CS-184: Computer Graphics

Lecture #8: Projection

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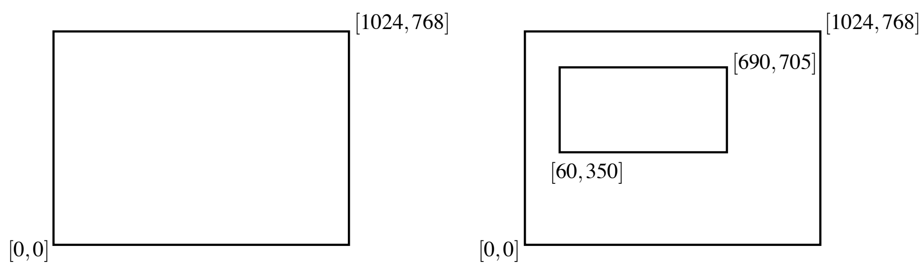
Today

- Windowing and Viewing Transformations
  - Windows and viewports
  - Orthographic projection
  - Perspective projection

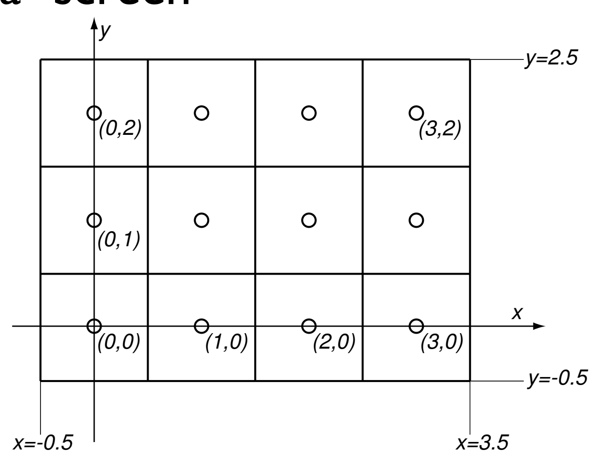
Screen Space

- Monitor has some number of pixels
  - *e.g.* 1024 x 768
- Some sub-region used for given program
  - You call it a window
  - Let's call it a viewport instead



Screen Space

- May not really be a "screen"
  - Image file
  - Printer
  - Other
- Little pixel details
- Sometimes odd
  - Upside down
  - Hexagonal



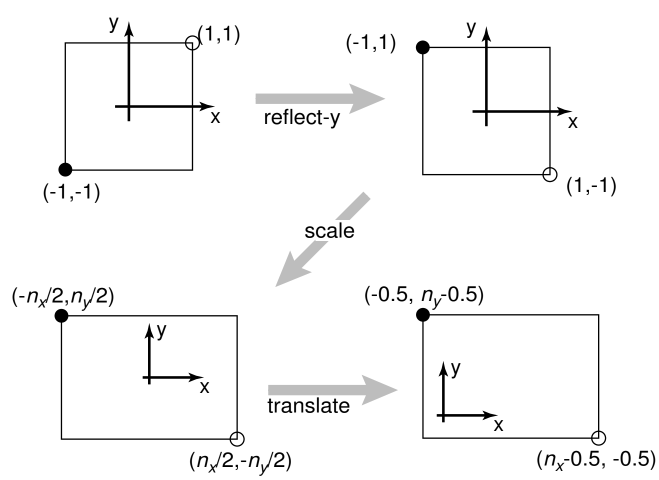
From Shirley textbook.

Screen Space

- Viewport is somewhere on screen
  - You probably don't care where
  - Window System likely manages this detail
  - Sometimes you care exactly where
- Viewport has a size in pixels
  - Sometimes you care (images, text, *etc.*)
  - Sometimes you don't (using high-level library)

Canonical View Space

- Canonical view region
  - 2D: [-1,-1] to [+1,+1]



From Shirley textbook.

Canonical View Space

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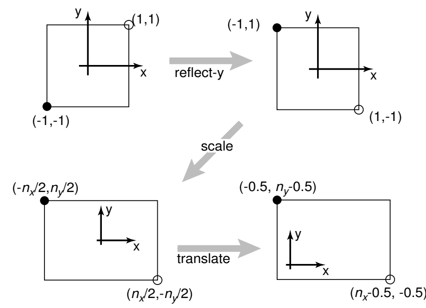


From Shirley textbook.

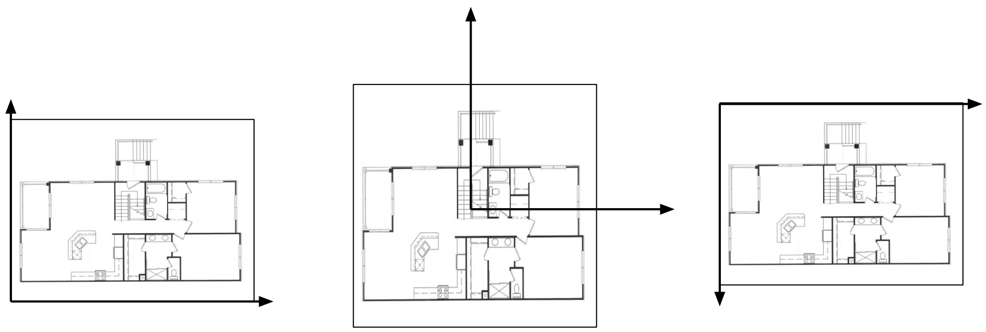
$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{n_x}{2} & 0 & \frac{n_x-1}{2} \\ 0 & -\frac{n_y}{2} & \frac{n_y-1}{2} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Canonical View Space

- Canonical view region
  - 2D: [-1,-1] to [+1,+1]
- Define arbitrary *window* and define objects
- Transform window to canonical region
- Do other things (we'll see clipping latter)
- Transform canonical to screen space
- Draw it.

From Shirley textbook.

Canonical View Space



World Coordinates (Meters) — Canonical — Screen Space (Pixels)

Note distortion issues...

Projection

- Process of going from 3D to 2D
- Studies throughout history (e.g. painters)
- Different types of projection
  - Linear
    - Orthographic
    - Perspective
  - Nonlinear

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Many special cases in books just one of these two...

Projection

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- Studies throughout history (e.g. painters)
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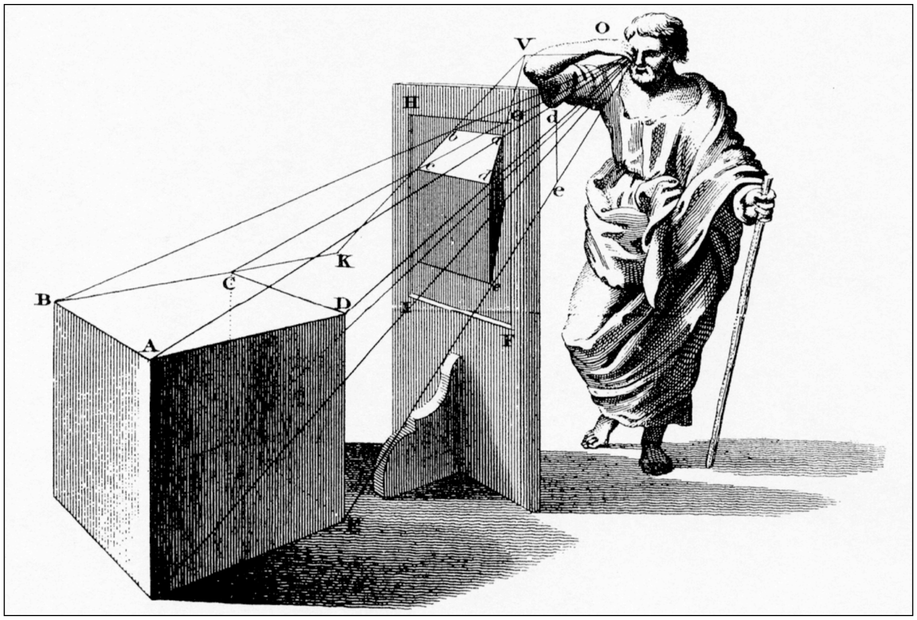
    - Orthographic
    - Perspective
  - Nonlinear

Many special cases in books just one of these two...

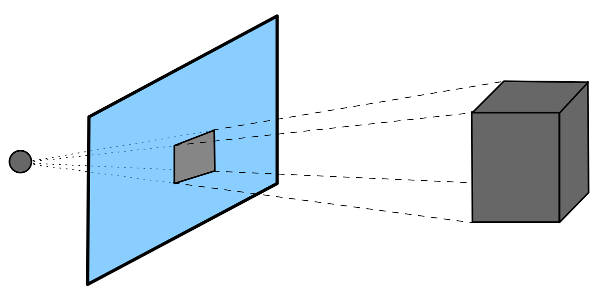
Orthographic is special case of perspective...

