta l}{l_0}$$

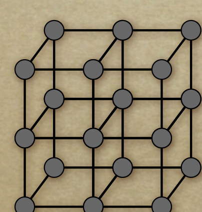
Nice and simple for 1D...

Structures from Springs

- Sheets



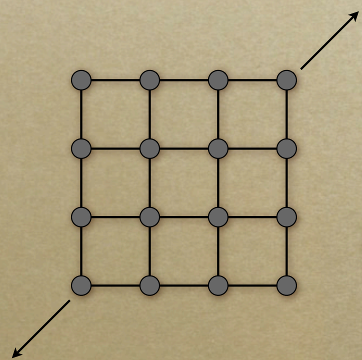
- Blocks



- Others

Structures from Springs

- They behave like what they are (obviously!)

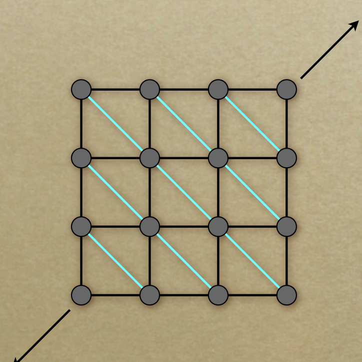


This structure will not resist shearing

This structure will not resist out-of-plane bending either...

Structures from Springs

- They behave like what they are (obviously!)

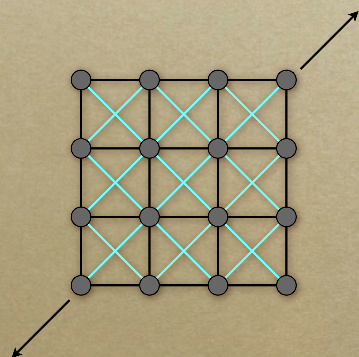


This structure will resist shearing but has anisotopic bias

This structure still will not resist out-of-plane bending

Structures from Springs

- They behave like what they are (obviously!)

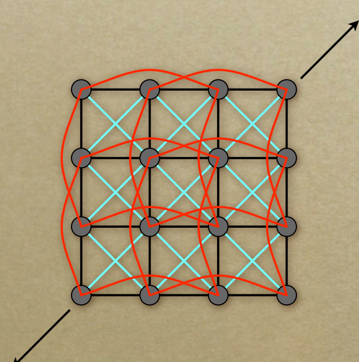


This structure will resist shearing
Less bias
Interference between spring sets

This structure still will not resist out-of-plane bending

Structures from Springs

- They behave like what they are (obviously!)

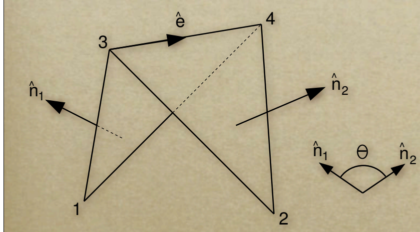


This structure will resist shearing
Less bias
Interference between spring sets

This structure will resist out-of-plane bending
Interference between spring sets
Odd behavior

How do we set spring constants?

Edge Springs



$$u_1 = |E|\frac{N_1}{|N_1|^2} \qquad u_2 = |E|\frac{N_2}{|N_2|^2}$$

$$u_3 = \frac{(x_1 - x_4)\cdot E}{|E|}\frac{N_1}{|N_1|^2} + \frac{(x_2 - x_4)\cdot E}{|E|}\frac{N_2}{|N_2|^2}$$

$$u_4 = -\frac{(x_1 - x_3)\cdot E}{|E|}\frac{N_1}{|N_1|^2} - \frac{(x_2 - x_3)\cdot E}{|E|}\frac{N_2}{|N_2|^2}$$

$$F_i^e = k^e \frac{|E|^2}{|N_1| + |N_2|}\sin(\theta/2)\, u_i$$

From Bridson *et al.*, 2003, also see Grinspun *et al.*, 2003

Suggested Reading

- Physically Based Modeling: Principles and Practice
  - Andy Witkin and David Baraff
  - http://www-2.cs.cmu.edu/~baraff/sigcourse/index.html
- Grinspun, Hirani, Desbrun, and Peter Schroder, "Discrete Shells," SCA 2003
- Bridson, Marino, and Fedkiw, "Simulation of Clothing with Folds and Wrinkles," SCA 2003
- O'Brien and Hodgins, "Graphical Modeling and Animation of Brittle Fracture," SIGGRAPH 99