CS-184: Computer Graphics

Lecture #25: Modeling w/ Points

Prof. James O'Brien
University of California, Berkeley

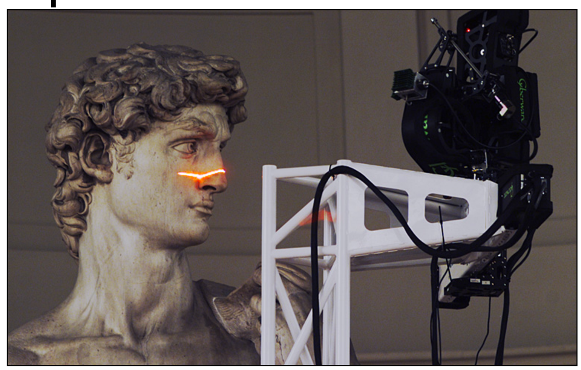
V2006F-15-1.0

Today

- Points as a graphics primitive

A Thought Experiment

- Laser scanners
  - Millions to *billions* of points
- Typical image
  - At most a few million pixels
- More points than pixels...



"Point-Based Graphics"

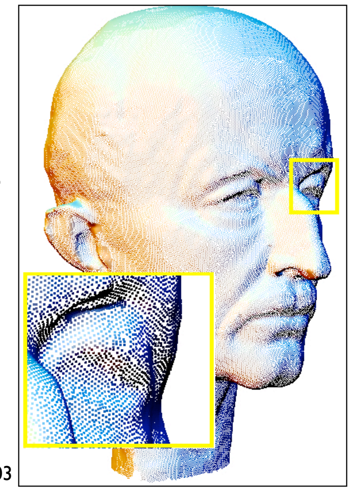
- Surfaces represented only by points
  - Maybe normals also
  - No topology
- How can we do
  - Rendering
  - Modeling opperations
  - Simulation

Rendering

- For each point draw a little "splat"
  - Use associated normal for shading
  - Possibly apply texture

If "splats" are small compared to spacing then gaps result

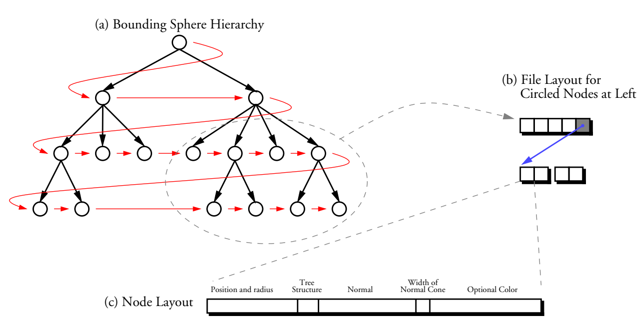
Splatting too many points would waste time



Ohtake, *et al.*, SIGGRAPH 2003

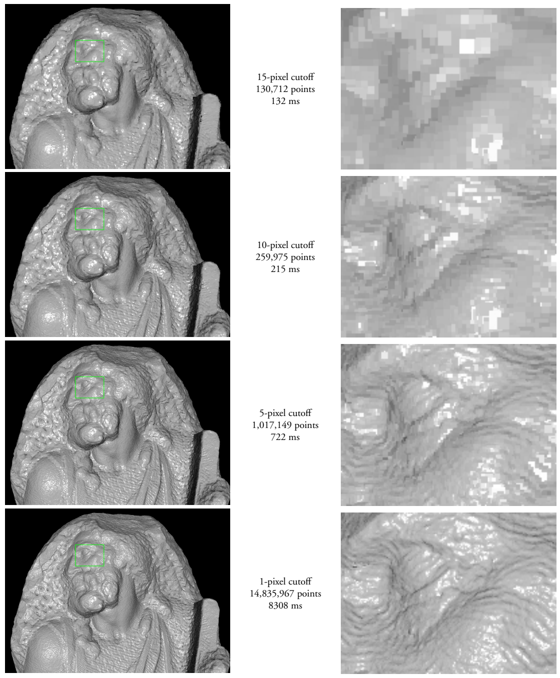
Rendering

- "QSplat" algorithm
  - Build hierarchical tree of the points
  - Use bounding spheres to estimate size of clusters
  - Render clusters based on screen size
  - Use cluster-normals for internal nodes