Perspective Projections



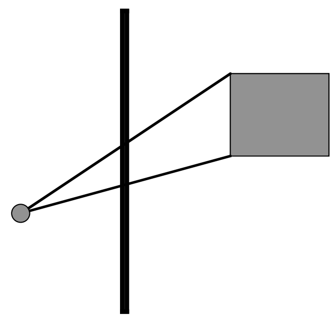
Linear Projection

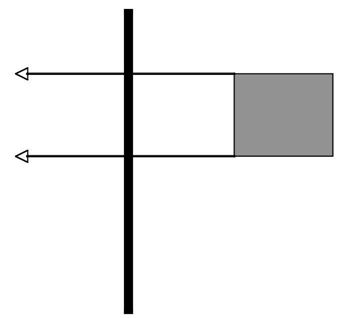
- Projection onto a planar surface
- Projection directions either
  - Converge to a point
  - Are parallel (converge at infinity)



Linear Projection

- A 2D view

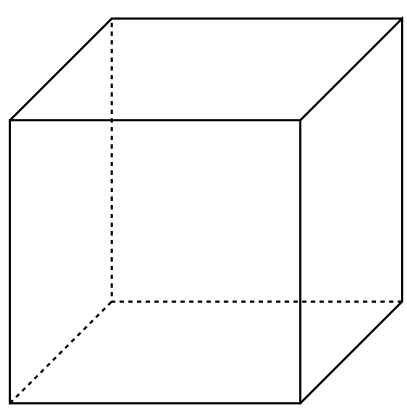




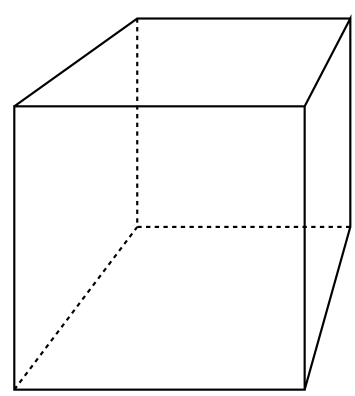
Perspective

Orthographic

Linear Projection

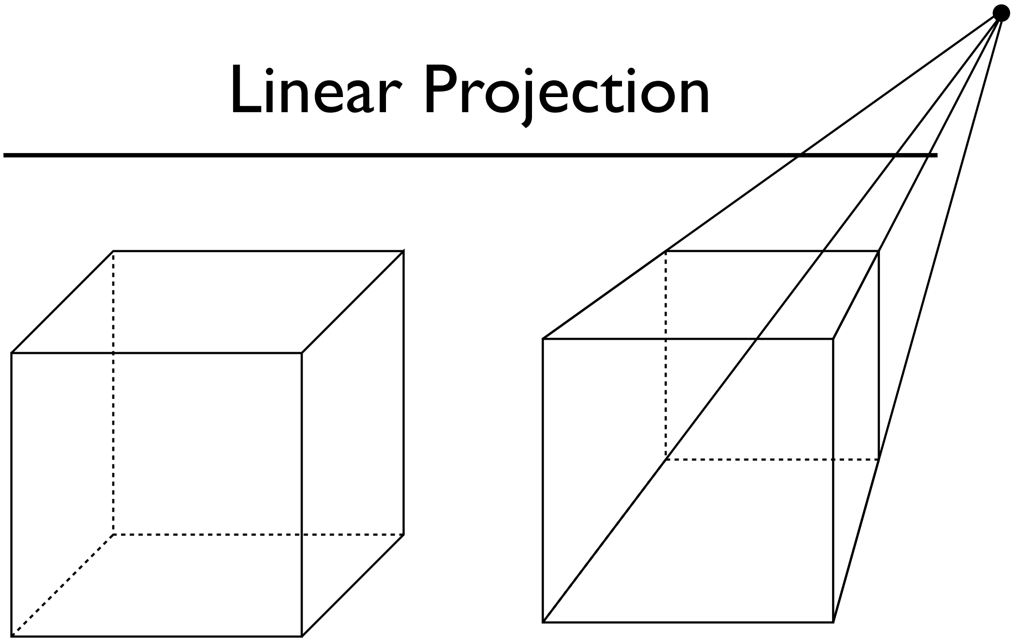


Orthographic



Perspective

Linear Projection



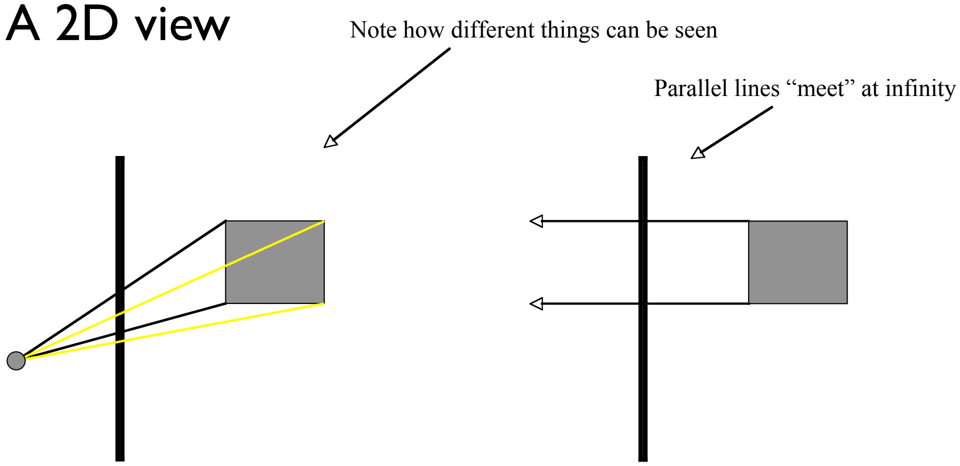
Orthographic

Perspective

15

Linear Projection

- A 2D view



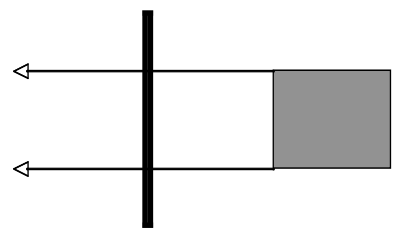
Perspective

Orthographic

16

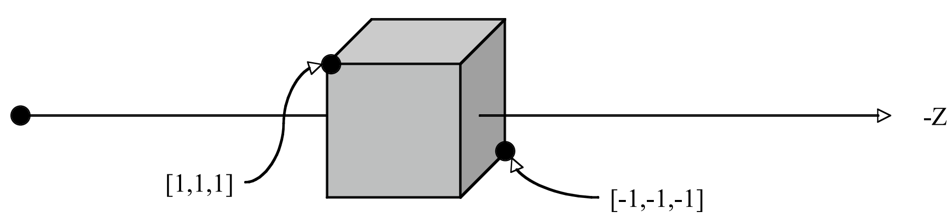
Orthographic Projection

- No foreshortening
- Parallel lines stay parallel
- Poor depth cues



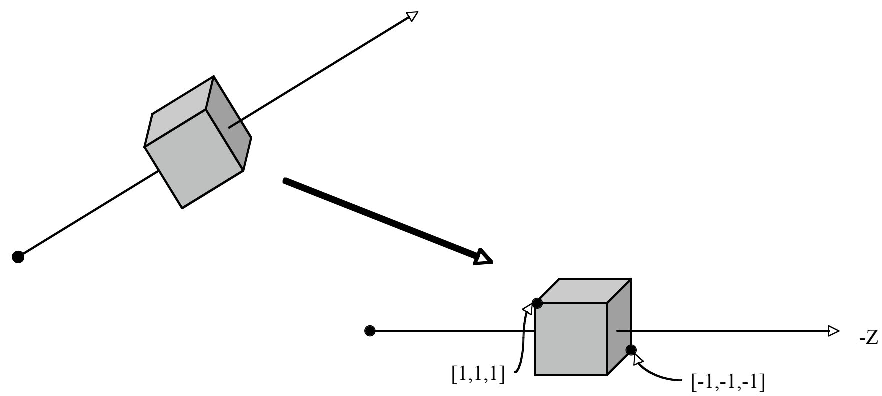
Canonical View Space

- Canonical view region
  - 3D: [-1,-1,-1] to [+1,+1,+1]
- Assume looking down *-Z* axis
  - Recall that "Z is in your face"

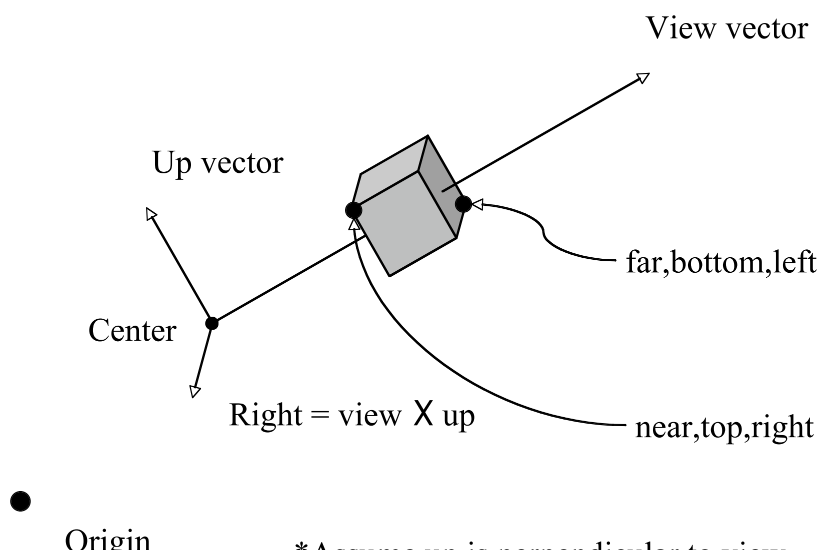


Orthographic Projection

- Convert arbitrary view volume to canonical



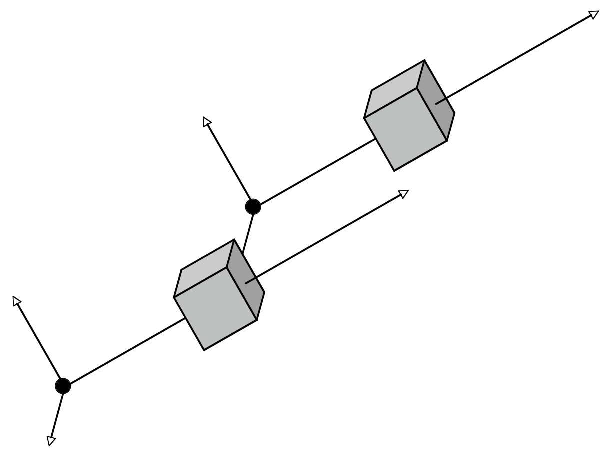
Orthographic Projection



*Assume up is perpendicular to view.

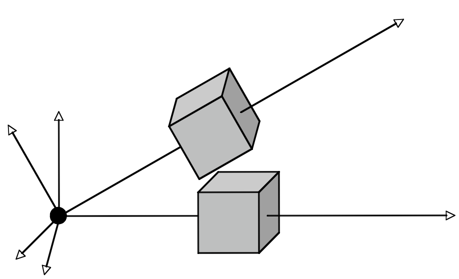
Orthographic Projection

- Step 1: translate center to origin



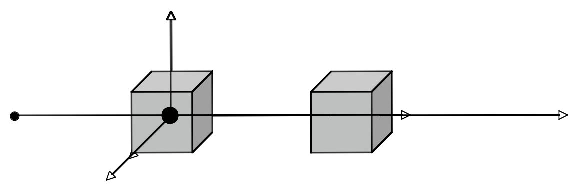
Orthographic Projection

- Step 1: translate center to origin
- Step 2: rotate *view* to **-Z** and *up* to **+Y**



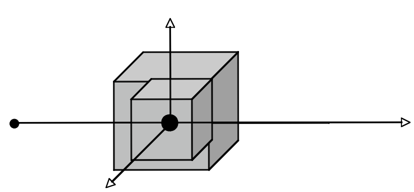
Orthographic Projection

- Step 1: translate center to origin
- Step 2: rotate *view* to **-Z** and *up* to **+Y**
- Step 3: center view volume



Orthographic Projection

- Step 1: translate center to origin
- Step 2: rotate *view* to **-Z** and *up* to **+Y**
- Step 3: center view volume
- Step 4: scale to canonical size



Orthographic Projection

- Step 1: translate center to origin
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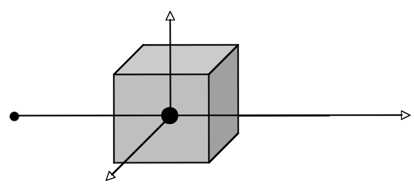
$$\mathbf{M} = \mathbf{S} \cdot \mathbf{T}_2 \cdot \mathbf{R} \cdot \mathbf{T}_1$$



Orthographic Projection

- Step 1: translate center to origin
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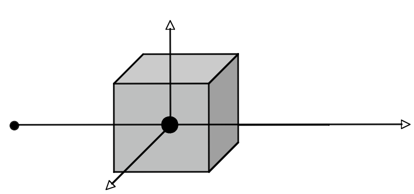
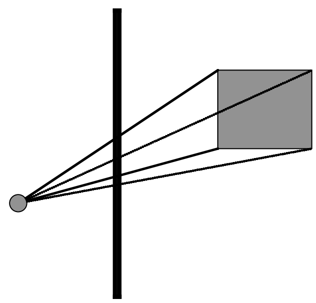
$$\mathbf{M} = \mathbf{S} \cdot \mathbf{T}_2 \cdot \mathbf{R} \cdot \mathbf{T}_1$$

$$\mathbf{M} = \mathbf{M}_o \cdot \mathbf{M}_v$$



Perspective Projection

- Foreshortening: further objects appear smaller
- Some parallel line stay parallel, most don't
- Lines still look like lines
- **Z** ordering preserved (where we care)



Perspective Projection

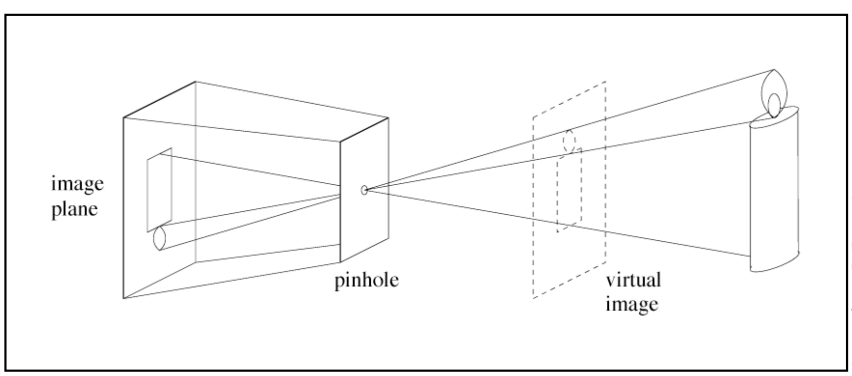


Image from D. Forsyth

Pinhole *a.k.a* center of projection

Perspective Projection

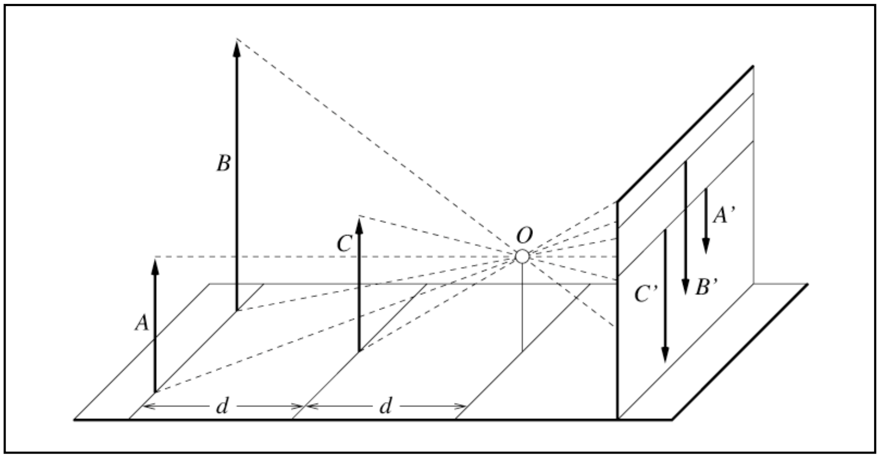
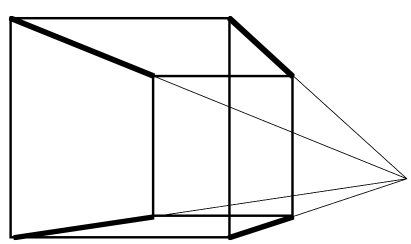


Image from D. Forsyth

Foreshortening: distant objects appear smaller

Perspective Projection

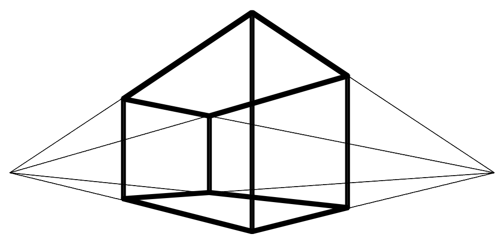
- Vanishing points
  - Depend on the scene
  - Not intrinsic to camera



"One point perspective"

Perspective Projection

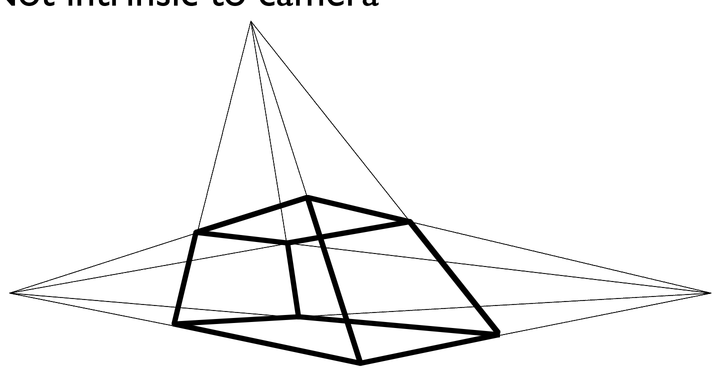
- Vanishing points
  - Depend on the scene
  - Nor intrinsic to camera



"Two point perspective"

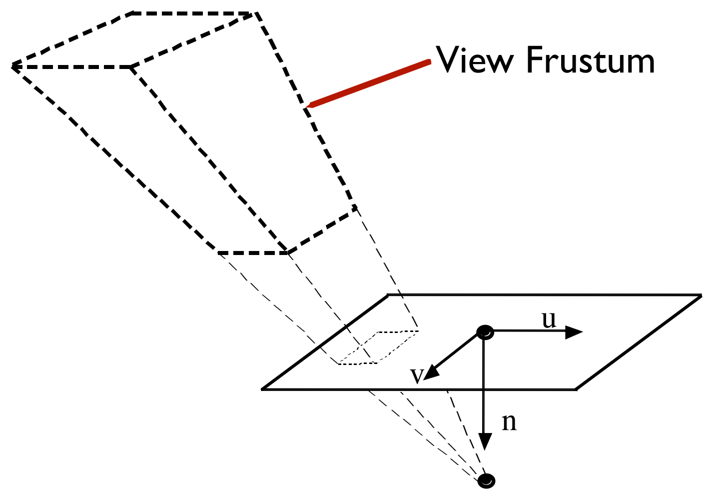
Perspective Projection

- Vanishing points
  - Depend on the scene
  - Not intrinsic to camera

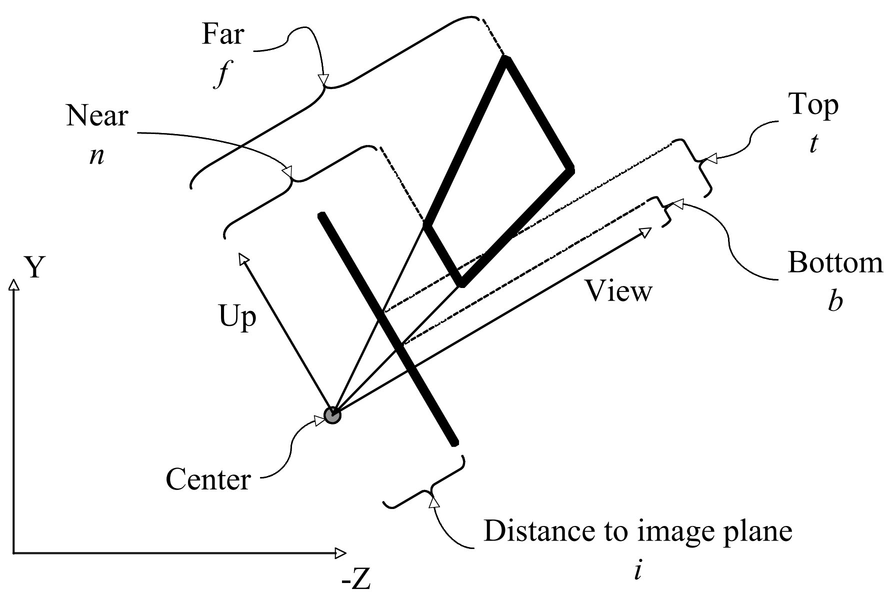


"Three point perspective"