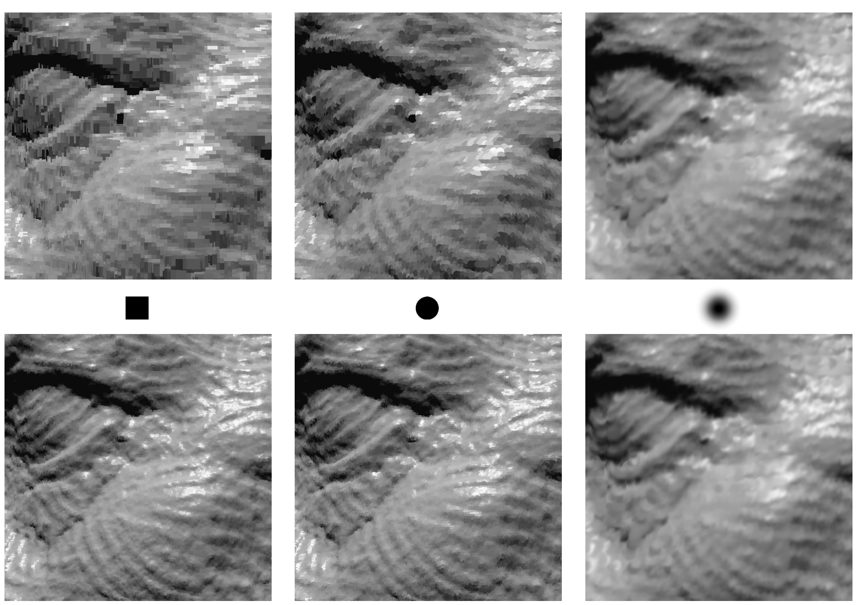
From Rusinkiewicz and Levoy, SIGGRAPH 2000.

Rendering



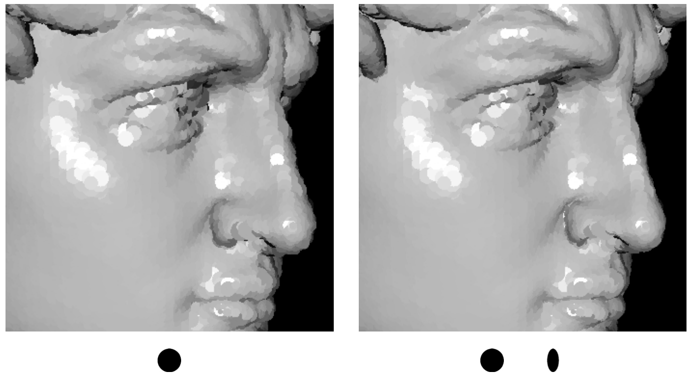
From Rusinkiewicz and Levoy, SIGGRAPH 2000.

Rendering



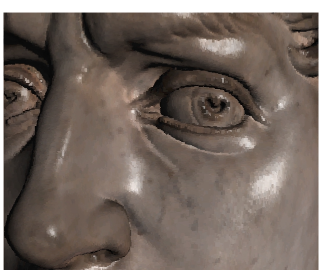
From Rusinkiewicz and Levoy, SIGGRAPH 2000.

Rendering

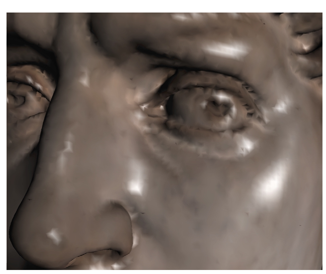


From Rusinkiewicz and Levoy, SIGGRAPH 2000.

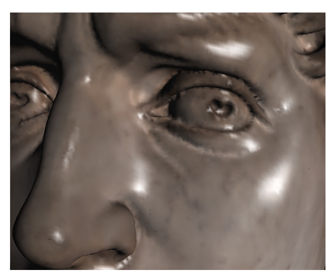
Rendering



(a)
Points



(b)
Polygons – same number of primitives as (a)
Same rendering time as (a)



(c)
Polygons – same number of vertices as (a)
Twice the rendering time of (a)

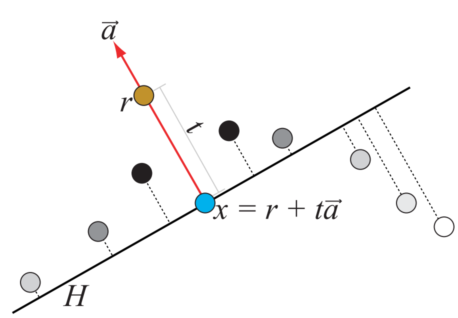
From Rusinkiewicz and Levoy, SIGGRAPH 2000.