Perspective Projection

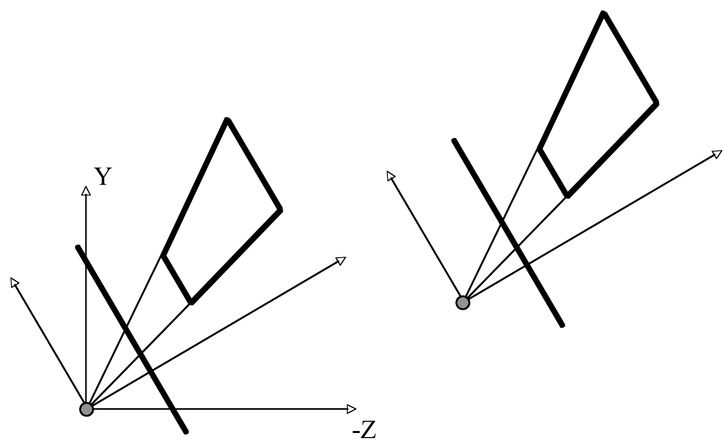


Perspective Projection



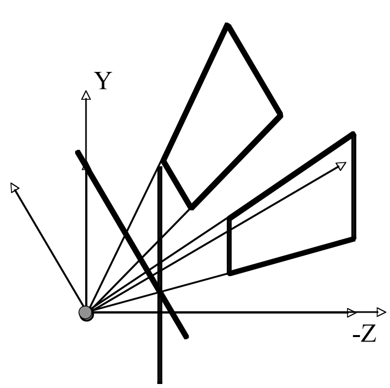
Perspective Projection

- Step 1: Translate *center* to origin



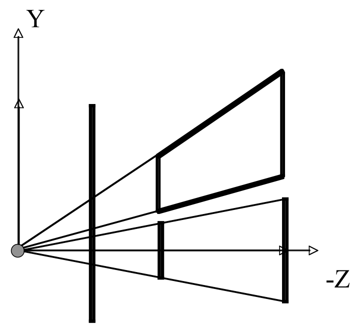
Perspective Projection

- Step 1: Translate *center* to origin
- Step 2: Rotate *view* to **-Z**, *up* to **+Y**



Perspective Projection

- Step 1: Translate *center* to origin
- Step 2: Rotate *view* to **-Z**, *up* to **+Y**
- Step 3: Shear center-line to **-Z** axis



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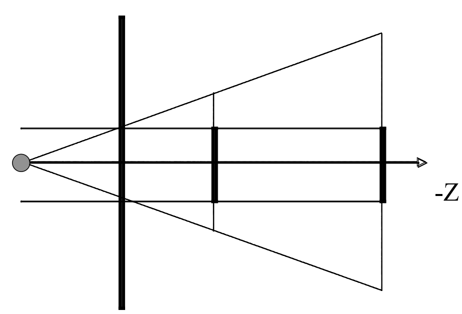

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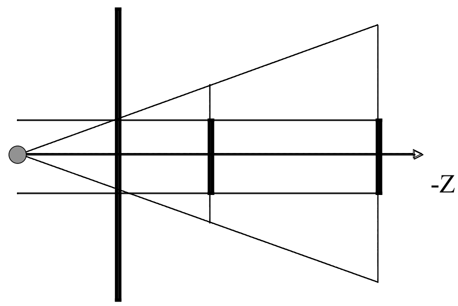
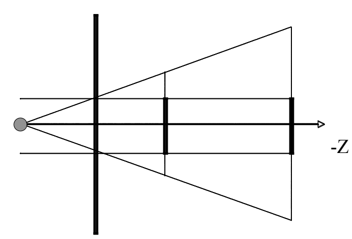
- Step 4: Perspective

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{i+f}{i} & f \\ 0 & 0 & \frac{-1}{i} & 0 \end{bmatrix}$$



Perspective Projection

- Step 4: Perspective
  - Points at $z=-i$ stay at $z=-i$
  - Points at $z=-f$ stay at $z=-f$
  - Points at $z=0$ goto $z=\pm\infty$
  - Points at $z=-\infty$ goto $z=-(i+f)$
  - x and y values divided by $-z/i$
  - Straight lines stay straight
  - Depth ordering preserved in $[-i,-f]$
  - Movement along lines distorted



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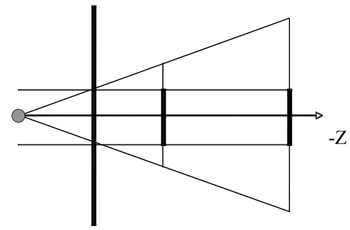
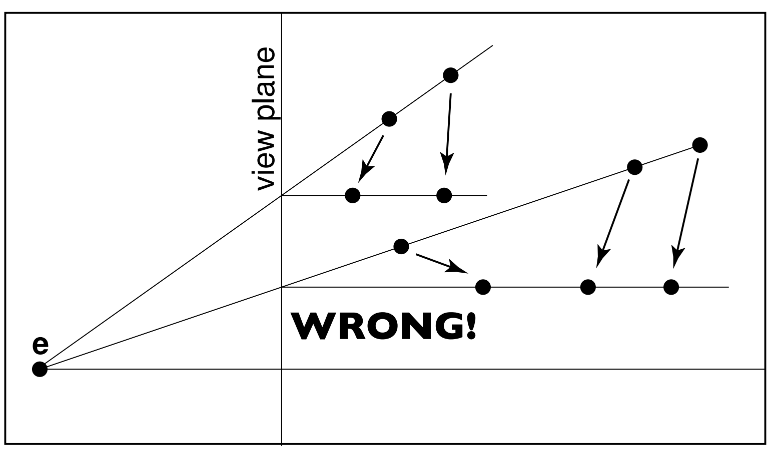


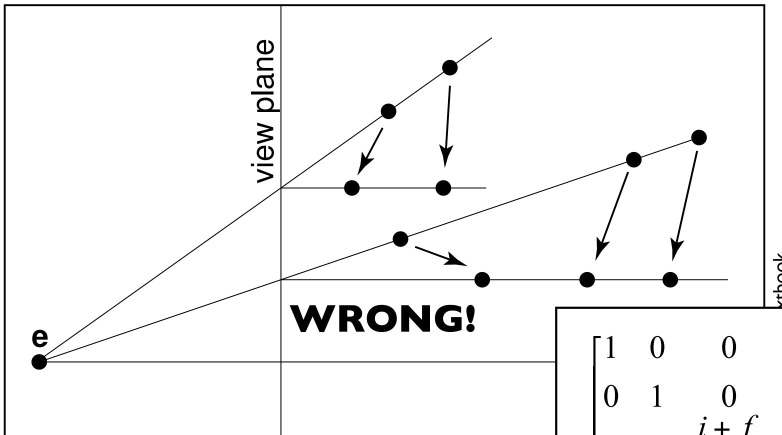
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{i+f}{i} & f \\ 0 & 0 & \frac{-1}{i} & 0 \end{bmatrix}$$

Perspective Projection



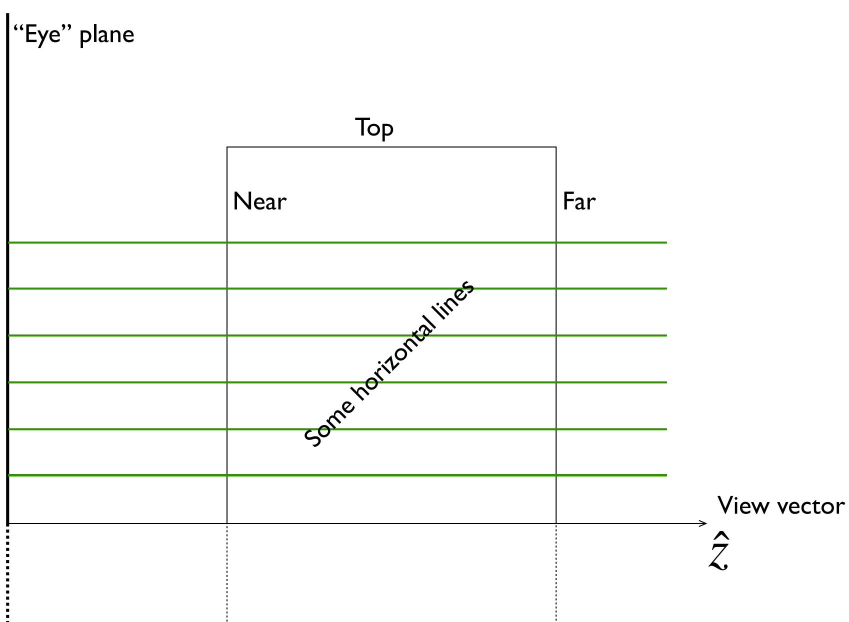
From Shirley textbook.

Perspective Projection

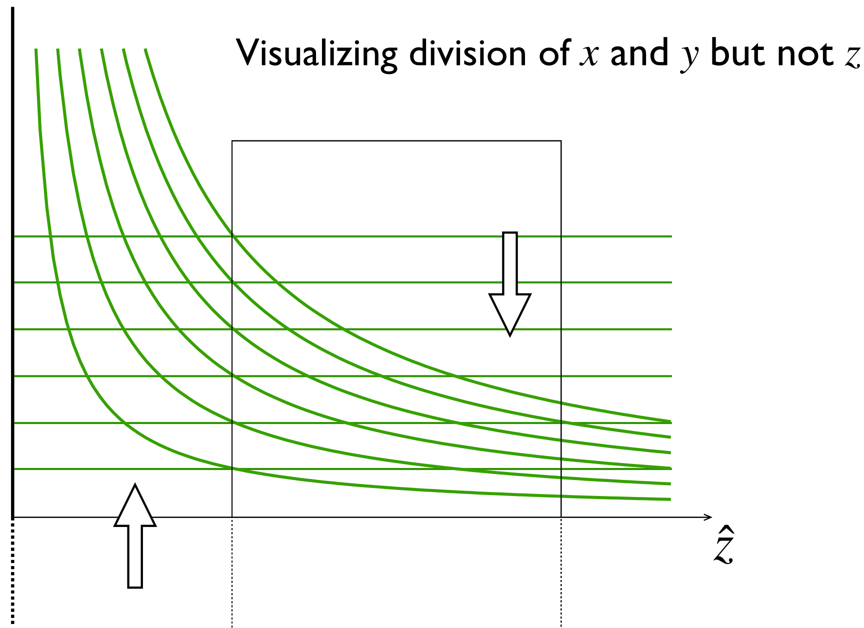


$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{i+f}{i} & f \\ 0 & 0 & \frac{-1}{i} & 0 \end{bmatrix}$$

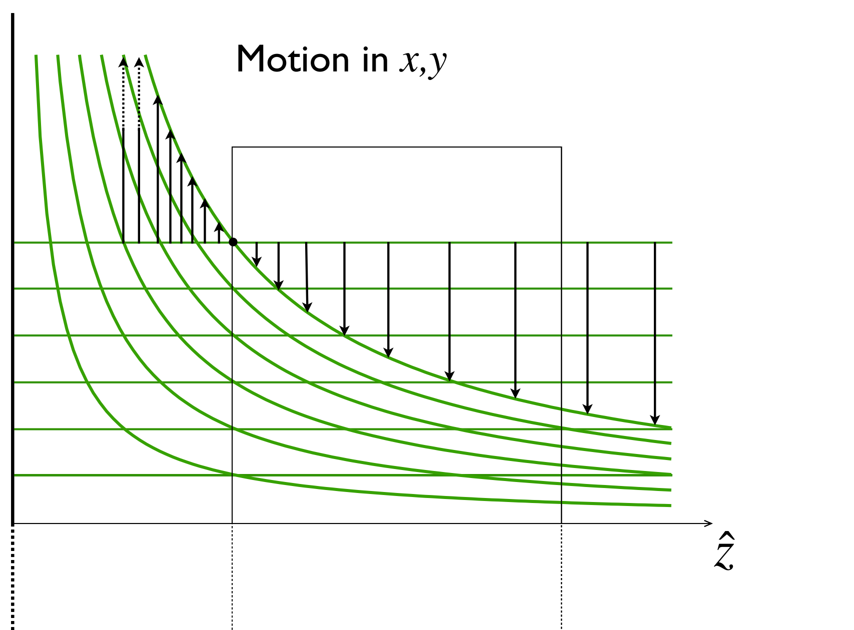
Perspective Projection



Perspective Projection

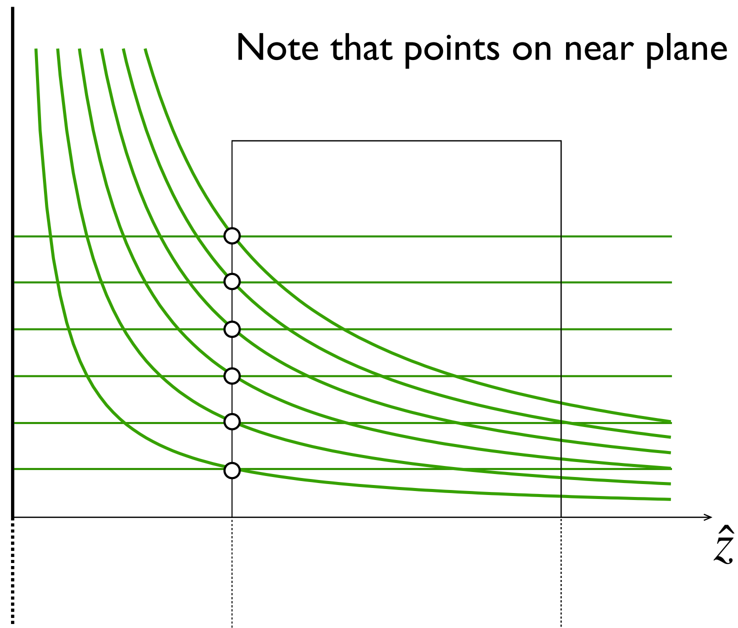


Perspective Projection



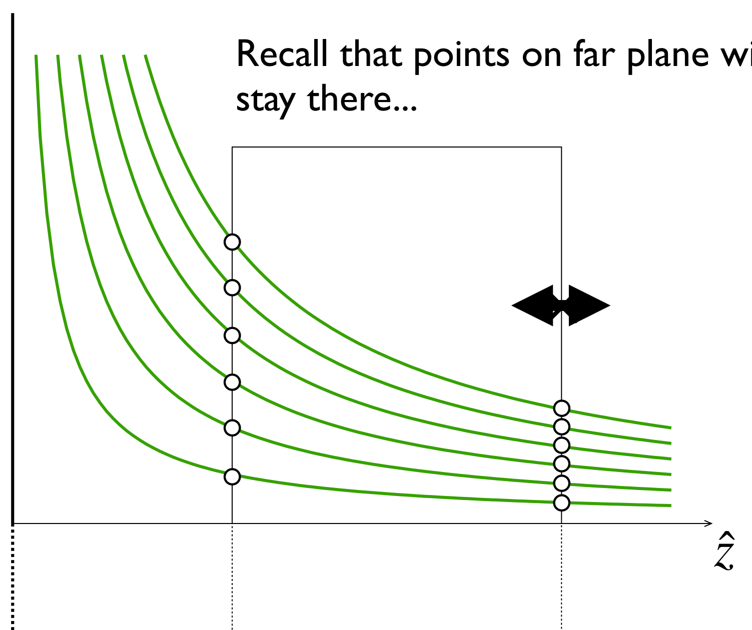
Perspective Projection

Note that points on near plane fixed



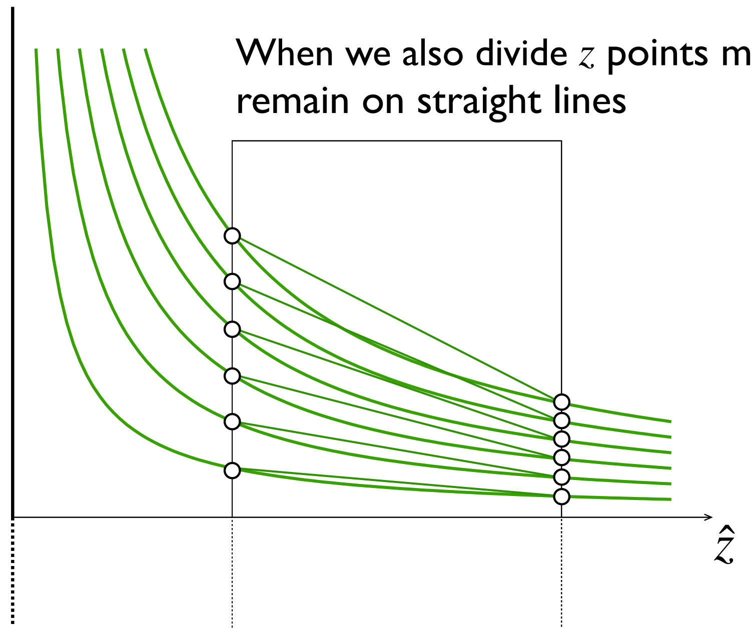
Perspective Projection

Recall that points on far plane will stay there...