Defining a Surface

- Two related methods
  - Surface is a point attractor
    - Point-set surfaces
  - Implicit surface
    - Multi-level Partition of Unity Implicits
    - Implicit Moving Least-Squares

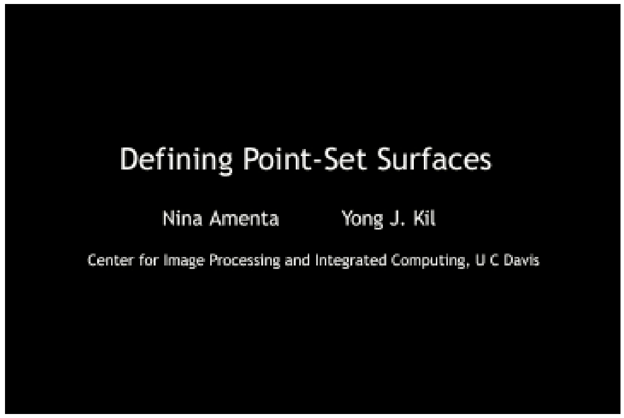
Point-Set Surfaces

- Surface is the attractor of a repeated projection process
  - Find nearby points
  - Fit plane (weighted)
  - Project into plane
  - Repeat
- Does it converge?
- How to weight points?

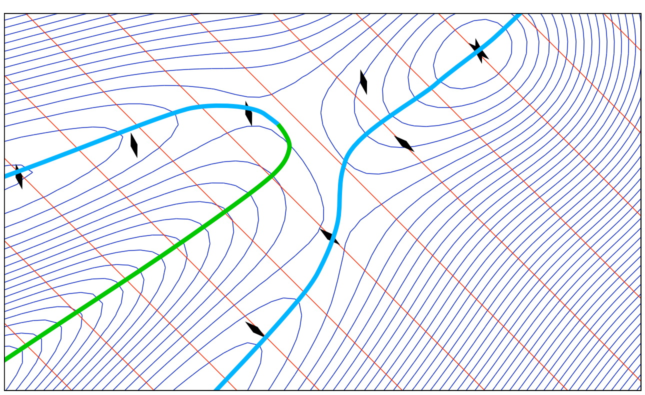


From Amenta and Kil, SIGGRAPH 2004.

Point-Set Surfaces



Point-Set Surfaces



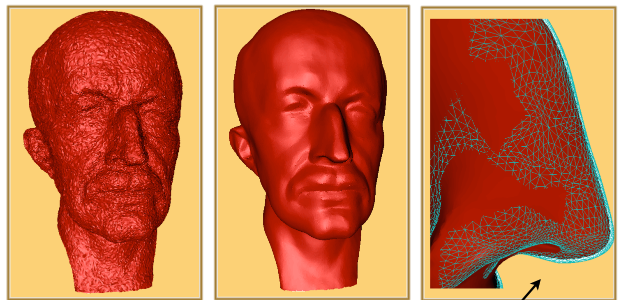
From Amenta and Kil, SIGGRAPH 2004.

Does this give us a good surface?

New “robust” methods existfor sharp features

Point-Set Surfaces

- Some examples

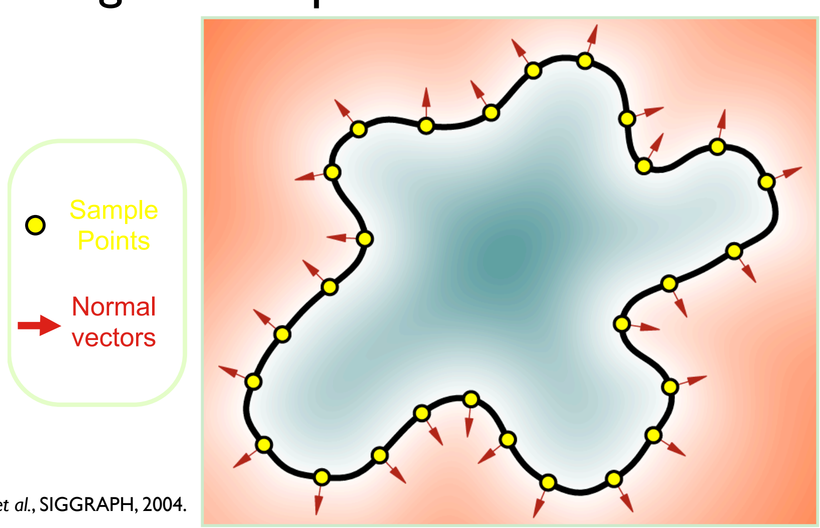


Note shrinkage

From Fleichman, Thesis, 2003.

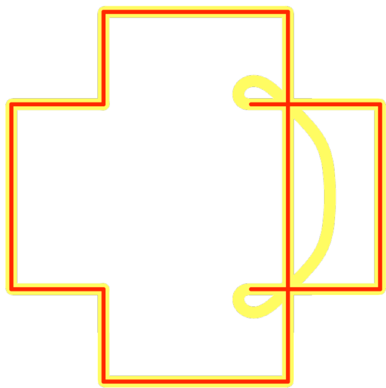
Implicit Moving Lest-Squares

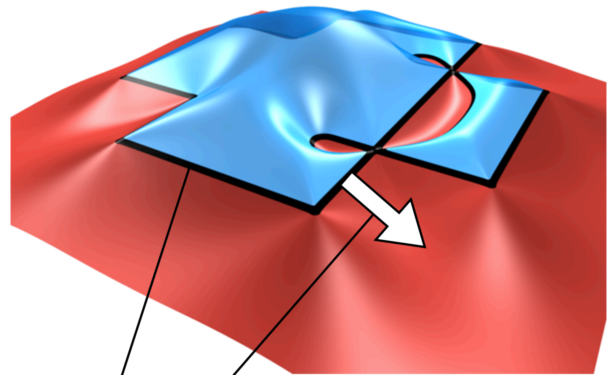
- Define a scalar function that is zero passing through all the points



From Shen, *et al.*, SIGGRAPH, 2004.

Implicit Moving Lest-Squares





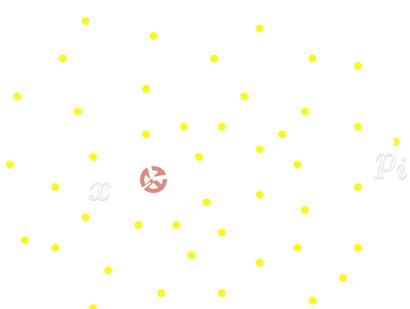
Function is zero on boundary
Decreases in outward direction

From Shen, *et al.*, SIGGRAPH, 2004.

Moving Least-Square Interpolation

Standard Least Square

$$\begin{bmatrix} b^\top(p_1) \\ \vdots \\ b^\top(p_N) \end{bmatrix} c = \begin{bmatrix} \phi_1 \\ \vdots \\ \phi_N \end{bmatrix}$$

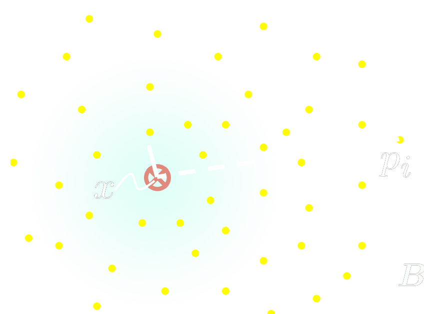


$$B^\top B\, c = B^\top \phi$$

Moving Least-Square Interpolation

Moving Least Square

$$\begin{bmatrix} w(x,p_1) & & \\ & \ddots & \\ & & w(x,p_N)\end{bmatrix}\begin{bmatrix} b^{\mathsf{T}}(p_1)\\ \vdots \\ b^{\mathsf{T}}(p_N)\end{bmatrix} c = \begin{bmatrix} w(x,p_1) & & \\ & \ddots & \\ & & w(x,p_N)\end{bmatrix}\begin{bmatrix}\phi_1\\ \vdots\\ \phi_N\end{bmatrix}$$

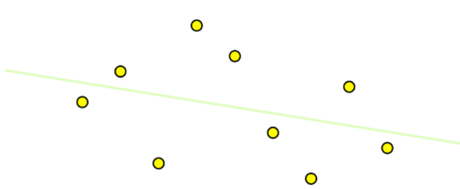


$$w(r) = \frac{1}{(r^2+\epsilon^2)}$$

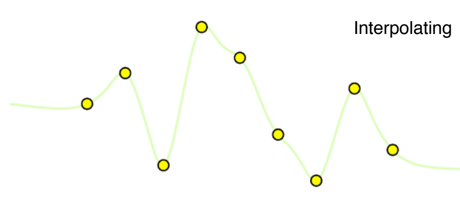
$$B^{\mathsf{T}}\ (W(x))^2\ B\ c(x) = B^{\mathsf{T}}\ (W(x))^2\ \phi$$