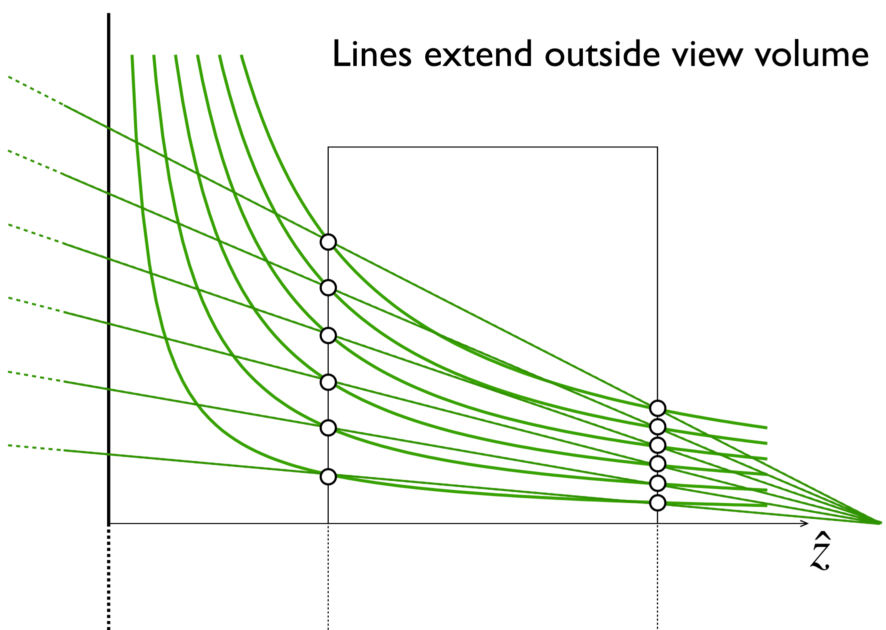
Perspective Projection

When we also divide z points must remain on straight lines

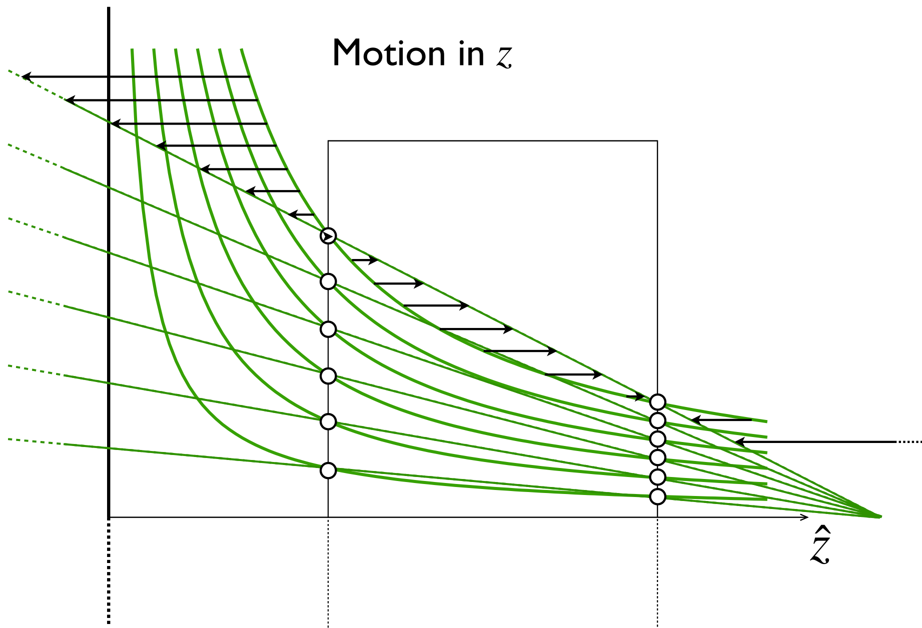


Perspective Projection

Lines extend outside view volume



Perspective Projection



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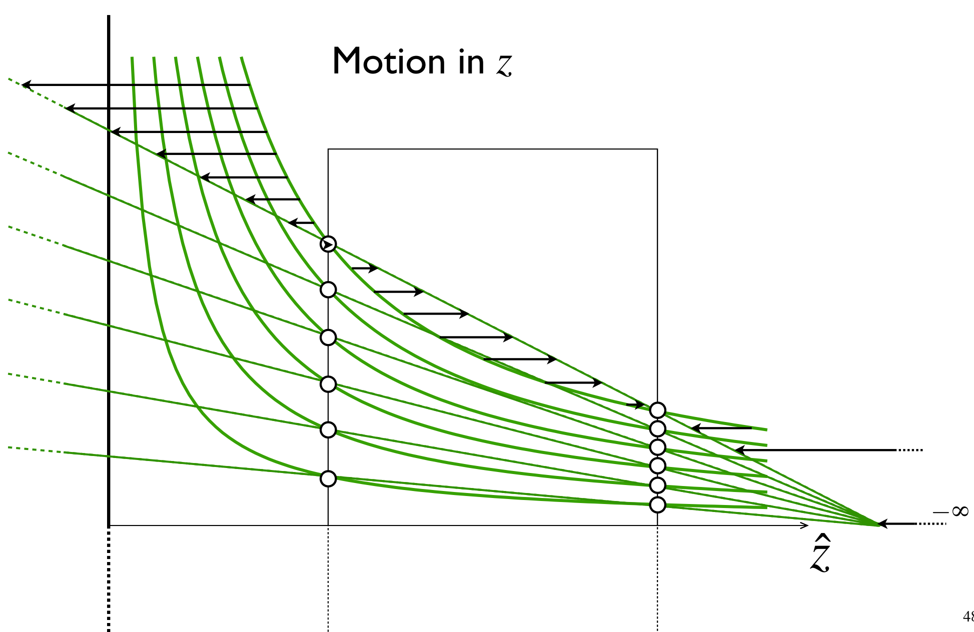
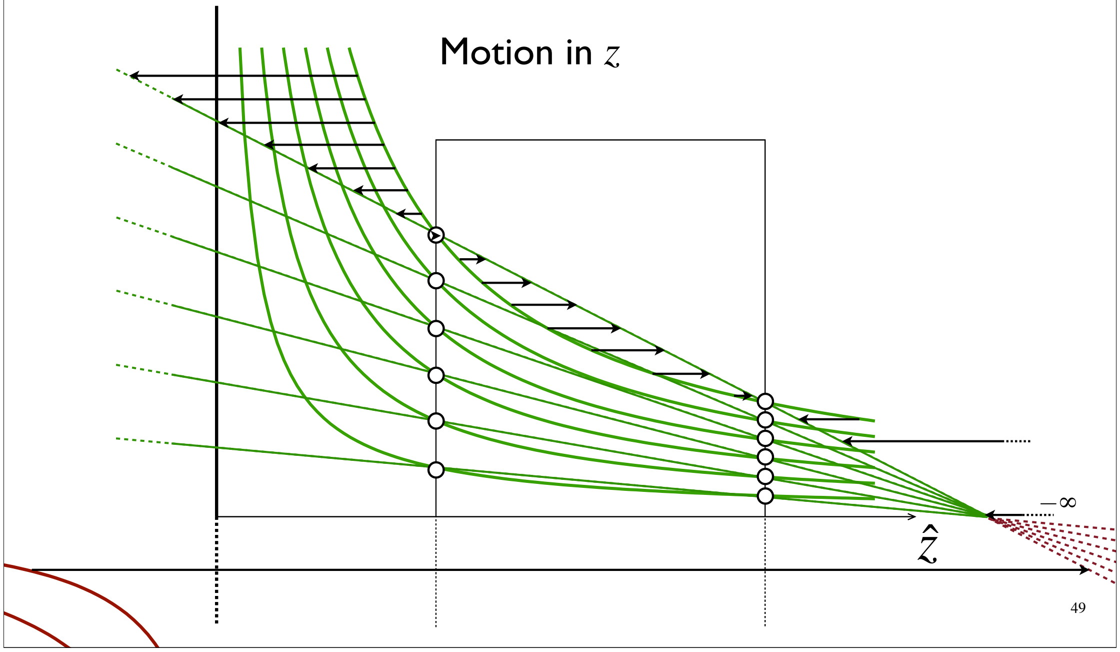
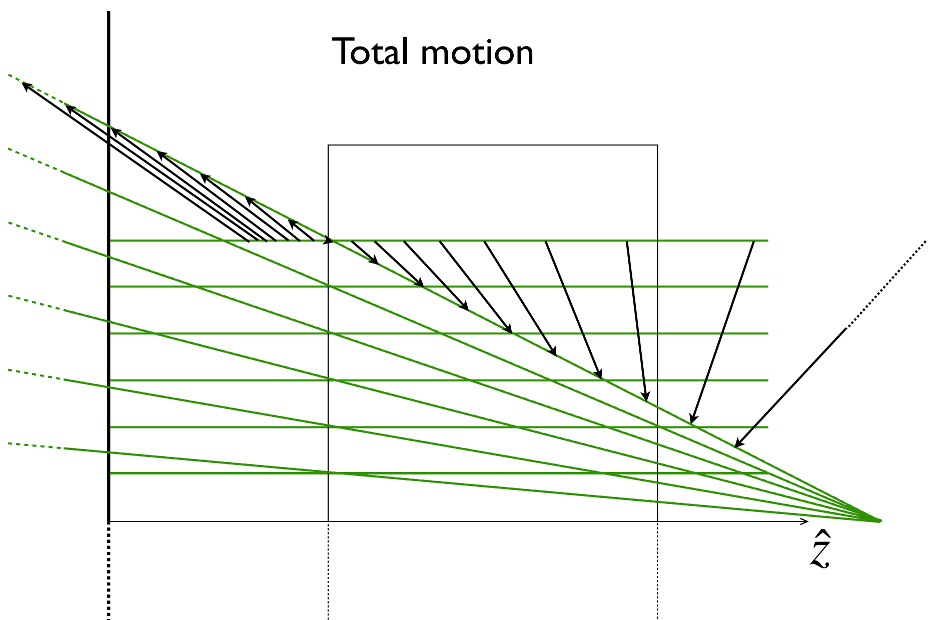


Perspective Projection

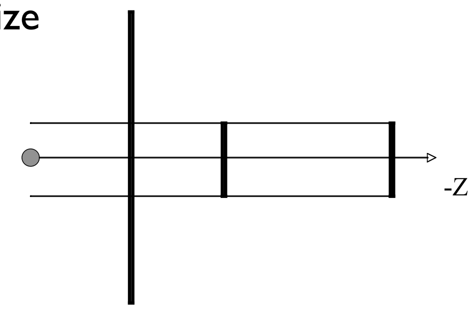


Perspective Projection



Perspective Projection

- Step 1: Translate *center* to orange
- Step 2: Rotate *view* to **-Z**, *up* to **+Y**
- Step 3: Shear center-line to **-Z** axis
- Step 4: Perspective
- Step 5: center view volume
- Step 6: scale to canonical size



Perspective Projection

- Step 1: Translate *center* to orange
- Step 2: Rotate *view* to **-Z**, *up* to **+Y**

$\} \mathbf{M}_v$

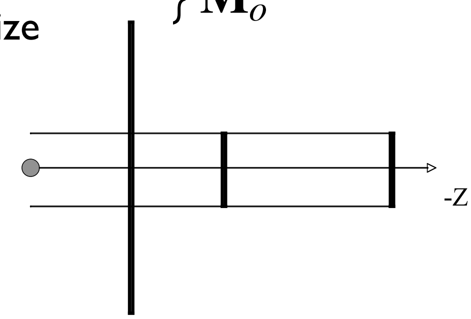
- Step 3: Shear center-line to **-Z** axis
- Step 4: Perspective

$\} \mathbf{M}_p$

- Step 5: center view volume
- Step 6: scale to canonical size

$\} \mathbf{M}_o$

$$\mathbf{M} = \mathbf{M}_o \cdot \mathbf{M}_p \cdot \mathbf{M}_v$$



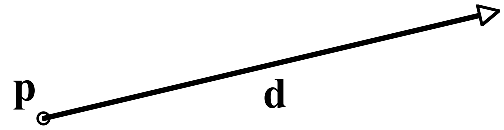
Perspective Projection

- There are other ways to set up the projection matrix
  - View plane at $z=0$ zero
  - Looking down another axis
  - *etc...*
- Functionally equivalent

Vanishing Points

- Consider a ray:

$$\mathbf{r}(t) = \mathbf{p} + t\,\mathbf{d}$$



Vanishing Points

- Ignore **Z** part of matrix
- **X** and **Y** will give location in image plane
- Assume image plane at z=-i

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ & \text{whatever} & & \\ 0 & 0 & -1 & 0 \end{bmatrix} \longrightarrow \begin{bmatrix} I_x \\ I_y \\ I_w \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Vanishing Points

$$\begin{bmatrix} I_x \\ I_y \\ I_w \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x \\ y \\ -z \end{bmatrix}$$

$$\begin{bmatrix} I_x / I_w \\ I_y / I_w \end{bmatrix} = \begin{bmatrix} -x/z \\ -y/z \end{bmatrix}$$