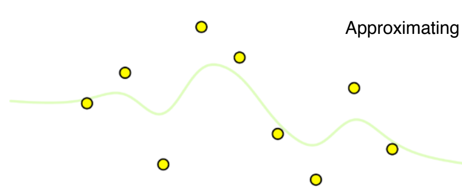
Moving Least-Square Interpolation

Least Square

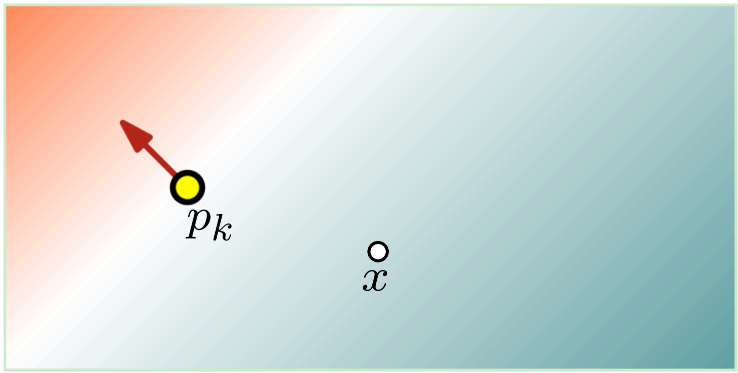


Moving Least Square



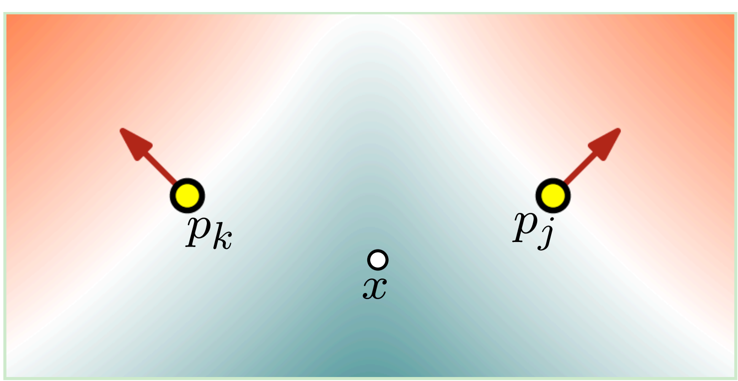


Interpolating Functions



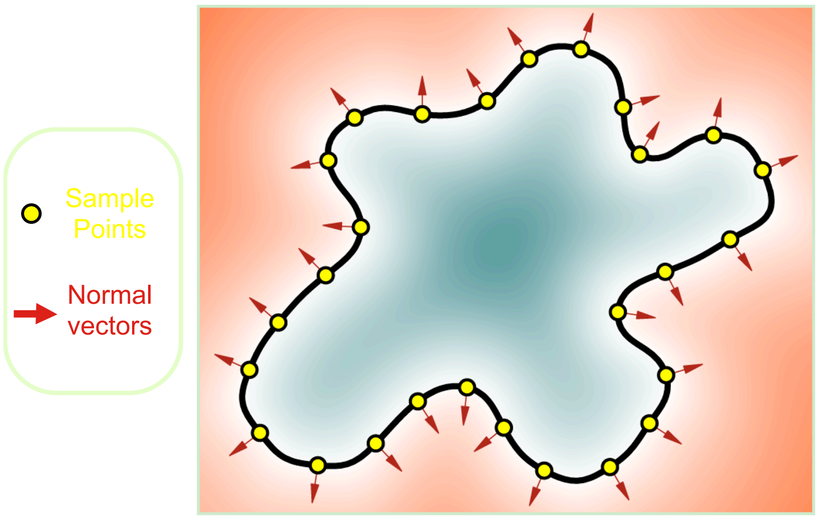
$$
\begin{aligned}
S_k(x) &= \phi_k + (x - p_k)^\top \hat{n}_k \\
&= \psi_{0k} + \psi_{xk}\, x + \psi_{yk}\, y + \psi_{zk}\, z
\end{aligned}
$$

Interpolating Functions



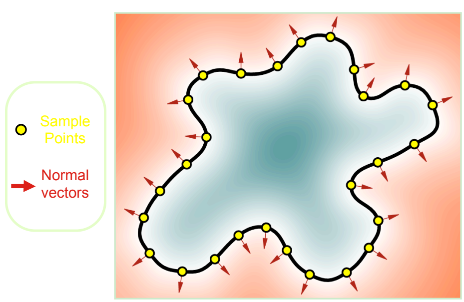
$$
\begin{bmatrix} w(x, p_1) \\ \vdots \\ w(x, p_i) \end{bmatrix} c_1 = \begin{bmatrix} w(x, p_1) & & \\ & \ddots & \\ & & w(x, p_N) \end{bmatrix} \begin{bmatrix} S_1(x) \\ \vdots \\ S_N(x) \end{bmatrix}
$$