Vanishing Points

- Assume $d_z = -1$

$$\begin{bmatrix} I_x / I_w \\ I_y / I_w \end{bmatrix} = \begin{bmatrix} -x/z \\ -y/z \end{bmatrix} = \begin{bmatrix} \dfrac{p_x + td_x}{-p_z + t} \\ \dfrac{p_y + td_y}{-p_z + t} \end{bmatrix}$$

$$\lim_{t \to \pm\infty} = \begin{bmatrix} d_x \\ d_y \end{bmatrix}$$

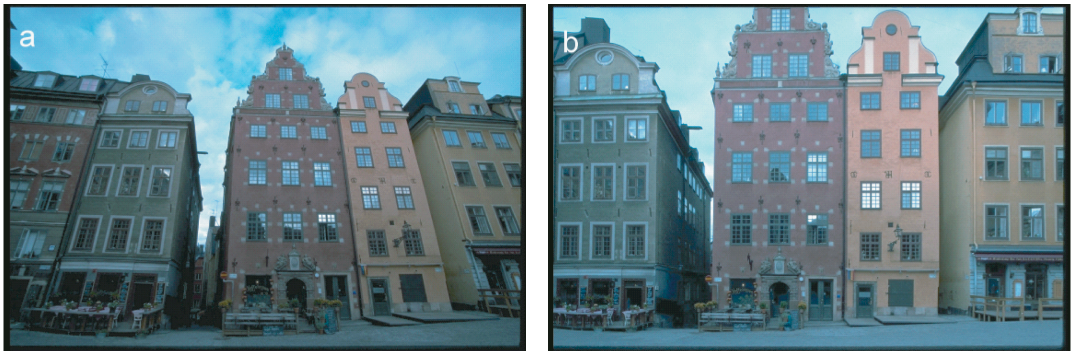
Vanishing Points

$$\lim_{t \to \pm\infty} = \begin{bmatrix} d_x \\ d_y \end{bmatrix}$$

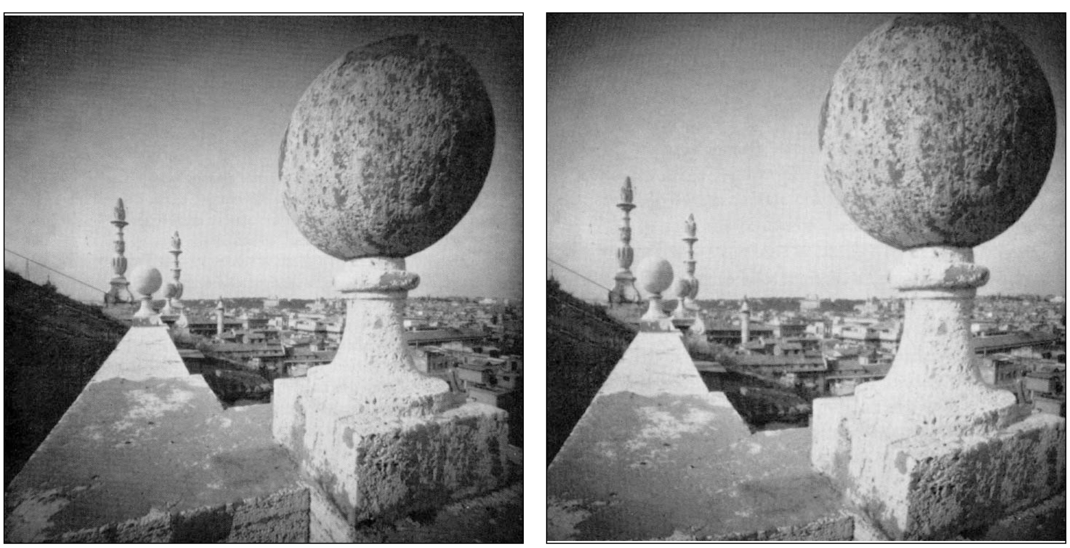
- All lines in direction $\mathbf{d}$ converge to same point in the image plane -- the vanishing point
- Every point in plane is a v.p. for some set of lines
- Lines parallel to image plane ($d_z = 0$) vanish at infinity

What's a horizon?

Perspective Tricks



Right Looks Wrong (Sometimes)



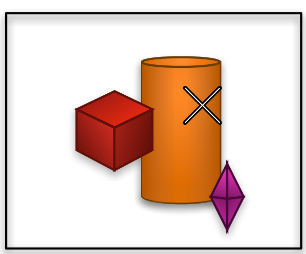
From *Correction of Geometric Perceptual Distortions in Pictures*, Zorin and Barr SIGGRAPH 1995

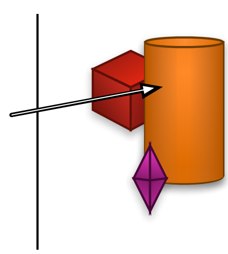
Strangeness

The Ambassadors
by Hans Holbein the Younger

Ray Picking

- Pick object by picking point on screen





- Compute ray from pixel coordinates.

Ray Picking

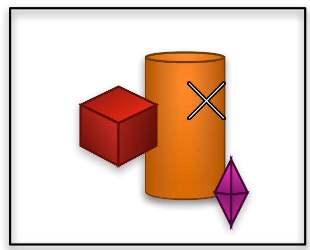
- Transform from World to Screen is:

$$\begin{bmatrix} I_x \\ I_y \\ I_z \\ I_w \end{bmatrix} = \mathbf{M} \begin{bmatrix} W_x \\ W_y \\ W_z \\ W_w \end{bmatrix}$$

- Inverse:

$$\begin{bmatrix} W_x \\ W_y \\ W_z \\ W_w \end{bmatrix} = \mathbf{M}^{-1} \begin{bmatrix} I_x \\ I_y \\ I_z \\ I_w \end{bmatrix}$$

- What $\mathbf{Z}$ value?



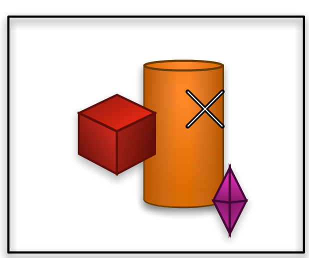
Ray Picking

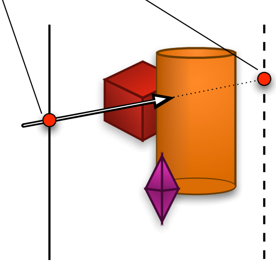
- Recall that:
  - Points at z=-i stay at z=-i
  - Points at z=-f stay at z=-f

Depends on screen details, YMMV
General idea should translate...

$$\mathbf{r}(t) = \mathbf{p} + t\,\mathbf{d}$$
$$\mathbf{r}(t) = \mathbf{a}_w + t(\mathbf{b}_w - \mathbf{a}_w)$$

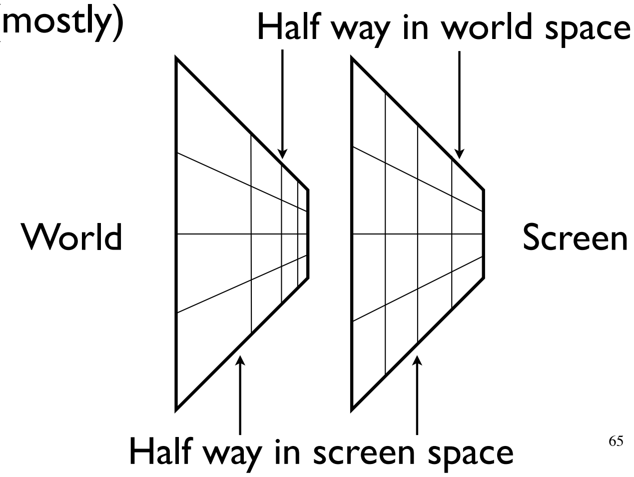
$$\mathbf{a}_s = [s_x, s_y, -i]$$
$$\mathbf{b}_s = [s_x, s_y, -f]$$



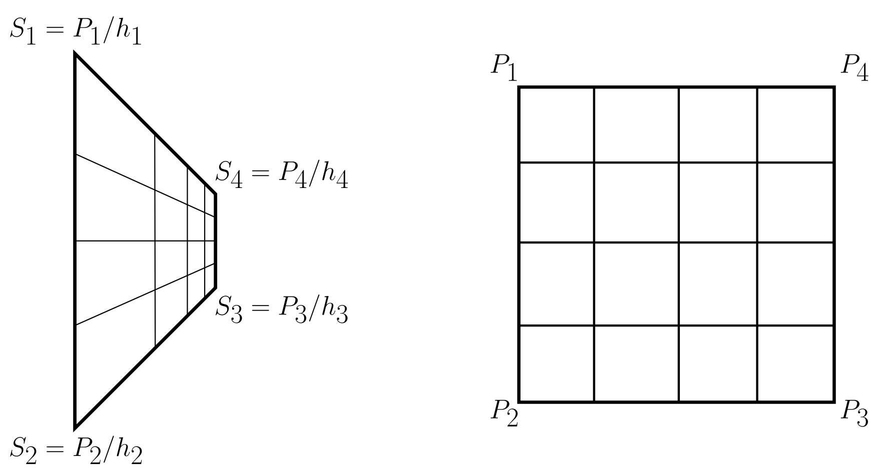


Depth Distortion

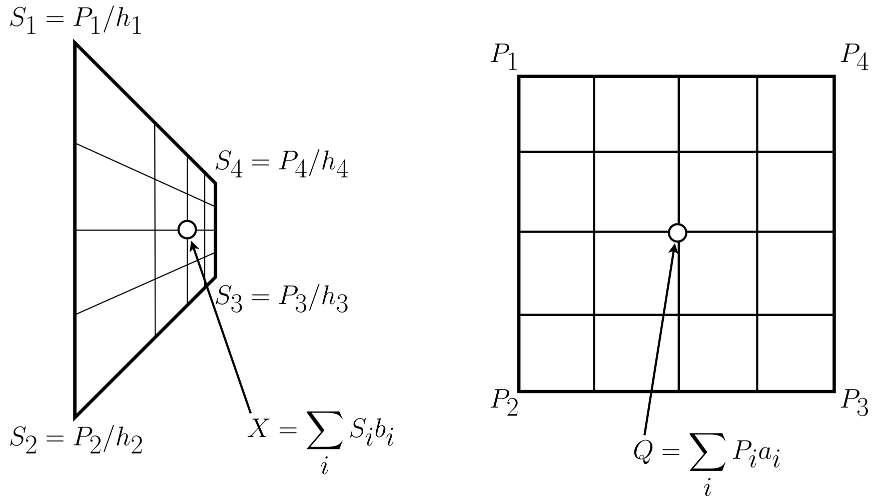
- Recall depth distortion from perspective
  - Interpolating in screen space different than in world
  - Ok, for shading (mostly)
  - Bad for texture