Implicit Moving Lest-Squares

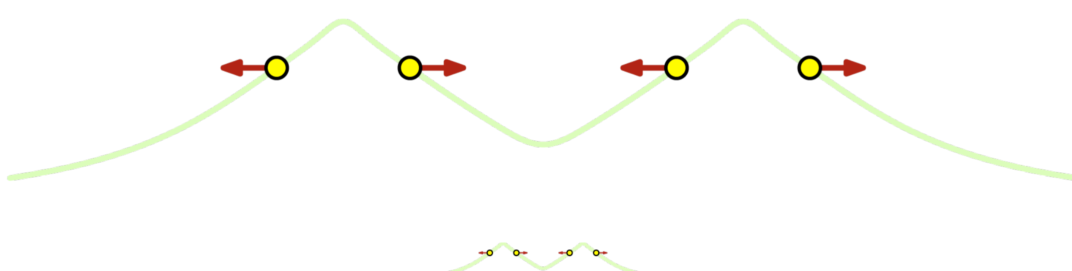


From Shen, *et al.*, SIGGRAPH, 2004.

Implicit Moving Lest-Squares



Proof of good behavior in Kolluri SODA 2005



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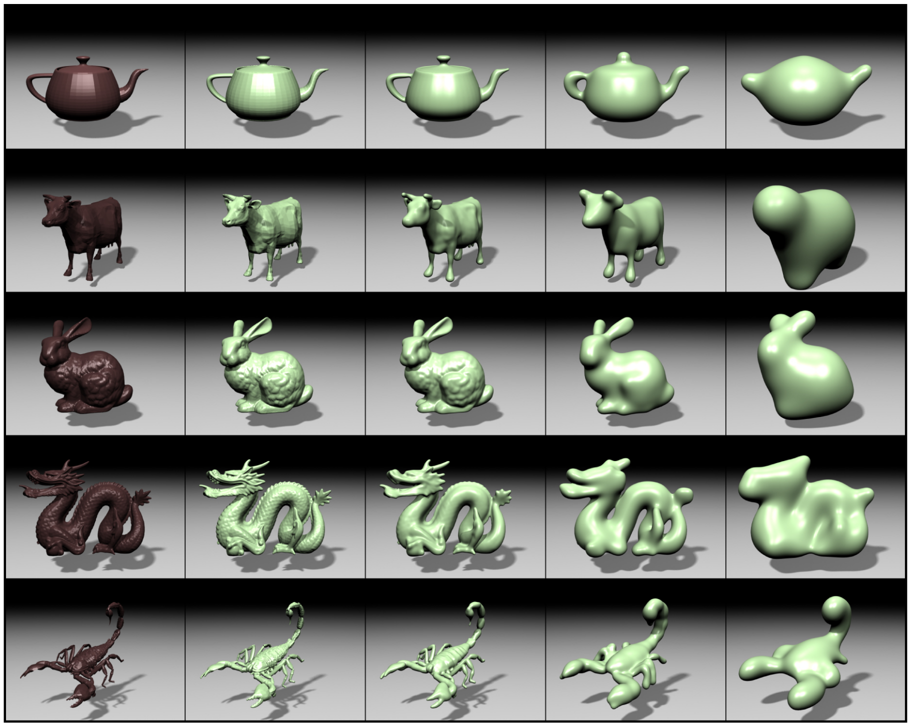
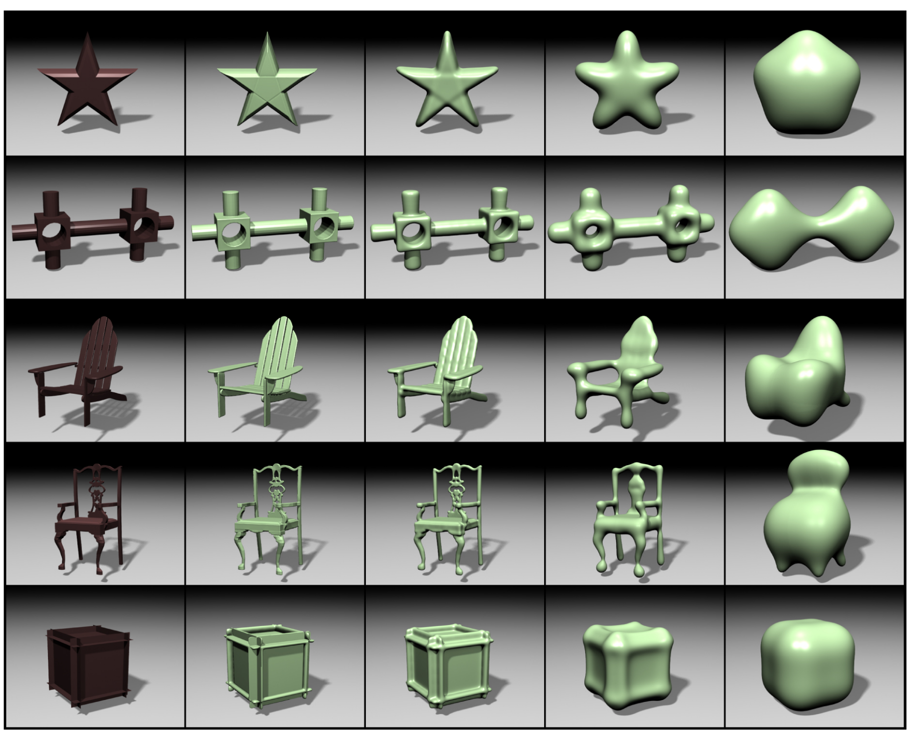