Depth Distortion

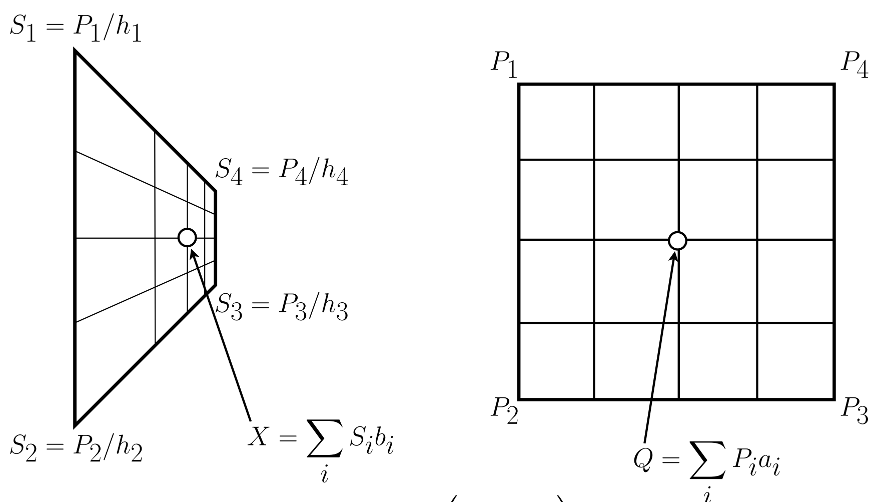


Depth Distortion



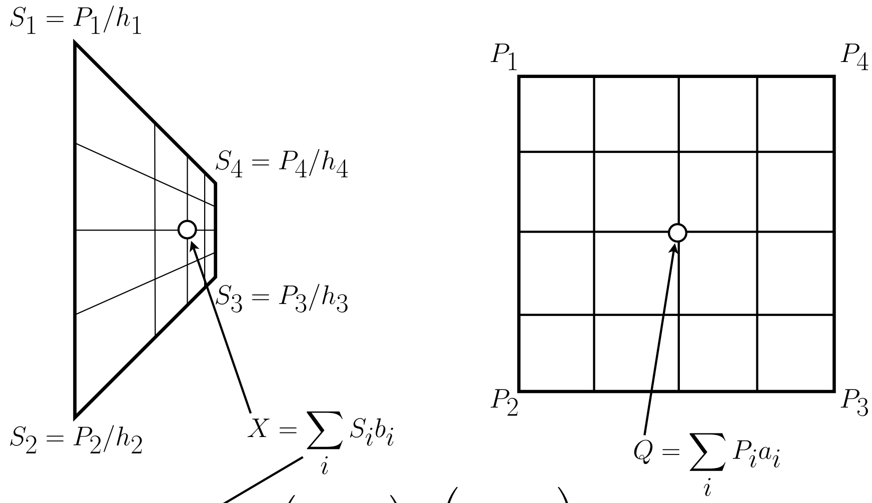
We know the S_i, P_i, and b_i, but not the a_i.

Depth Distortion



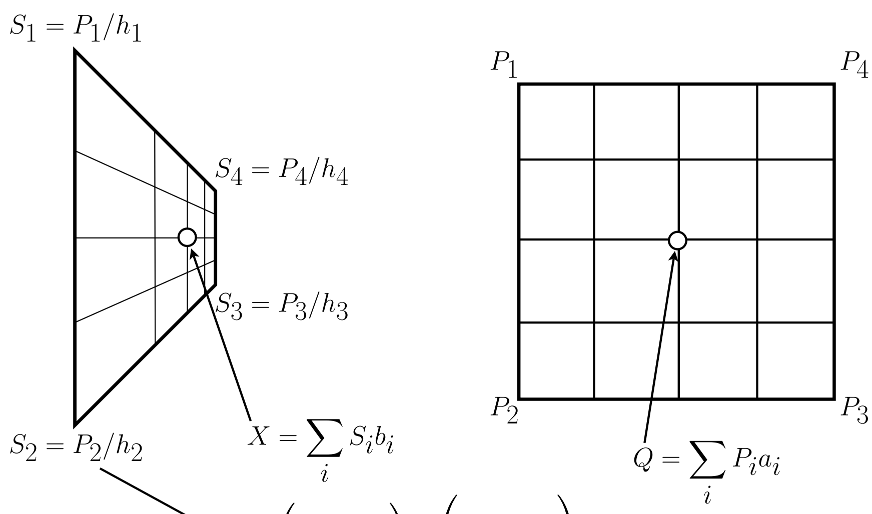
$$X = Q/h = \left(\sum_i P_i a_i \right) / \left(\sum_j h_j a_j \right)$$

Depth Distortion



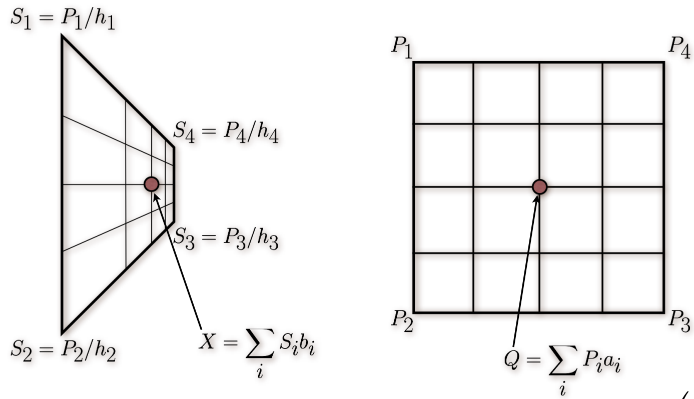
$$\sum_i S_i b_i = \left(\sum_i P_i a_i \right) / \left(\sum_j h_j a_j \right)$$

Depth Distortion



$$\sum_i P_i b_i / h_i = \left(\sum_i P_i a_i \right) / \left(\sum_j h_j a_j \right)$$

Depth Distortion

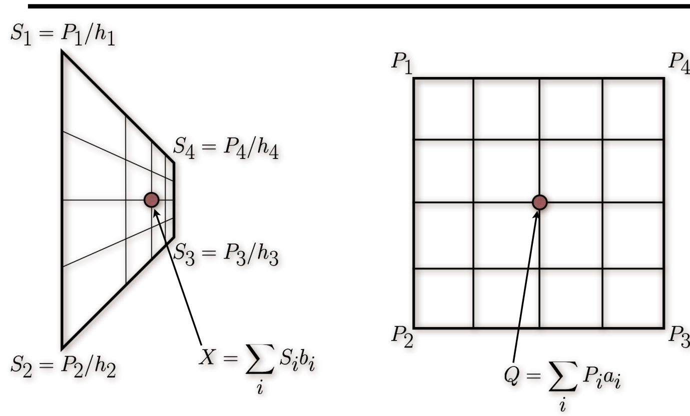


Independent of given vertex locations.

$$\sum_i P_i b_i / h_i = \left(\sum_i P_i a_i \right) / \left(\sum_j h_j a_j \right)$$

$$b_i / h_i = a_i / \left(\sum_j h_j a_j \right) \quad \forall i$$

Depth Distortion

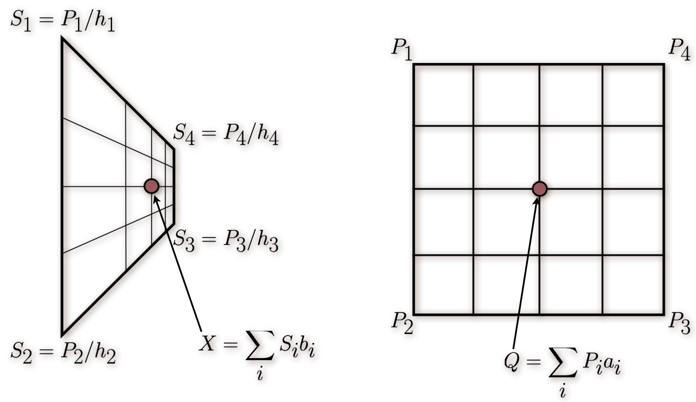


$$b_i / h_i = a_i / \left(\sum_j h_j a_j \right) \quad \forall i$$

Linear equations in the a_i.

$$\left(\sum_j h_j a_j \right) b_i / h_i - a_i = 0 \quad \forall i$$

Depth Distortion



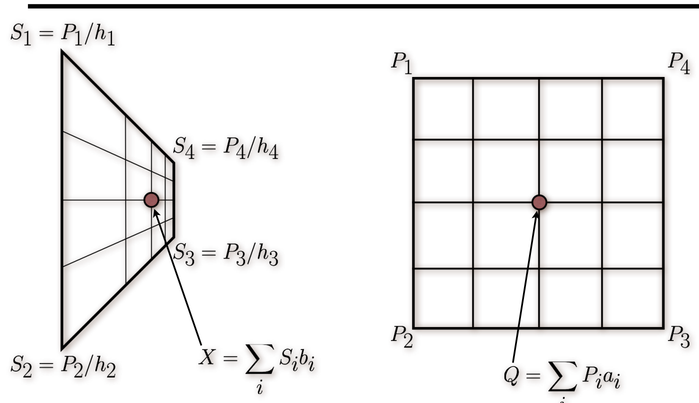
Linear equations in the a_i .

$$\left(\sum_j h_j a_j\right) b_i/h_i - a_i = 0 \quad \forall i$$

Not invertible so add some extra constraints.

$$\sum_i a_i = \sum_i b_i = 1$$

Depth Distortion



For a line: $a_1 = h_2 b_i/(b_1 h_2 + h_1 b_2)$

For a triangle: $a_1 = h_2 h_3 b_1/(h_2 h_3 b_1 + h_1 h_3 b_2 + h_1 h_2 b_3)$

Obvious Permutations for other coefficients.