From Shen, *et al.*, SIGGRAPH, 2004.
(Actually based on polygon constraints... but same idea.)



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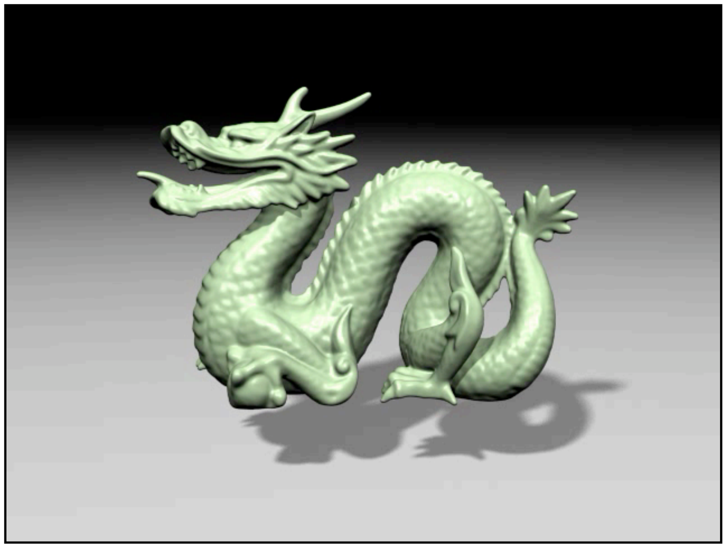

From Shen, *et al.*, SIGGRAPH, 2004.
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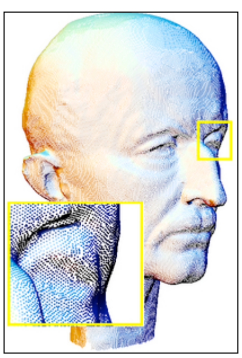
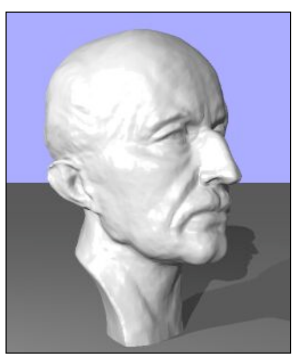
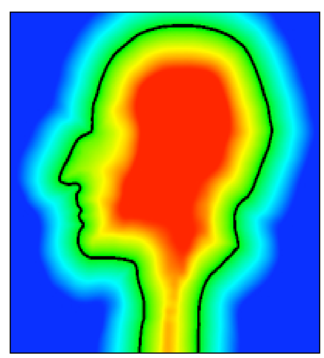
Partition of Unity Method







Ohtake, *et al.,* SIGGRAPH 2003

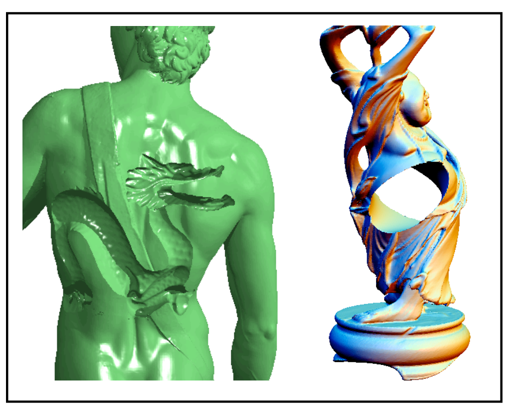
Partition of Unity is a special case of Moving Least-Squares

Partition of Unity Method

Ohtake, *et al.,* SIGGRAPH 2003

Editing Operations

- Implicit function can be
  - Combined w/ booleans
  - Warped
  - Offset
  - Composed
  - And more...



Ohtake, *et al.,* SIGGRAPH 2003

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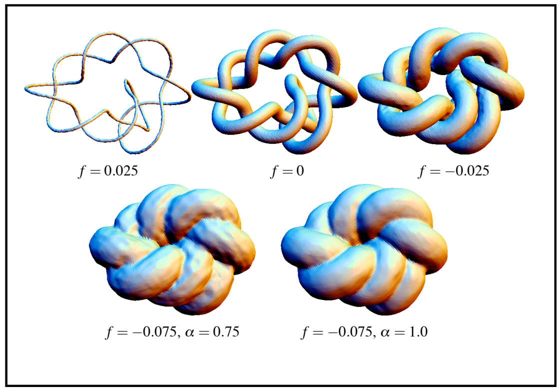

Ohtake, *et al.*, SIGGRAPH 2003

Editing Operations

- Implicit function can be
  - Combined w/ booleans
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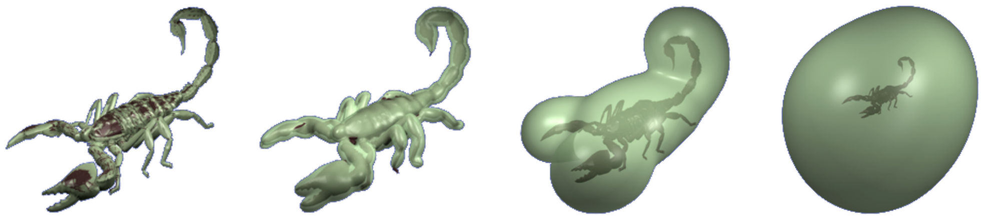
  - Offset
  - Composed
  - And more...

Ohtake, *et al.*, SIGGRAPH 2003

Smoothing

Simple Smoothing

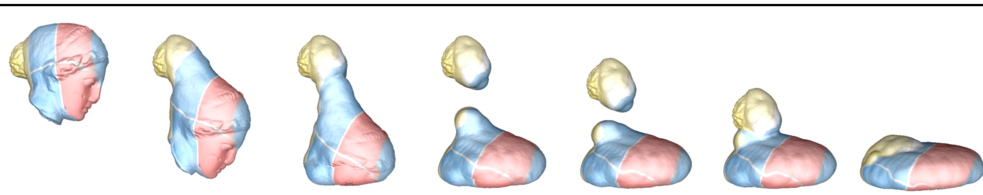


Adjustment Smoothing

From Shen, *et al.*, SIGGRAPH, 2004.

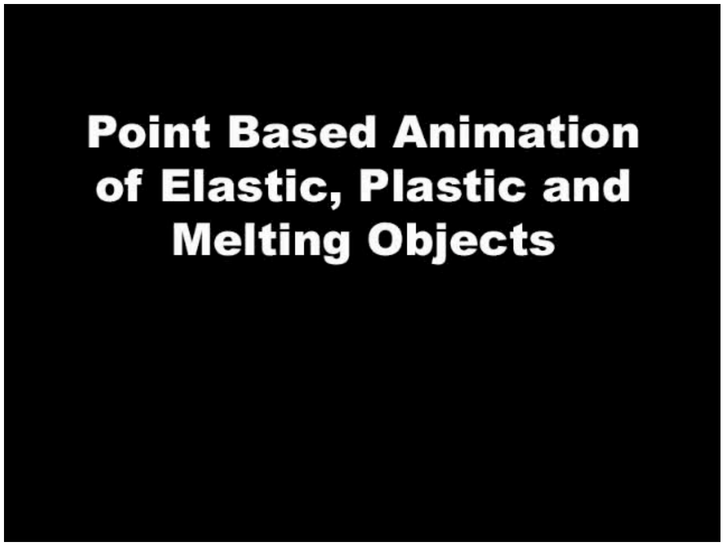
Point-Based Simulation

- MLS originated in mechanics literature
- Natural use in graphics for animation



From Mueller, *et al.*, SCA, 2004.

Point-Based Simulation



From Mueller, *et al.*, SCA, 2004.

Suggested Reading

- “QSplat: A Multiresolution Point Rendering System for Large Meshes” by Szymon Rusinkiewicz and Marc Levoy, SIGGRAPH 2000.
- “Multi-level Partition of Unity Implicits” by Yutaka Ohtake and colleagues, SIGGRAPH 2003.
- “Point Based Animation of Elastic, Plastic and Melting Objects” by Mueller and colleagues, SCA 2004.
- “Defining point-set surfaces” by Nina Amenta and Yong Joo Kil, SIGGRAPH 2004.
- "Interpolating and Approximating Implicit Surfaces from Polygon Soup" by Chen Shen, James O’Brien, and Jonathan Shewchuk, SIGGRAPH 2004.